



MANUAL

ABNEY LEVELS

***Uses and Applications
with
Appropriate Tables***

50¢

KEUFFEL & ESSER CO.



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Appropriate Tables*

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K+Σ

Abney Levels

The Abney Level is a device for measuring slope. It can be used to determine the heights of trees, poles, overhead wires and the like, to measure ground elevations, to reduce measurements made on a slope to their horizontal equivalents, and to run lines of levels.

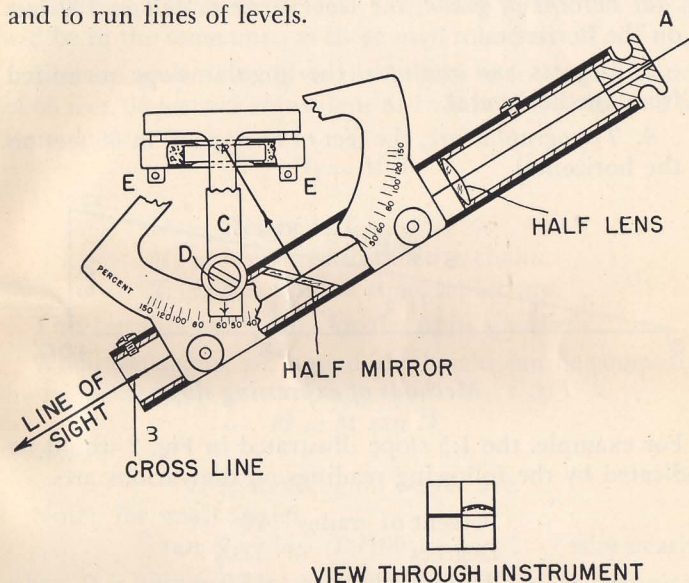


Fig. 1 Measuring a 57.8% slope

Fig. 1 illustrates the principles of the instrument. The observer, looking through the peep hole at *A* (Fig. 1) sees, at the left side of the tube, the cross line at *B* which he aligns with the object sighted. At the right side of the tube he sees the bubble vial reflected in a 45° mirror. By moving the index arm *C*, the bubble can be centered in the vial. The position of the index on the graduated circular arc then gives the slope.

When the bubble is centered, the knurled nut *D* is tightened to hold the index in place.

The eyepiece may be slid in and out to suit the eyesight of the observer.

Four types of arc graduations are used for indicating slope, as follows:

1. *Slope*, the ratio of rise or fall to distance.
2. *Percent of grade*, the feet of rise or fall in 100 feet on the horizontal.
3. *Degrees and minutes*, the angular slope measured from the horizontal.
4. *Topographic arc*, the feet of rise or fall in 66 feet on the horizontal.

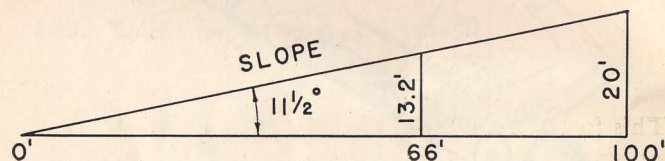


Fig. 2 Methods of expressing slope

For example, the 1:5 slope illustrated in Fig. 2 would be indicated by the following readings on the various arcs.

Percent of grade	20
Degrees	11½
Topographic arc	13.2

The following K&E instruments provide these scales in the combinations shown:

No. N5710	Degrees and slopes
No. N5710P	Degrees and percent of grade
No. 5713PD	Degrees and percent of grade
No. 5713TP	Percent of grade and topographic arc

It is clear from Fig. 2 that, at a distance of 100 feet, the reading on the *percent arc* will be the *actual height* of the

object in feet above the observer's eye. At other distances, the height can be computed by proportion. The formula is:

$$H = \frac{M}{100} P$$

Where

H = Height,

M = Measured distance,

P = Reading on percent arc.

The formula holds for meters, links, or any other unit of measurement as well as feet. The value computed for *H* will be in the same units as those used to express *M*.

The *topographic arc* gives the heights in *feet* at a distance of 66 feet. 66 feet are equivalent to 1 chain or 100 links. The formula is:

$$H_f = M_c T$$

Where

H_f = Height in feet,

M_c = Measured distance in chains,

T = Reading on topographic arc.

This formula holds only for the units specified.

When the *degree arc* is used, the height can be computed by trigonometry.

$$H = M \tan D$$

Where

D = Reading on degree arc.

Note: for small angles,

$$\tan D = 1\frac{3}{4} (D/100) \quad \text{very nearly,}$$

when *D* is expressed in degrees. The error in *H* computed by this formula is less than 1/1000 *M* up to 9°.

Downward slopes can be converted to differences in height in a similar manner.

When distances are measured on sloping ground, it is usually necessary to determine their horizontal equivalents. The slope of the ground is measured with the Abney level and corrections to the measured lengths are made accordingly. Simple methods of determining these corrections have been devised. They are described later.

There are a number of ways of determining the increase in length or the correction, depending on the arc used in measuring the slope. Methods of determining these corrections are given in the following paragraphs.

The percent arc. The percent of slope read on the percent arc can be readily converted into the correction required for a slope measurement of 100 units, by an approximate formula, thus:

$$C = \frac{P^2}{200} \quad (\text{approx.})$$

Where C = the correction

P = the percent of slope

Example: assume a slope of 12%.

$$C = \frac{(12)^2}{200} = 0.72$$

The following equivalents are then available:

Measurement on 12% slope	Horizontal Equivalents
100.72 feet	100 feet
100.72 meters	100 meters
100.00 feet	99.28 feet
100.00 meters	99.28 meters

The error in this formula is negligible for most slopes. As the slope increases, the error in the formula also increases, but even at a 23% slope the error is less than 1 part in 1000, or 0.1 foot in 100 feet.

If desired, Table A can be used to obtain these corrections. For example, in the table 12% falls between 11.4% and 12.3%. The correction then is 0.7 per hundred.

If the distance measured is other than 100 units, the correction is changed proportionally. For example, if 325 feet is measured on a 12% slope, the correction is $3.25 \times 0.72 = 2.34'$ and the horizontal equivalent is $325' - 2.34' = 322.66'$.

The degree arc. When the slope is measured in degrees, the correction can be obtained from Table A. If desired, the natural cosine table or an accurate slide rule can be used to reduce the measured values directly to their horizontal equivalents.

Example: 325 feet measured on a 10° slope.

From Table A, correction per 100 = 1.5,

$$3.25' \times 1.5 = 4.9',$$

$$325' - 4.9' = 320.1'.$$

By cosines,

$$\cos 10^\circ = 0.9848,$$

$$0.9848 \times 325' = 320.1'.$$

Measurement procedure. When the correction has been obtained by one of the methods described, the actual measurement on the slope may be made in one of two ways.

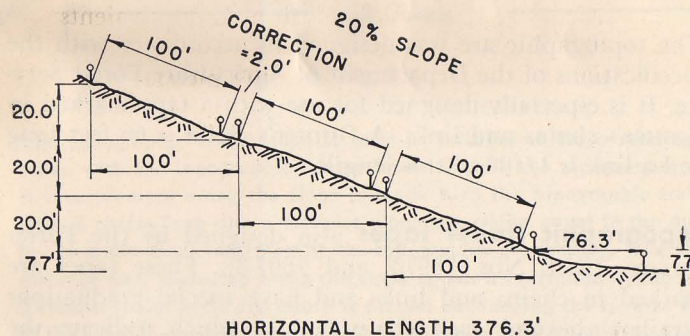


Fig. 6 Measurement correcting each tape length

Method 1. Applicable to slopes up to $10^\circ = 25\%$ = Top. arc 16.5 and all slopes with trailer tape. Each 100 feet (or 100 links) measured may be lengthened by the proper correction. Then the last fractional tape length should be measured on the horizontal to avoid corrections (Fig. 6).

TO MEASURE THE SLOPE OF THE GROUND

To measure the slope of the ground it is customary to measure the slope of a line parallel to the ground, either at eye height or at a fixed height above the ground, such as 5 feet. Three methods are suggested below.

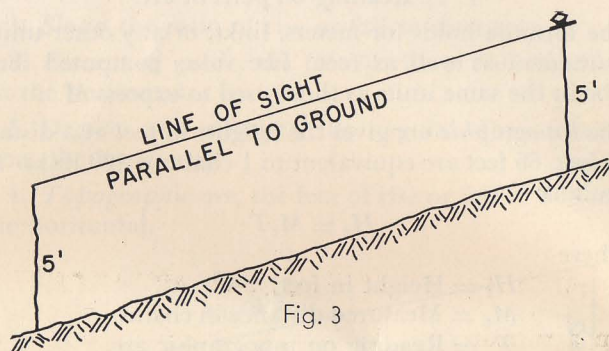


Fig. 3

Instrument supported. For accurate measurement, notch two sticks or cut two forked sticks so that they will hold the instrument exactly 5 feet above the ground. Rest the instrument on one stick and sight to the top of the other, held by an assistant at a point on the slope. (Fig. 3.) For maximum accuracy, the assistant should check by sighting back with a second instrument. The average of the two readings should be used.

Eye height method. Determine how high the observer's eye comes on the assistant when standing close to him. When he is in position on the slope, sight that point on his body. (Fig. 4.)

Estimation. Merely sight at a point estimated to be as far above the ground as the eye.

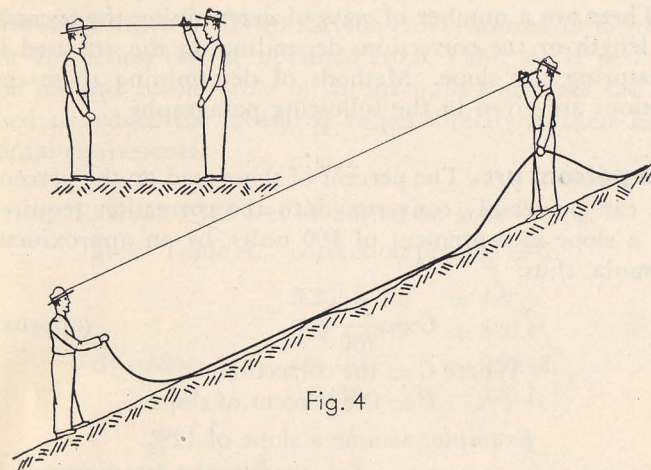


Fig. 4

TO MEASURE HORIZONTAL DISTANCES

When a measurement is made on a slope, it is common practice either to measure slightly more than the distance desired (Fig. 5a), so that the horizontal equivalent will be

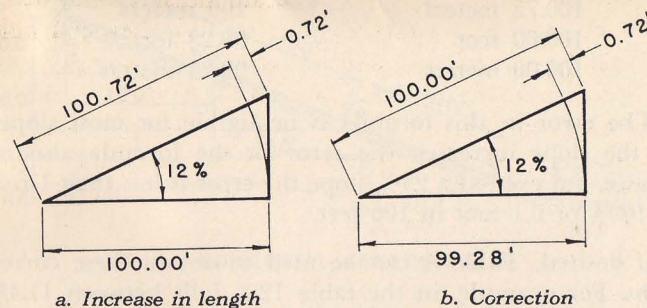


Fig. 5

correct, or to measure the slope distance exactly and then to subtract a correction (Fig. 5b). For all practical purposes the value of the increase in length for any slope is the same size as the correction to the length.

Method 2. Applicable to all uniform slopes. The entire length may be measured in one operation and corrected by computation (Fig. 7).

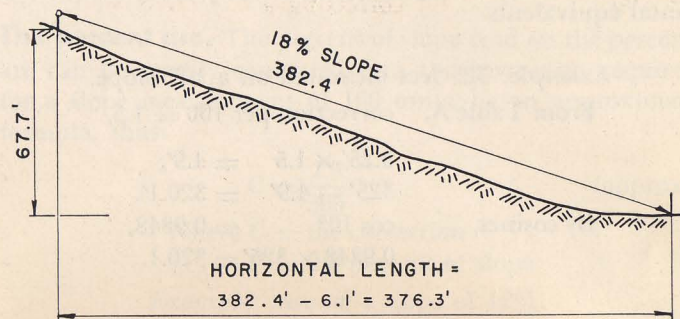


Fig. 7 Continuous measurement

THE TOPOGRAPHIC ARC

The topographic arc was designed in accordance with the specifications of the Department of Agriculture Forest Service. It is especially designed for use with a tape marked in Gunter's chains and links. A Gunter's chain is 66 feet long and a link is 1/100 of this length.

Topographic trailer tapes also designed by the Forest Service (K&E Nos. 7697-2 and 7697-3). These tapes are marked in chains and links and have special graduations (trailers) beyond each chain length, which indicate the increase in length that must be measured on a slope to establish a desired horizontal distance. These graduations are numbered in accordance with the topographic arc (Fig. 8).

K&E No. 7697-2 Topographic Trailer Tape is a steel tape, 1/4 inch wide, two chains (132 feet) long with a trailer of about 33 feet. The face is graduated every link for the first

two chains. The trailer, which extends beyond the two chain mark, contains the special graduations for correcting a two chain slope measurement. The reverse is graduated every link for one chain. Beyond the one chain mark are the special graduations for correcting a one chain slope measurement.

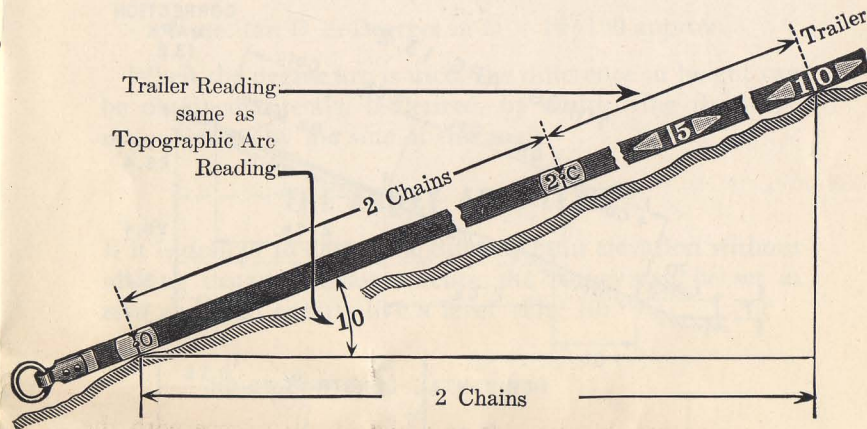


Fig. 8

The topographic trailer tape is used as follows: A slope reading is taken with the topographic Abney level, reading the topographic arc. A measurement along the slope is made with the topographic trailer tape, 2 chains long plus a distance along the trailer equal to the number of graduations indicated by the topographic arc reading. The total distance thus measured along the slope equals a horizontal distance of 2 chains. If the same procedure is carried out reading the reverse face of the tape, the measurement along the slope will be equal to the horizontal distance of 1 chain.

K&E No. 7697-3 Topographic Trailer Tape is a steel tape, 1/4 inch wide, three chains (198 feet) long. The face is graduated every link for three chains. The reverse, beyond the 50 link, one chain and two chain marks, are the corresponding slope correction graduations.

To measure along a slope, the value of the slope is first determined with the topographic arc. The rear chainman then holds the graduation marked with this value, while the head chainman holds the zero of the tape. When this length is measured on the slope, the desired horizontal distance is established. (See Fig. 9.)

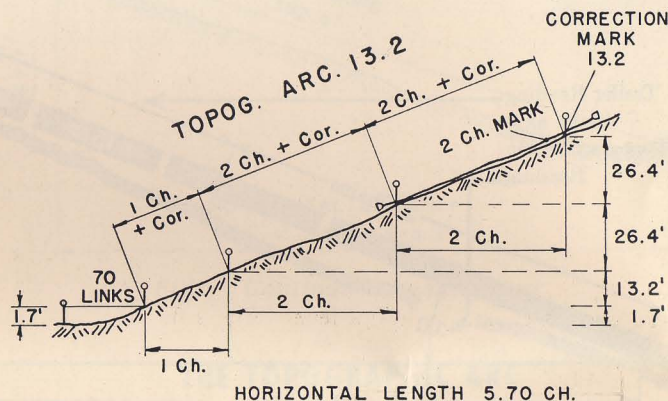


Fig. 9 Measurement with a trailer tape

MEASUREMENTS OF DIFFERENCES IN HEIGHT BETWEEN GROUND POINTS

When a slope measurement is made along the ground, the differences in height between the various points marked can be easily obtained from the corresponding horizontal distance. For example, in Fig. 9 the difference in height for a horizontal length of one chain is the value given by the topographic arc, 13.2 feet. In Figs. 6 and 7, the percent slope gives the values. When the horizontal distance has been determined, the difference in height can always be computed quickly from the slope reading. The examples given in the table below include the examples in Figs. 9, 6 and 7.

Instrument Read.	Hor. Dist.	Computation of Diff. in Ht.
Topographic Arc 13.2	2 ch.	$2 \times 13.2 = 26.4'$
13.2	1 ch.	$1 \times 13.2 = 13.2'$
Percent Arc 20	100 ft.	$100 \times 0.20 = 20.0'$
18	376.3	$376.3 \times 0.18 = 67.7'$
Degree Arc 10°	200 ft.	$200 \times \tan 10^\circ = 35.3'$
5°	100 ft.	$100 \times \tan 5^\circ = 8.7'$

Note: $\tan D = \text{Degrees in } D \times 1\frac{3}{4}/100$ approx.

When the degree arc is used, the difference in height can be obtained directly, if desired, by multiplying the slope measurement by the sine of the angle.

THE ABNEY AS A LEVEL

If it is desired to determine differences in elevation without making distance measurements, the Abney can be set at zero and used exactly like a level (Fig. 10).

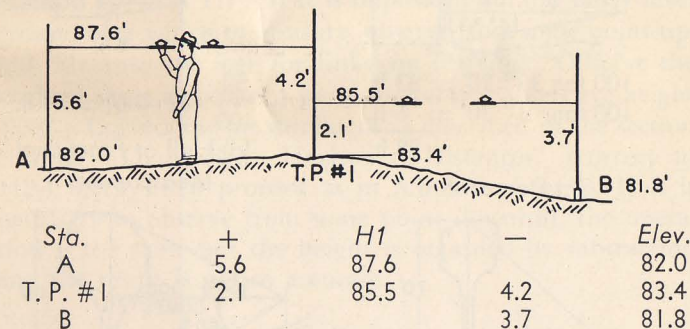
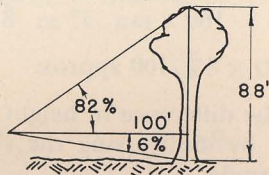


Fig. 10

Assume that A has an elevation of 82. It is desired to find the elevation of B. A graduated (leveling) rod is held on A. The level line of sight strikes at 5.6 feet. This is added to 82 to find the height of instrument, 87.6. A sight on a point called a turning point in the direction desired yields 4.2. This is subtracted from the height of instrument to obtain the elevation 83.4. This sequence can be repeated indefinitely. The usual form of notes is shown in the figure.

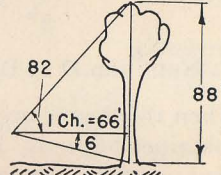
DETERMINATION OF HEIGHTS OF TREES AND POLES

The Abney level is especially adapted to the determination of the heights of trees, poles, chimneys, overhead wires



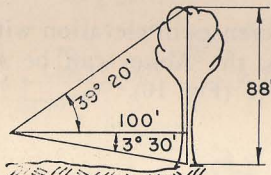
$$82 + 6 = 88 \text{ ft.}$$

(a) Percent Arc



$$82 + 6 = 88 \text{ ft.}$$

(b) Topog. Arc

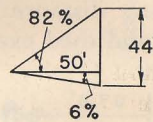


$$100 \tan 39^\circ 20' = 82 \text{ ft.}$$

$$100 \tan 3^\circ 30' = 6 \text{ ft.}$$

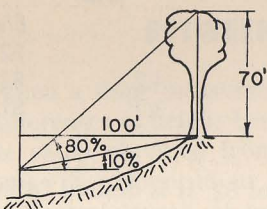
$$88 \text{ ft.}$$

(c) Degree Arc



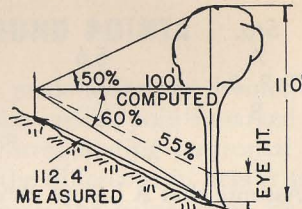
$$\frac{50}{100} (82 + 6) = 44 \text{ ft.}$$

(d) Half or Fractional Measurement



$$80 - 10 = 70 \text{ ft.}$$

(e) Uphill Measurement



$$50 + 60 = 110 \text{ ft.}$$

(f) Slope Measurement

Fig. 11 Methods of Measuring Height

and the like. There are several methods of accomplishing this. They are described in the order of their simplicity.

If possible, use the percent arc or the topographic arc. Measure the ground distance in feet for the percent arc, in links for the topographic arc. In either case the height will be in feet. The process is identical for both.

Method A. (Figs. 11a, 11b and 11c). Measure horizontally 100 feet (or links) from the tree. Standing at this point the percent (or topographic) arc reads 82 to the top of the tree and 6 to the bottom. The height is $82 + 6 = 88$ feet. (Subtract the reading to the bottom of the tree if it is above the eye.) (See Fig. 11e.)

Method B. (Fig. 11d). When the object is not very tall or the space is restricted, measure horizontally 50 feet (or links). Add the two readings and divide by 2. (Subtract the reading to the bottom of the tree if it is above the eye.)

Method C. (Fig. 11f). If it is impossible to find fairly level ground over which to measure, observe from some point uphill. Measure 100 feet (or links) on the slope. Observe the angle of slope aiming at a point on the tree about eye height (55%). Correct the measurement as described in the section entitled "To Measure Horizontal Distances" (correct to 112.4 feet). Then proceed as in Methods A or B. If it is necessary to observe from some point downhill, the operation is the same but the height is obtained by subtraction and the result is not so accurate.

APPLICATIONS OF THE ABNEY LEVEL

It is evident that any surveying operation desired may be carried out, if, in addition to an Abney and a tape, a pocket compass is available to obtain directions. Among such instruments made by K&E are No. 5340 Forester's and Geologist's Compass, No. 5368 Pocket Transit and No. 5600 Forestry Compass. The reader is referred to any of the numerous texts on surveying for possible applications.

TRAVERSES

A fundamental operation with the Abney is the measurement of a traverse for mapping. The procedure is briefly described here.

Several intervisible points are selected in the area to be mapped (ABCDEF in Fig. 12).

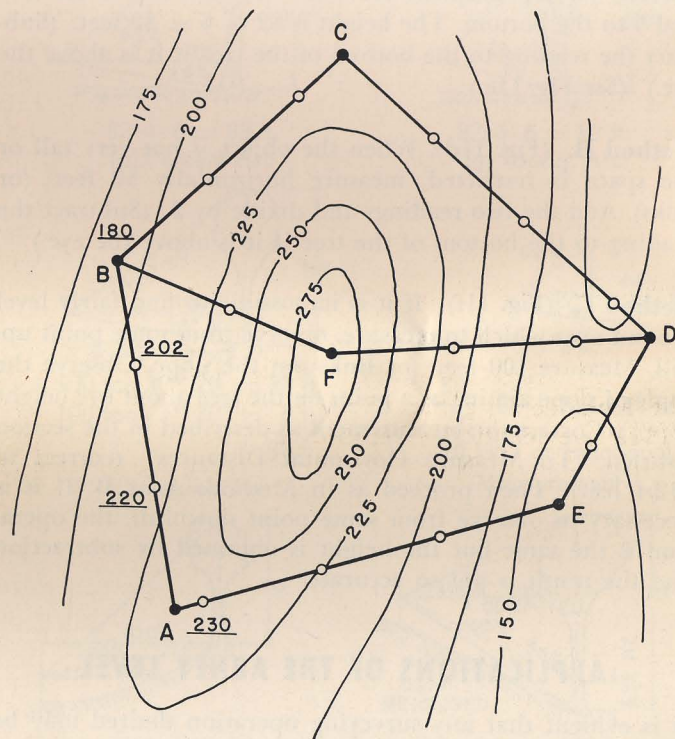


Fig. 12

From A, some object is selected in the desired direction of B and a compass bearing is taken on it. The head tapeman, holding the zero of the tape, moves off in this direction.

When the tape has been run out, the observer takes an Abney reading and the corrected distance is carefully marked. The difference in elevation is computed and, from the known elevation of A (230 feet), the elevation of the end of the tape is computed (220 feet). The 225 foot contour is interpolated between these values and sketched in the notes showing how it crosses the line and how it runs on each side. The process is then repeated as often as necessary and B is finally set at an even tape length or chain length, if possible. The other traverse lines are measured in the same way. When the elevations are brought back to a known elevation as at A, a check is obtained. When the traverse is plotted, position checks are obtained. In the figure, the line EA should terminate at A and the line FD should terminate at D.

After the traverse is plotted, the contours are sketched from the notes and filled in between traverse lines in accordance with the appearance of the topography.

TYPES OF ABNEY LEVELS THE TOPOGRAPHIC ABNEY LEVEL*

The K&E Topographic Abney Levels Nos. 5713PD and 5713TP are identical except for the graduations on their arcs (See Fig. 13). No. 5713PD is graduated on one side of its arc in percent of grade and on the other to single degrees. No. 5713TP is graduated on one side in percent of grade and in topographic arc on the other. To change from the graduations on one side to those on the other, the arc can be removed and turned around. An adjusting pin is provided for this purpose.

To change the arcs proceed as follows. Remove the capstan friction screw and slip out the level assembly. Remove the two nuts F and F', turn the scale around and replace F and F'. In replacing the level assembly, make sure the pin in the brass nut, into which the friction nut is threaded, is fitted into its hole in the limb. Tighten the friction screw only enough to provide the desired friction.

* As made for the U. S. Forest Service.

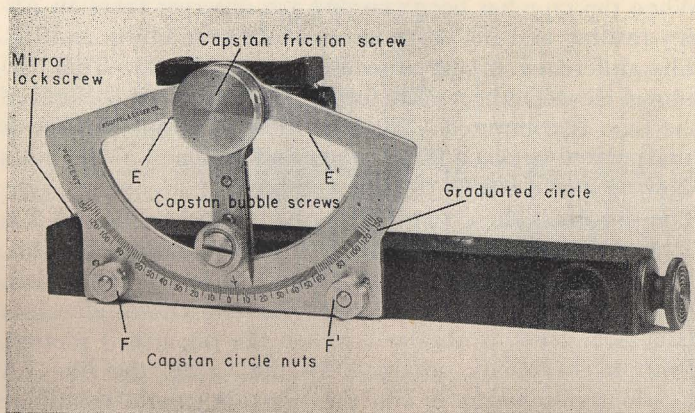


Fig. 13

Adjustment. There are three major adjustments of the instrument: (1) mirror slide, (2) height of vial, (3) bubble. They should be made in the order named.

1. *Mirror slide.* To make the zero mark on the vial appear opposite the cross line. Set the index arm at zero. Loosen the mirror lock screw. Slide the screw backward or forward, as required, and tighten it.

2. *Height of vial.* To center the vial zero mark on the horizontal axis. This adjustment prevents the zero mark from apparently moving away from the cross line when the index arm is moved. The zero mark is etched on the curved surface of the vial and accordingly it is slightly curved. Its curvature becomes apparent when viewed diagonally (Fig. 14). This always occurs when a slope is measured. The refraction of the fluid partly compensates for this effect but it is always apparent at high slopes. The height of the vial should be adjusted so that the ends of the zero mark appear to remain at the level of the cross line as shown in Fig. 14.

Turn capstan screws E and E' equal amounts in the same direction until the desired condition is obtained.

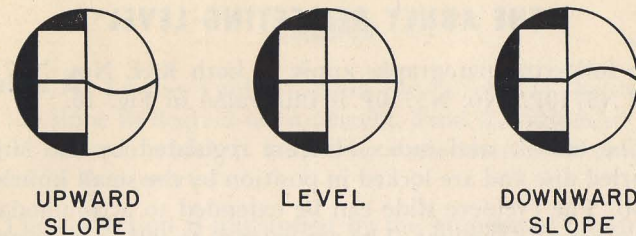


Fig. 14 Correct adjustment of vial height

3. *Bubble.* To make the bubble center when the index is at zero and the line of sight is horizontal (Fig. 15). Select two trees about 30 to 50 feet apart. Pin a strip of paper for a target on one of them. Call this target A. Set the instrument at zero, steady it against the side of the second tree and, by trial, find a point apparently at the same elevation as A.

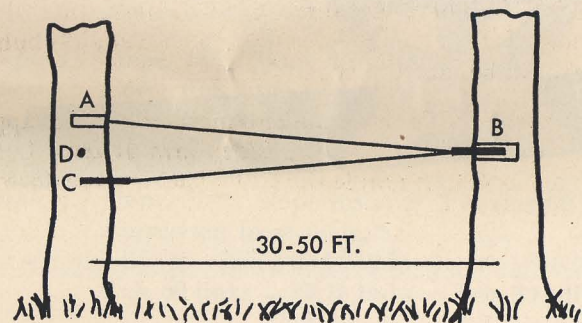


Fig. 15

Mark this point with target B. In the same manner find a point C on the first tree apparently at the elevation of B. Mark a point D on the first tree half way between A and C. D will be at the same elevation as B, no matter how much the instrument may be out of adjustment.

Turn the capstan screws E and E' equal amounts in opposite directions until, when the instrument is held level at D, the line of sight will strike B.

THE ABNEY REFLECTING LEVEL

The following paragraphs apply to both K&E Nos. N5710 and N5710P. No. N5710P is illustrated in Fig. 16.

The bubble and index arm are regulated by the large knurled disc and are locked in position by the small knurled knob. The eyepiece slide can be extended to accommodate the observer's eyesight. The arc extends to 90°. The instrument may be used as a contact level or as a surface clinometer.

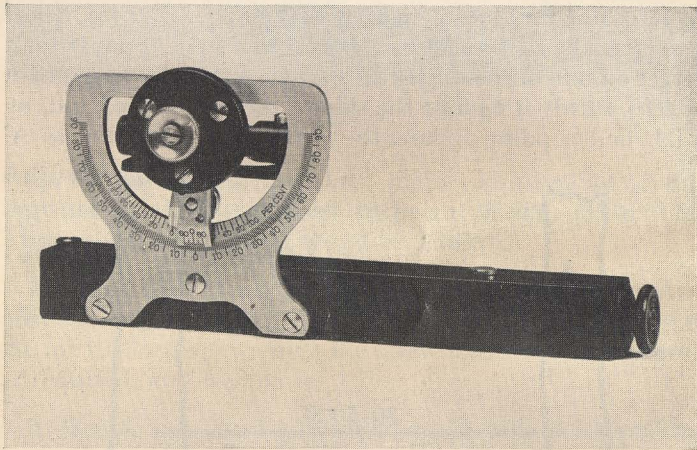


Fig. 16

Adjustment. Only the bubble adjustment is required for this instrument. Except that only one screw is used to regulate the bubble, the adjustment is performed exactly as described for the Topographic Abney.

The adjusting screw is at that end of the bubble tube which is nearest the eyepiece. A small horizontal brass screw in the end of the tube also identifies the end at which the adjustment is made. The screw at the other end must be kept tight.

TABLES

Table A. To use Table A, enter the table with the value of the slope in degrees or in percent. Find the values in the table between which it falls. The correction to be used is listed in this interval.

The correction is multiplied by the number of hundreds in the slope distance and the result subtracted from the slope distance to give the true horizontal distance.

The maximum error in the table is 0.05 units in 100 units of slope distance.

Example 1. Slope, 15°. Slope distance 250 feet.

Correction from table, 3.4

$$2.5 \times 3.4 = 8.5 \text{ feet}$$

$$250 - 8.5 = 241.5 \text{ feet Hor. Dist.}$$

Example 2. Slope, 18%. Slope distance 300 feet.

Correction from table, 1.6

$$3 \times 1.6 = 4.8$$

$$300 - 4.8 = 295.2 \text{ feet Hor. Dist.}$$

Example 3. Slope, 35%. Slope distance 3 chains 60 links.

Correction from table, 5.6

$$3.6 \times 5.6 = 20.16 \text{ links}$$

$$3 \text{ ch. } 60 \text{ links} - 20.16 \text{ links} = 3 \text{ ch. } 39.8 \text{ links.}$$

Table B. Table B gives the natural sines, cosines, tangents, and cotangents of angles, at intervals of 30 minutes to four decimal places.

TABLE A
CORRECTIONS TO SLOPE DISTANCES

$$\text{Hor. Dist.} = \text{Slope Dist.} - \frac{\text{Slope Dist.}^2}{100} \text{ Cor.}$$

Deg. Arc	Cor.	% Arc	Deg. Arc	Cor.	% Arc	Deg. Arc	Cor.	% Arc	Deg. Arc	Cor.	% Arc
1°49'		3.2	15°06'		27.0	21°20'		39.0	26°10'		49.1
3°08'	0.1	5.5	15°19'	3.5	27.4	21°29'	6.9	39.4	26°18'	10.3	49.4
4°03'	0.2	7.1	15°32'	3.6	27.8	21°39'	7.0	39.7	26°26'	10.4	49.7
4°48'	0.3	8.4	15°44'	3.7	28.2	21°48'	7.1	40.0	26°33'	10.5	50.0
5°26'	0.4	9.5	15°57'	3.8	28.6	21°57'	7.2	40.3	26°41'	10.6	50.3
6°01'	0.5	10.5	16°09'	3.9	29.0	22°06'	7.3	40.6	26°49'	10.7	50.5
6°32'	0.6	11.4	16°22'	4.0	29.4	22°15'	7.4	40.9	26°56'	10.8	50.8
7°01'	0.7	12.3	16°34'	4.1	29.7	22°24'	7.5	41.2	27°04'	10.9	51.1
7°29'	0.8	13.1	16°46'	4.2	30.1	22°33'	7.6	41.5	27°11'	11.0	51.4
7°54'	0.9	13.9	16°58'	4.3	30.5	22°42'	7.7	41.8	27°19'	11.1	51.6
8°19'	1.0	14.6	17°09'	4.4	30.9	22°51'	7.8	42.1	27°26'	11.2	51.9
8°42'	1.1	15.3	17°21'	4.5	31.2	23°00'	7.9	42.4	27°34'	11.3	52.2
9°04'	1.2	16.0	17°32'	4.6	31.6	23°09'	8.0	42.7	27°41'	11.4	52.5
9°26'	1.3	16.6	17°44'	4.7	32.0	23°18'	8.1	43.0	27°49'	11.5	52.7
9°47'	1.4	17.2	17°55'	4.8	32.3	23°26'	8.2	43.3	27°56'	11.6	53.0
10°06'	1.5	17.8	18°06'	4.9	32.7	23°35'	8.3	43.6	28°03'	11.7	53.3
10°25'	1.6	18.4	18°17'	5.0	33.0	23°43'	8.4	43.9	28°11'	11.8	53.6
10°44'	1.7	19.0	18°28'	5.1	33.4	23°52'	8.5	44.2	28°18'	11.9	53.8
11°02'	1.8	19.5	18°39'	5.2	33.7	24°00'	8.6	44.5	28°25'	12.0	54.1
11°20'	1.9	20.0	18°50'	5.3	34.1	24°09'	8.7	44.8	28°32'	12.1	54.4
11°37'	2.0	20.6	19°00'	5.4	34.4	24°17'	8.8	45.1	28°39'	12.2	54.7
11°54'	2.1	21.1	19°11'	5.5	34.8	24°26'	8.9	45.4	28°47'	12.3	54.9
12°11'	2.2	21.6	19°21'	5.6	35.1	24°34'	9.0	45.7	28°54'	12.4	55.2
12°27'	2.3	22.1	19°31'	5.7	35.5	24°42'	9.1	46.0	29°01'	12.5	55.5
12°43'	2.4	22.6	19°42'	5.8	35.8	24°50'	9.2	46.3	29°08'	12.6	55.7
12°58'	2.5	23.0	19°52'	5.9	36.1	24°58'	9.3	46.6	29°15'	12.7	56.0
13°13'	2.6	23.5	20°02'	6.0	36.5	25°07'	9.4	46.9	29°22'	12.8	56.3
13°28'	2.7	23.9	20°12'	6.1	36.8	25°15'	9.5	47.1	29°29'	12.9	56.5
13°43'	2.8	24.4	20°22'	6.2	37.1	25°23'	9.6	47.4	29°36'	13.0	56.8
13°57'	2.9	24.8	20°32'	6.3	37.4	25°31'	9.7	47.7	29°43'	13.1	57.1
14°11'	3.0	25.3	20°41'	6.4	37.8	25°39'	9.8	48.0	29°50'	13.2	57.3
14°25'	3.1	25.7	20°51'	6.5	38.1	25°47'	9.9	48.3	29°57'	13.3	57.6
14°39'	3.2	26.1	21°01'	6.6	38.4	25°54'	10.0	48.6	30°04'	13.4	57.9
14°52'	3.3	26.6	21°10'	6.7	38.7	26°02'	10.1	48.9	30°10'	13.5	58.1
15°06'	3.4	27.0	21°20'	6.8	39.0	26°10'	10.2	49.1	30°17'	13.6	58.4

TABLE B
NATURAL TRIGONOMETRIC FUNCTIONS

	sine	d	tan	d	cotan	d	cosine	d	
0°00'	0.0000	87	0.0000	87			1.0000	0	90°00'
0°30'	0.0087	88	0.0087	88	114.589		1.0000	2	89°30'
1°00'	0.0175	87	0.0175	87	57.290		0.9998	1	89°00'
1°30'	0.0262	87	0.0262	87	38.188		0.9997	3	88°30'
2°00'	0.0349	87	0.0349	87	28.636		0.9994	3	88°00'
2°30'	0.0436	87	0.0437	88	22.904		0.9990	4	87°30'
3°00'	0.0523	87	0.0524	87	19.081		0.9986	4	87°00'
3°30'	0.0610	88	0.0612	88	16.350		0.9981	5	86°30'
4°00'	0.0698	87	0.0699	88	14.301		0.9976	7	86°00'
4°30'	0.0785	87	0.0787	88	12.706		0.9969	7	85°30'
5°00'	0.0872	86	0.0875	88	11.430		0.9962	7	85°00'
5°30'	0.0958	86	0.0963	88	10.385		0.9954	8	84°30'
6°00'	0.1045	87	0.1051	88	9.514	871	0.9945	9	84°00'
6°30'	0.1132	87	0.1139	88	8.777	737	0.9936	9	83°30'
7°00'	0.1219	86	0.1228	89	8.144	633	0.9925	11	83°00'
7°30'	0.1305	87	0.1317	89	7.596	548	0.9914	11	82°30'
8°00'	0.1392	86	0.1405	88	7.115	481	0.9903	11	82°00'
8°30'	0.1478	86	0.1495	90	6.691	424	0.9890	13	81°30'
9°00'	0.1564	86	0.1584	89	6.314	377	0.9877	13	81°00'
9°30'	0.1650	86	0.1673	89	5.976	338	0.9863	14	80°30'
10°00'	0.1736	86	0.1763	90	5.671	305	0.9848	15	80°00'
10°30'	0.1822	86	0.1853	90	5.396	275	0.9833	15	79°30'
11°00'	0.1908	86	0.1944	91	5.145	251	0.9816	17	79°00'
11°30'	0.1994	86	0.2035	91	4.915	230	0.9799	17	78°30'
12°00'	0.2079	85	0.2126	91	4.705	210	0.9781	18	78°00'
12°30'	0.2164	85	0.2217	91	4.511	194	0.9763	18	77°30'
13°00'	0.2250	86	0.2309	92	4.331	180	0.9744	19	77°00'
13°30'	0.2334	84	0.2401	92	4.165	166	0.9724	20	76°30'
14°00'	0.2419	85	0.2493	92	4.011	154	0.9703	21	76°00'
14°30'	0.2504	85	0.2586	93	3.867	144	0.9681	22	75°30'
15°00'	0.2588	84	0.2679	93	3.732	135	0.9659	22	75°00'
15°30'	0.2672	84	0.2773	94	3.606	126	0.9636	23	74°30'
16°00'	0.2756	84	0.2867	94	3.487	119	0.9613	23	74°00'
16°30'	0.2840	84	0.2962	95	3.376	111	0.9588	25	73°30'
17°00'	0.2924	84	0.3057	95	3.271	105	0.9563	25	73°00'
17°30'	0.3007	83	0.3153	96	3.172	99	0.9537	26	72°30'
18°00'	0.3090	83	0.3249	96	3.078	94	0.9511	26	72°00'
18°30'	0.3173	83	0.3346	97	2.989	89	0.9483	28	71°30'
19°00'	0.3256	83	0.3443	97	2.904	85	0.9455	28	71°00'
19°30'	0.3338	82	0.3541	98	2.824	80	0.9426	29	70°30'
20°00'	0.3420	82	0.3640	99	2.747	77	0.9397	29	70°00'
20°30'	0.3502	82	0.3739	99	2.675	72	0.9367	30	69°30'
21°00'	0.3584	82	0.3839	100	2.605	70	0.9336	31	69°00'
21°30'	0.3665	81	0.3939	100	2.539	66	0.9304	32	68°30'
22°00'	0.3746	81	0.4040	101	2.475	64	0.9272	32	68°00'
22°30'	0.3827	81	0.4142	102	2.414	61	0.9239	33	67°30'
	cosine	d	cotan	d	tan	d	sine	d	

TABLE B (cont.)
NATURAL TRIGONOMETRIC FUNCTIONS

	sine	d	tan	d	cotan	d	cosine	d	
22°30'	0.3827		0.4142		2.414		0.9239		67°30'
23°00'	0.3907	80	0.4245	103	2.356	58	0.9205	34	67°00'
23°30'	0.3987	80	0.4348	103	2.300	56	0.9171	34	66°30'
24°00'	0.4067	80	0.4452	104	2.246	54	0.9135	36	66°00'
24°30'	0.4147	80	0.4557	105	2.194	52	0.9100	35	65°30'
25°00'	0.4226	79	0.4663	106	2.145	49	0.9063	37	65°00'
25°30'	0.4305	79	0.4770	107	2.097	48	0.9026	37	64°30'
26°00'	0.4384	79	0.4877	107	2.050	47	0.8988	38	64°00'
26°30'	0.4462	78	0.4986	109	2.006	44	0.8949	39	63°30'
27°00'	0.4540	78	0.5095	109	1.963	43	0.8910	39	63°00'
27°30'	0.4617	77	0.5206	111	1.921	42	0.8870	40	62°30'
28°00'	0.4695	78	0.5317	111	1.881	40	0.8829	41	62°00'
28°30'	0.4772	77	0.5430	113	1.842	39	0.8788	41	61°30'
29°00'	0.4848	76	0.5543	113	1.804	38	0.8746	42	61°00'
29°30'	0.4924	76	0.5658	115	1.767	37	0.8704	42	60°30'
30°00'	0.5000	76	0.5774	116	1.732	35	0.8660	44	60°00'
30°30'	0.5075	75	0.5890	116	1.698	34	0.8616	44	59°30'
31°00'	0.5150	75	0.6009	119	1.664	34	0.8572	44	59°00'
31°30'	0.5225	75	0.6128	119	1.632	32	0.8526	46	58°30'
32°00'	0.5299	74	0.6249	121	1.600	32	0.8480	46	58°00'
32°30'	0.5373	74	0.6371	122	1.570	30	0.8434	46	57°30'
33°00'	0.5446	73	0.6494	123	1.540	30	0.8387	47	57°00'
33°30'	0.5519	73	0.6619	125	1.511	29	0.8339	48	56°30'
34°00'	0.5592	73	0.6745	126	1.483	28	0.8290	49	56°00'
34°30'	0.5664	72	0.6873	128	1.455	28	0.8241	49	55°30'
35°00'	0.5736	72	0.7002	129	1.428	27	0.8192	49	55°00'
35°30'	0.5807	71	0.7133	131	1.402	26	0.8141	51	54°30'
36°00'	0.5878	71	0.7265	132	1.376	26	0.8090	51	54°00'
36°30'	0.5948	70	0.7400	135	1.351	25	0.8039	51	53°30'
37°00'	0.6018	70	0.7536	136	1.327	24	0.7986	53	53°00'
37°30'	0.6088	70	0.7673	137	1.303	24	0.7934	52	52°30'
38°00'	0.6157	69	0.7813	140	1.280	23	0.7880	54	52°00'
38°30'	0.6225	68	0.7954	141	1.257	23	0.7826	54	51°30'
39°00'	0.6293	68	0.8098	144	1.235	22	0.7771	55	51°00'
39°30'	0.6361	68	0.8243	145	1.213	22	0.7716	55	50°30'
40°00'	0.6428	67	0.8391	148	1.192	21	0.7660	56	50°00'
40°30'	0.6494	66	0.8541	150	1.171	21	0.7604	56	49°30'
41°00'	0.6561	67	0.8693	152	1.150	21	0.7547	57	49°00'
41°30'	0.6626	65	0.8847	154	1.130	20	0.7490	57	48°30'
42°00'	0.6691	65	0.9004	157	1.111	19	0.7431	59	48°00'
42°30'	0.6756	65	0.9163	159	1.091	20	0.7373	58	47°30'
43°00'	0.6820	64	0.9325	162	1.072	19	0.7314	59	47°00'
43°30'	0.6884	64	0.9490	165	1.054	18	0.7254	60	46°30'
44°00'	0.6947	63	0.9657	167	1.036	18	0.7193	61	46°00'
44°30'	0.7009	62	0.9827	170	1.018	18	0.7133	60	45°30'
45°00'	0.7071	62	1.0000	173	1.000	18	0.7071	62	45°00'
	cosine	d	cotan	d	tan	d	sine	d	



*Drafting, Reproduction, Surveying,
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