C-105 Drag Estimation

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February 23, 1955

AREA RULE PROGRESS REPORT - FEBRUARY 1955

1. INTRODUCTION

The area rule program at Avro has been carried out in four phases:

(a) Investigation of Numerical Procedure
(b) Model Program
(c) Application of Method to Drag Reduction of C-105 Aircraft at the Design Mach Number.
(d) Comparison with Experiment.

Phases (a) and (b) were carried out more or less simultaneously, while work on phases (c) and (d) is still continuing.

It has been found that the result obtained depends very strongly on the methods used, both in the determination of the area distributions of the models and in the numerical calculations. Results differing by as much as a factor of 5 can be obtained using different methods on wing-body combinations having high slopes in the area distribution curves. Also, if the method of approximating the slope of the area distribution by a Fourier series is used to calculate the drag, the result depends quite strongly on the number of terms in the series as well as the degree of accuracy obtained in representing the area distribution of the model.

The method finally adopted at Avro is an extension of the method outlined by Holdaway in reference 1, and consists of approximating the slope of the area distribution curve by a 100 term Fourier series. In order to obtain a good degree of accuracy in the determination of the Fourier Coefficients, the slope curve is represented by 300 points. (In this way, more than 100 terms in the series may be calculated without too great a loss in accuracy). A high speed computing machine is required to carry out the integrations in a reasonable time. The area curve is obtained by cutting oblique slices from an accurate model of the aircraft, and measuring the resulting areas by means of a planimeter. In regions of rapidly changing slopes, larger scale models of the regions are used, in order to improve the accuracy of the representation.

The wave drags calculated by this method have been used as a basis for suggesting possible changes to reduce the drag of the C-105 aircraft at the design Mach number. By means of two small modifications to the aircraft lines in the regions of the intakes and the rear nacelles, and the addition of a larger fairing between the tail pipes, a drag reduction of 37% has been realized. Comparison of these results with optimum bodies (2) of the general configuration has also been carried out.

2. INVESTIGATION OF NUMERICAL PROCEDURES

The fundamental numerical problem involved is to develop a method for evaluating the integral

$$D/q = -\frac{1}{2\pi}\int_0^1 A''(x)\int_0^1 A''(\xi)\log\left|x-\xi\right| d\xi\, dx \qquad (1)$$

$A''(x)$ is the second derivative of the area curve with respect to the length along the body, x. Three methods have been investigated in an attempt to choose the one most suited to this program. The area distribution chosen for this purpose was the unmodified C-105 cross-sectional area, corresponding to M = 1.0.

(a) Legendres Integral Form

In reference 3, it is shown that the form (1) may be replaced by the non-singular form given below, which is better suited for numerical integration: ?

$$D/q = \frac{1}{2\pi}\int_0^1 \frac{[A'(x)]^2\,dx}{x(1-x)} + \frac{1}{4\pi}\int_0^1 \left[\frac{A'(x)-A'(\xi)}{x-\xi}\right]^2 d\xi dx \qquad (2)$$

Although the integrals in (2) are non-singular, some difficulty is still met for ξ very close to x. Carrying out the integration, the value of D/q is 67.5 ft.^{2}. The high value of this integral is due to the high slopes of the area distribution used, in the regions of the intakes and the tail. It was felt that the value of the drag obtained by this method should prove to be the upper limit of the drags calculated by Fourier series methods, providing a sufficiently large number of terms was taken.

In view of the time required to carry out a numerical integration such as (2), only two examples were worked out. It would have been interesting to compare results with Fourier series evaluations for smoother bodies, but time limitations have prevented this. The general trend was established, however, that the Fourier series method to be outlined in section (c) did tend to the upper limit given by (2). It should be pointed out that no approximations are made to the area distribution of the body being considered in this method.

(b) R.A.E. Method of Fourier Analysis

A method for evaluating (1) has been proposed by the R.A.E., Farnborough in Reference 4. In this method, the area distribution curve is approximated by a 36 term Fourier series, and the resulting simplification introduced in equation (1). The drag calculated by this method for the unmodified C-105 was found to be one fifth that calculated by integral for the same area distribution. This is due to two effects; firstly, the number of points used is insufficient to represent an irregular body such as the C-105, and secondly, the fact that the area distribution is represented by the Fourier series and then analytically differentiated rather than representing the actual slope of the area curve by the Fourier series as in Holdaways Method. The representation of any curve by means of a Fourier series always tends to smooth out irregularities in the curve, thus the slope of any body approximated

(b) R.A.E. Method Of Fourier Analysis (Continued)

by a Fourier series, as in this method, will always be less than the actual slope. A greater number of terms might well have been used in this calculation, however, at the time that this work was one, no set-up was available on the high speed computer to handle the additional work involved, so an attempt to use a larger number of terms was not made. It is felt, however, that the R.A.E. method should give good results for bodies which have very smooth area distributions.

(c) Holdaway's Method of Fourier Analysis

Holdaways method[1] approximates the slope of the area distribution curve by a 24 term Fourier series. The drag can than be calculated from the simple formula

$$\frac{D}{q} = \frac{\pi}{4} \sum_{1}^{24} n a_n^2 \qquad (3)$$

Carrying out the calculation for the 24 term series, the value is found to be $D/q = 46$ ft.^{2}, which is 32% lower than that found by Legendres method. For a 100 term series, passing through 300 points, the total $D/q = 55$ ft.^{2}, about 18% lower. (It should be noted that the 24 term result was also evaluated by approximating the slope curve by 300 points, in an attempt to pick up all the peaks accurately. If only 24 points are used on the slope curve, the results cannot be expected to be very reliable because it is very likely that the Fourier series curve so approximated will completely miss some of the important regions of high slopes).

If the $\Sigma n a_n^2$ series is examined term by term, it can be seen that the series is gradually converging. Presumably, the final convergent answer, taking an infinite number of terms will be the same as that given by the Legendres formula. It is felt that the Fourier series approximations would converge more rapidly to the final values of D/q for smoother wing-body combinations than that used in this investigation.

The method finally decided on for further calculations was the 100 term Fourier series, modified Holdaway method. With the high speed computer available, a complete calculation can be carried out in just under three hours, given a table of 300 values of the area curve slopes. The fact that 100 terms may not be sufficient to converge to the complete wave drag coefficient is allowed for when evaluating the results. It is usually possible to estimate the degree of convergence by examining the $\Sigma n a_n^2$ series term by term.

In most cases, with relatively smooth bodies, convergence is fairly good. However, it should be noted that on the average the drag calculated for 24 terms is about 18% lower than that calculated using 100 terms. This fact is used to help analyze the experimental results presented by Holdaway in references 1 and 5, in order to ascertain the agreement between the theory and experiment.

3. MODEL PROGRAM

The model program consisted of taking an accurate .03 scale model of the aircraft and making plastic copies of it. These copies, painted black, were set on dowels to predetermined locations within a box and in turn encased in plastic. This resulted in a rectangular block of plastic which was conveniently handled. The cut lines desired were scribed on the surface of the block, and cutting was carried out on a bandsaw to these lines. After some practise, very good results were obtained.

The required outline of the embedded model in the slice was sharply defined by the paint line. This area was measured by means of a planimeter.

In regions where the slope was changing very rapidly, 1/16 scale models were made, and the procedure repeated. This gave much closer estimates of these critical areas. In the case of the fairing between the jet tail pipes, owing to the lack of suitable models, geometrical layouts were made to determine the sections. This latter technique is considered to be practical only as an interim phase in preliminary drag estimates, owing to the difficulty of laying out accurate oblique sections of complex shapes.

Photographs showing the number of models cut up to calculate the area distributions for two Mach numbers, with five rotation angles each, are enclosed in the appendix. Also shown are the details of some of the models used and examples of the oblique sections obtained.

4. APPLICATION OF THE METHOD TO DRAG REDUCTION OF C-105 AIRCRAFT

Typical area distribution curves for the C-105 aircraft are shown in Figure 2, for $M = 1.50$. The area of the stream tube entering the intakes has been added to the fuselage area in front of the ducts, and an area corresponding to the jet exhaust area has been added to the rear. These areas are calculated to be 11 $ft.^2$ and 22.4 $ft.^2$ respectively, at $M=1.50$.

The curves shown have had modifications made to the intake lips and a larger fairing added between the tail pipes, as illustrated by figure 3. The drag coefficients, based on the wing area, are given as a function of the angle of rotation θ in figure 4, both for the original and modified fuselages.

Integrating figure 4, the wave drag figures obtained are:-

Unmodified $\Delta C_D = .0190$

Modified $\Delta C_D = .0119$

Based on the unmodified figure, a reduction of 37% in the wave drag has been achieved. The major contribution to this lower drag has been the reduction of the slopes in the area distribution curves shown in figure 2.

In reference 5, Holdaway reports that changes in the wave drag coefficient resulting from area rule modifications of a given wing-body combination tend to be underestimated when comparing the theory with the experiment. It is thus expected that the reduction in the wave drag coefficient noted above should be fully realized.

5. COMPARISON WITH EXPERIMENT

It was felt that the results of the 100 term Fourier series method should be compared with as much experimental evidence as was available, in order to obtain a rational basis for estimating the total drag of the aircraft. Experimental results are given in references 1 and 5 for a series of wing-body combinations, for which area rule drag calculations have been carried out using 24 term Fourier series. In Figure 5, the results have been replotted showing the measured D/q as a function of Mach number for the four models. Superimposed on these graphs are Holdaway's calculated wave drag figures, added to the calculated turbulent skin friction at the corresponding Mach number. In addition, the wave drag calculated using a 100 term series for the scaled down C-105, added to the calculated turbulent skin friction, is shown for comparison with model A.

It can be seen that the resulting calculated supersonic drag is too low, as compared to the measured values, in all cases.

In carrying out the calculations for the C-105 drag analysis, it was found that the drag obtained by means of a 100 term Fourier series was approximately 18% higher than that obtained for a 24 term series, over the complete range of Mach numbers. For smoother bodies, such as those tested by Holdaway, the 100 term result is expected to give a drag only about 10% higher than that shown on the graph, due to the more rapid convergence of the series. This would indicate that, at the higher Mach numbers, the wave drag as calculated by the 100 term Fourier series will be approximately 10% lower than the experimental result. The calculated wave drag has thus been factored up by 10% in the following two examples.

This hypothesis was borne out when the drag of an 1/8 scale "crude model" of the C-105 was compared to that calculated by area rule at M = 1.50. The experimental results were obtained from free flight rocket firings made by the Avro Aircraft Company, and are shown in Figure 6. The drag estimate is broken down as follows:-

Estimate	A	B
1. Wave Drag (100 term series)	.018	.022
2. Turbulent Skin Friction (M = 1.5)	.006	.006
3. Base Drag (based on N.A.C.A. TR 1051)	.007	.007
Total C_{D_0}	.031	.035

Two estimates are shown for the wave drag, to cover a range of possible values. As oblique cuts were made on this model for only 3 rotation angles, some uncertainty arose as to the final calculated wave drag. Both the most pessimistic and the most optimistic values are shown in estimate A and B above. Had cuts been made at five rotation angles, which is the normal procedure, very little uncertainty would have been possible.

Adding 10% of the calculated wave drag to the average of the two estimates, the total C_{D_0} is .035. This agrees well with the experimental results.

6. DRAG ESTIMATE FOR C-105

In the light of the forgoing analysis, it is considered that a reasonable estimate of the drag at zero lift of an aircraft at supersonic speeds can be obtained by adding the wave drag calculated by Area rule to the skin friction drag at that Mach number, plus any correction required for the base drag.

For the modified C-105, at M = 1.50, this breakdown is as follows:-

1.	Wave Drag (100 term series)	.0119
2.	Turbulent Skin Friction (M = 1.5)	.0053
3.	Base Drag (based on Ref. 6)	-.0010
	Total C_{D_o} (theoretical)	.0161

Allowing for a 10% underestimation of the wave drag, the total C_{D_o} arrived at is .0173. This is considered to be a reasonable estimate of the total drag at this time and is supported by the limited number of experimental results available. Further rocket firings of 1/8 scale C-105 models are planned, and it is expected that the results so obtained will agree well with that obtained by the area rule method.

REFERENCES

1. Holdaway G.H., - Comparison of Theoretical and Experimental Zero-Lift Drag Rise Characteristics of Wing-Body-Tail Combinations near the Speed of Sound. - N.A.C.A. RM A53H17, 1953.

2. Lord, W.T., and Eminton E., - Slender Bodies of Minimum Wave Drag - Readers Forum Journal of Aeronautical Science, Vol. 21, No. 8 August 1954, p. 569-70.

3. Legendre, R.M., - Limite Sonique de la Resistance d'ondes d'un aeronef - Comptes Rendus, Vol. 236, No. 26, June 1953, p. 2479-80.

4. Warren, C.H.E. - Recent N.A.C.A. information on the Area Rule, T.N. Aero 2309, 1954.

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6. Englert, G.W., Vargo D.J. and Cubbison R.W. - Effect at Jet Nozzle Expansion Ratio on Drag of Parabolic Afterbodies - N.A.C.A. RM E54B12, 1954.

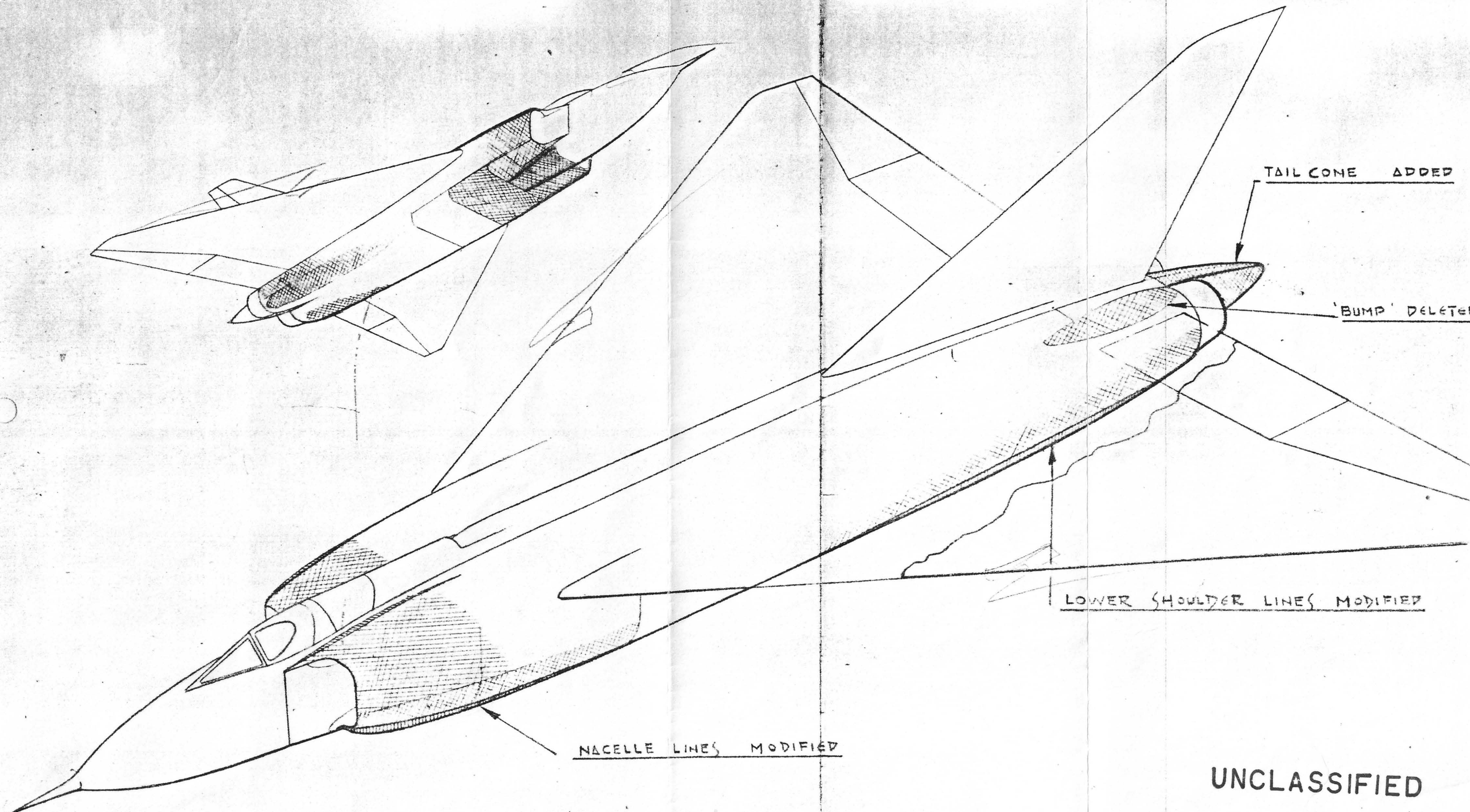

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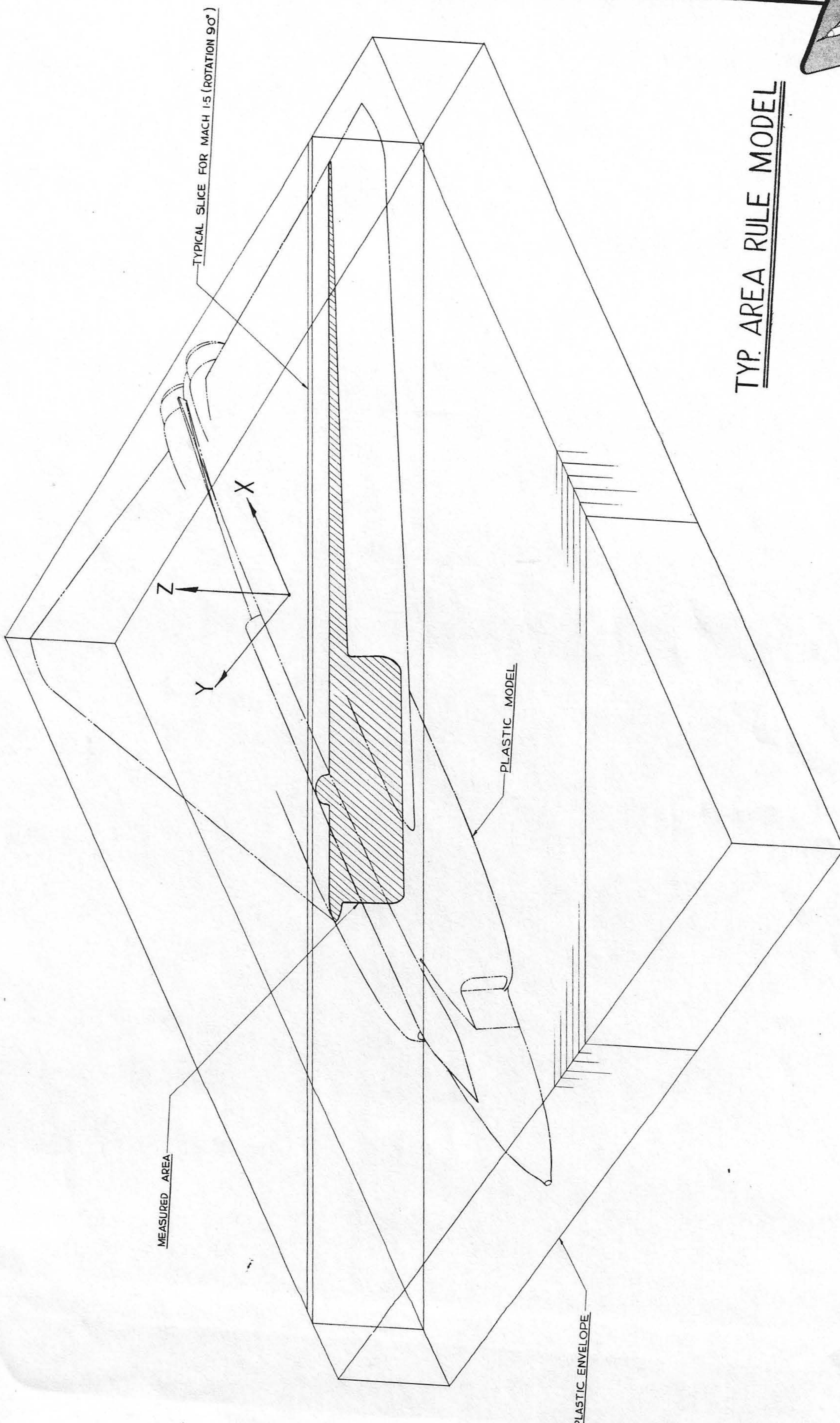
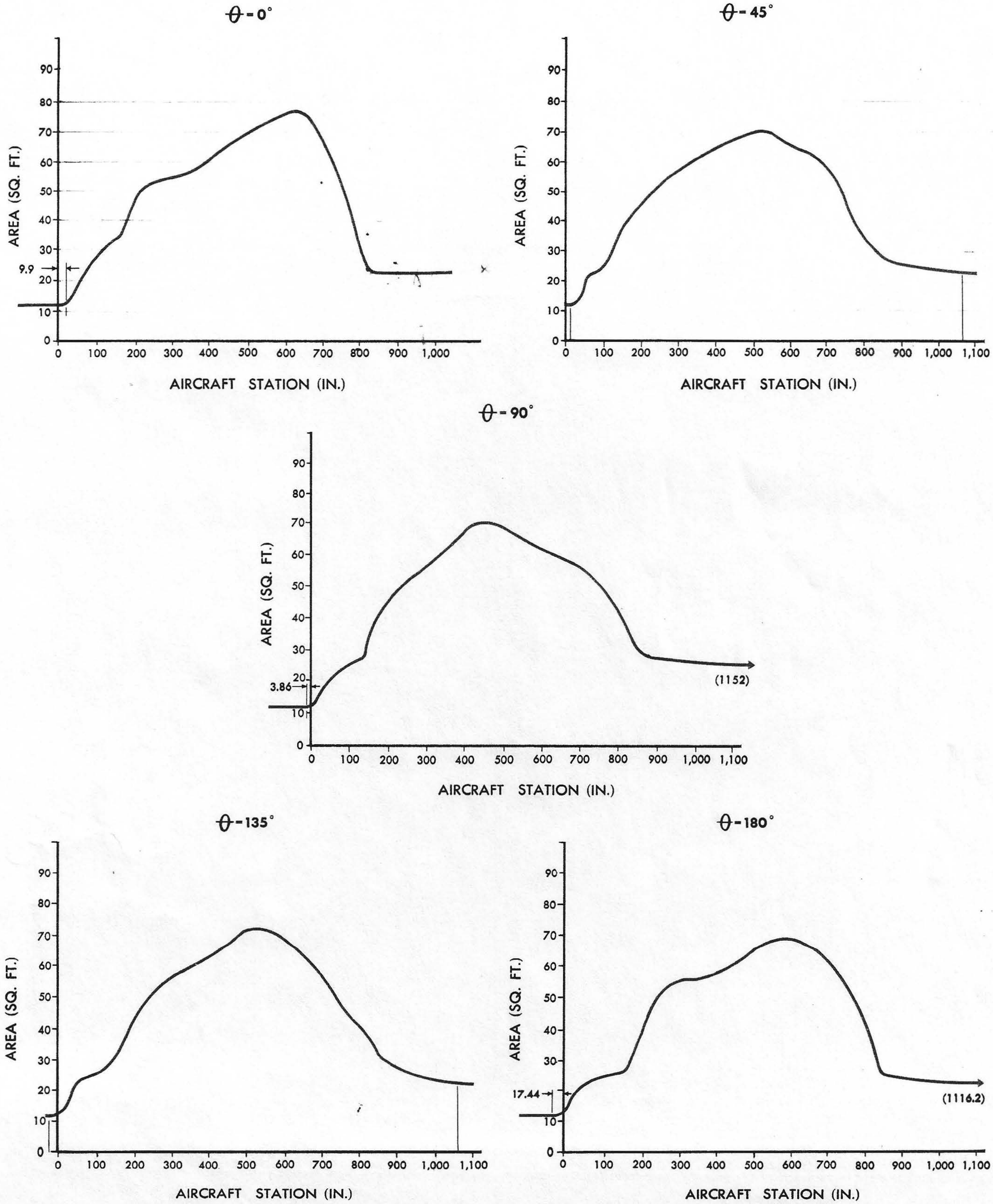
fig 3

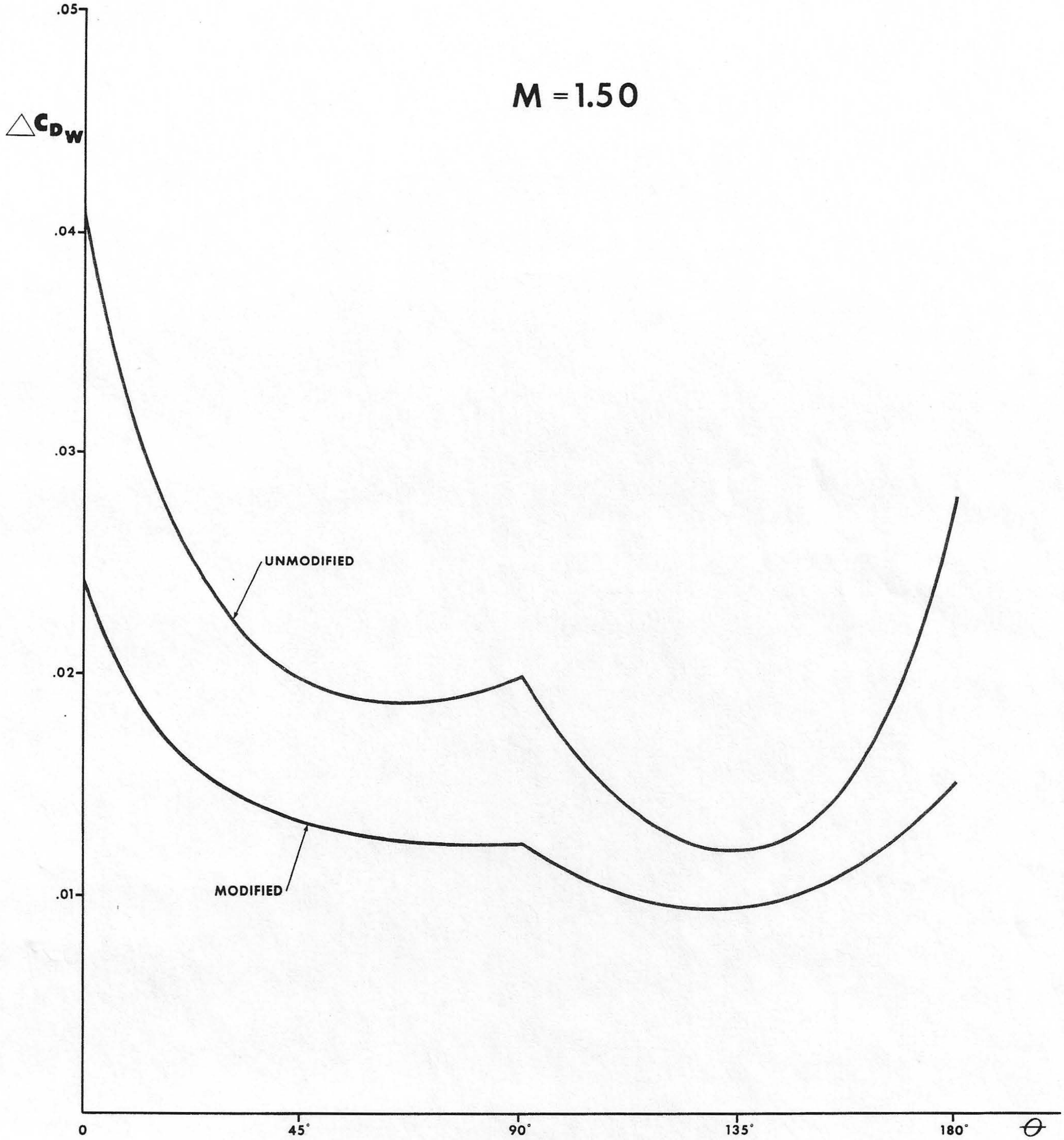
MODIFICATIONS TO FUSELAGE
INCORPORATING 'AREA RULE'

TYP. AREA RULE MODEL



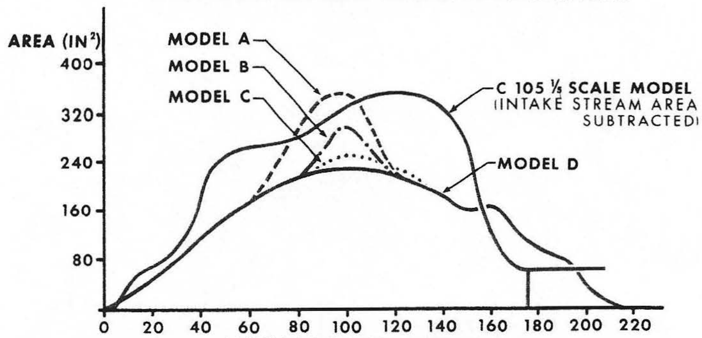


AREA DISTRIBUTION - MACH=1.50

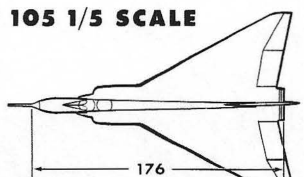


WAVE DRAG COEFFICIENT AS A FUNCTION OF ROTATION

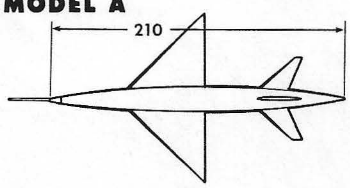
CROSS-SECTIONAL AREA DISTRIBUTIONS OF MODELS



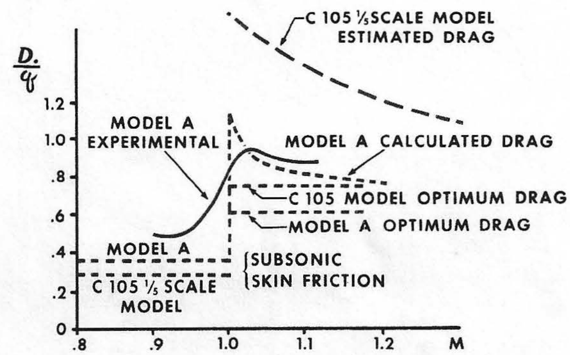
C 105 1/5 SCALE



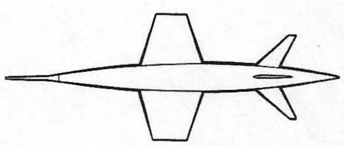
MODEL A



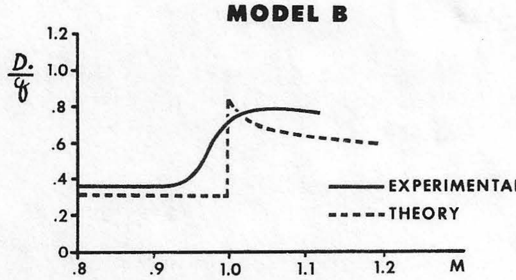
MODEL A



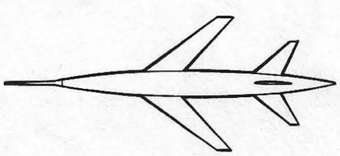
MODEL B



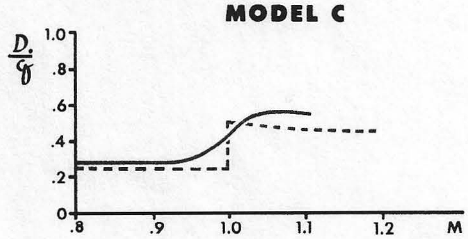
MODEL B



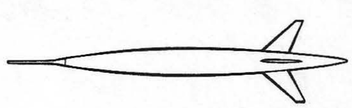
MODEL C



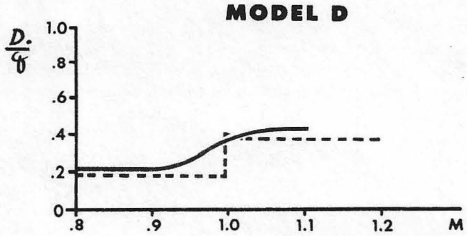
MODEL C



MODEL D



MODEL D



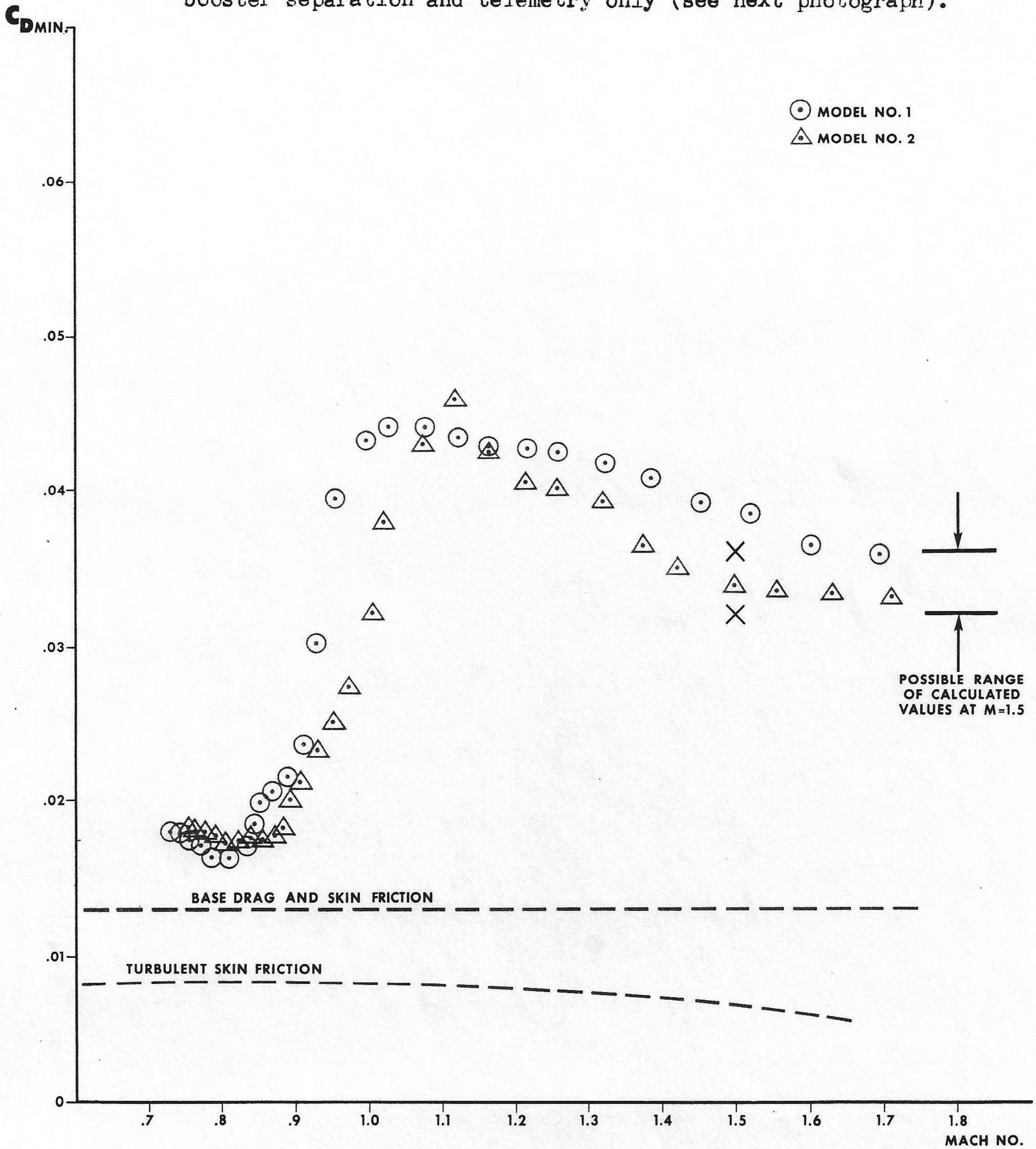
HOLDAWAY'S RESULTS

(N. A. C. A. RM A 53 H-17)

(NOTE:- 24 TERM FOURIER SERIES USED)

C-105 CRUDE MODEL TEST RESULTS

Note: These very crude models do not bear any dimensional relationship to the CF-105 except for general configuration, and were used to check launching techniques, booster separation and telemetry only (see next photograph).



C-105 FREE FLIGHT MODEL

CRUDE MODELS NOS. 1 & 2

$C_{D\ MIN.}$ VS MACH No.

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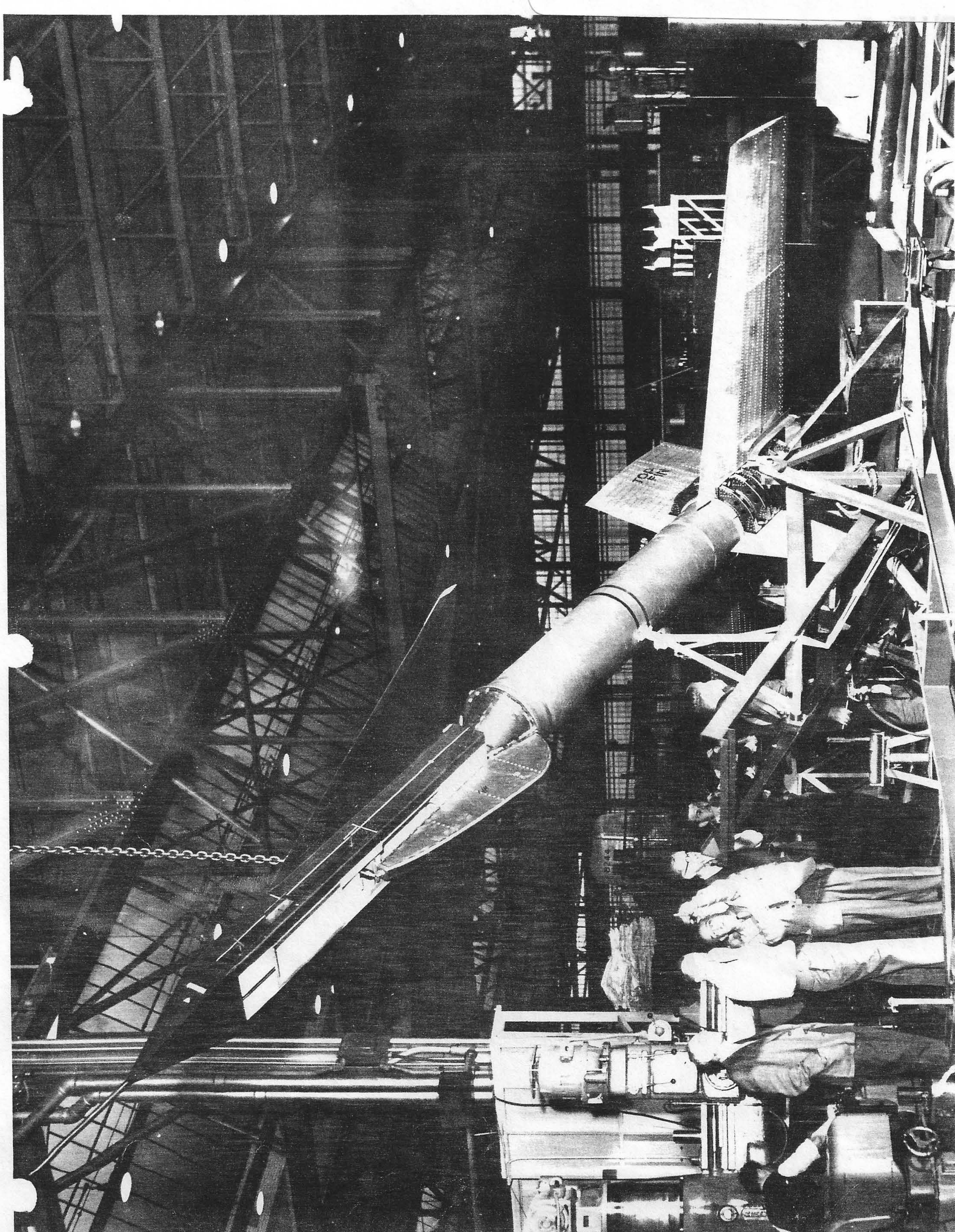

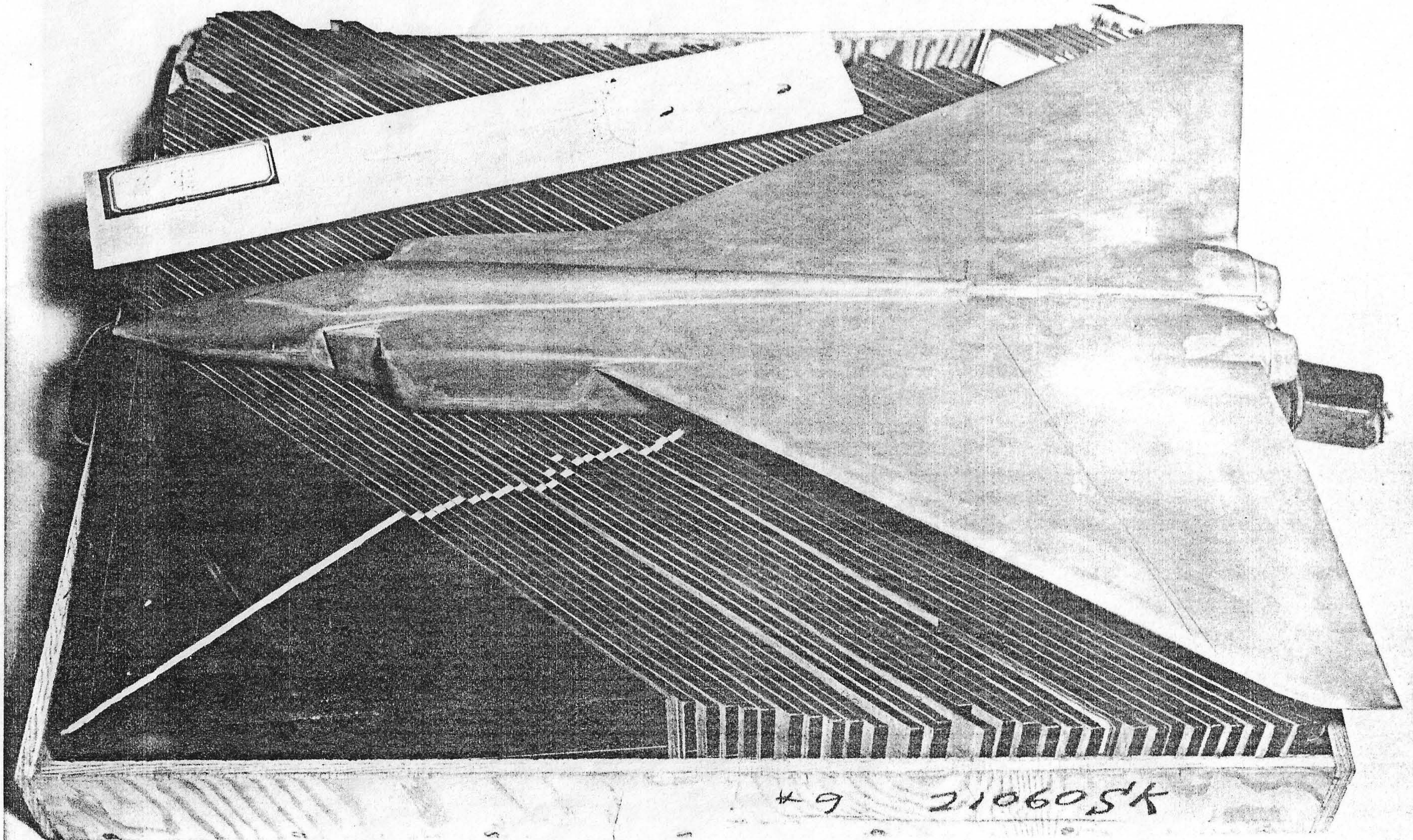

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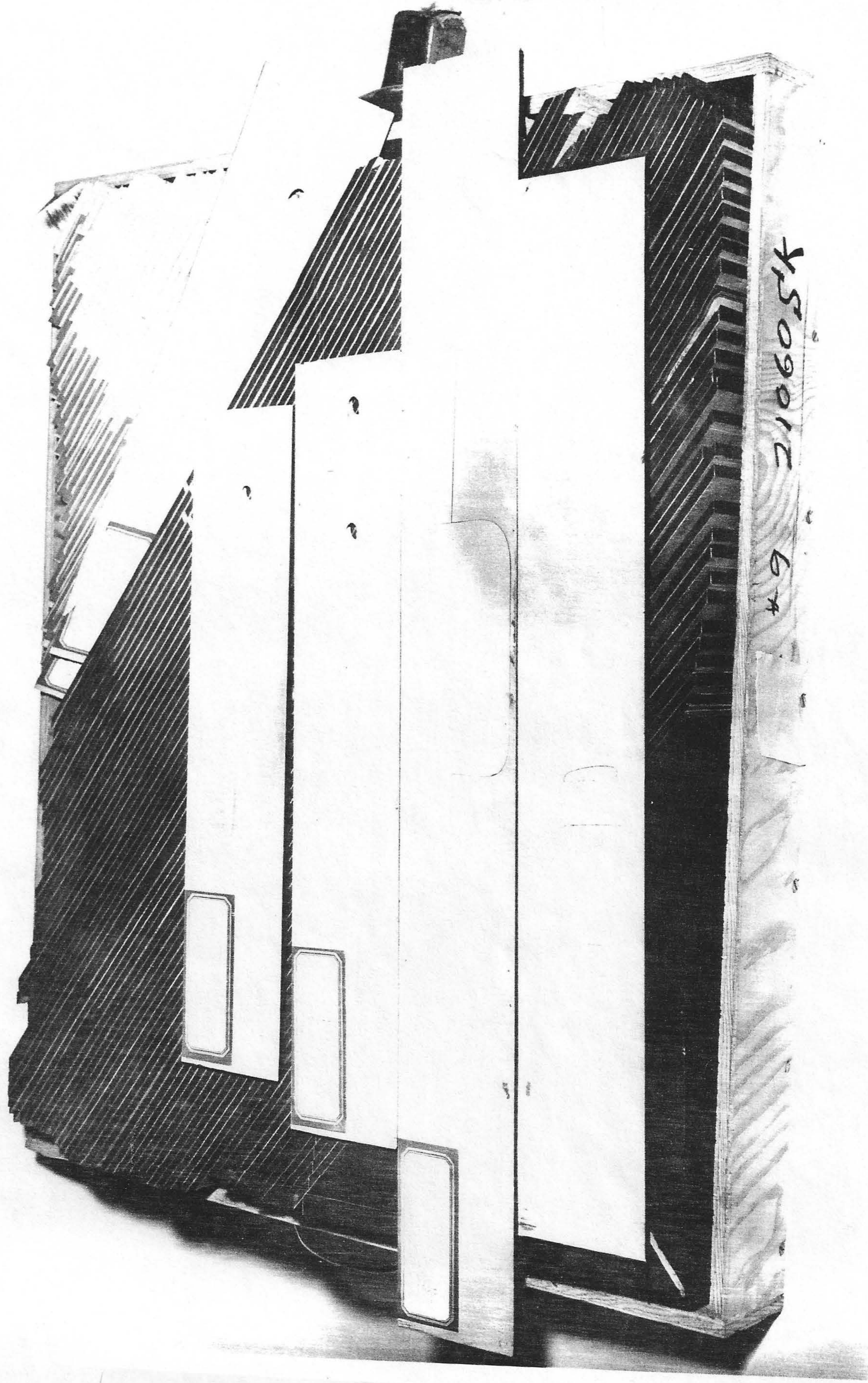

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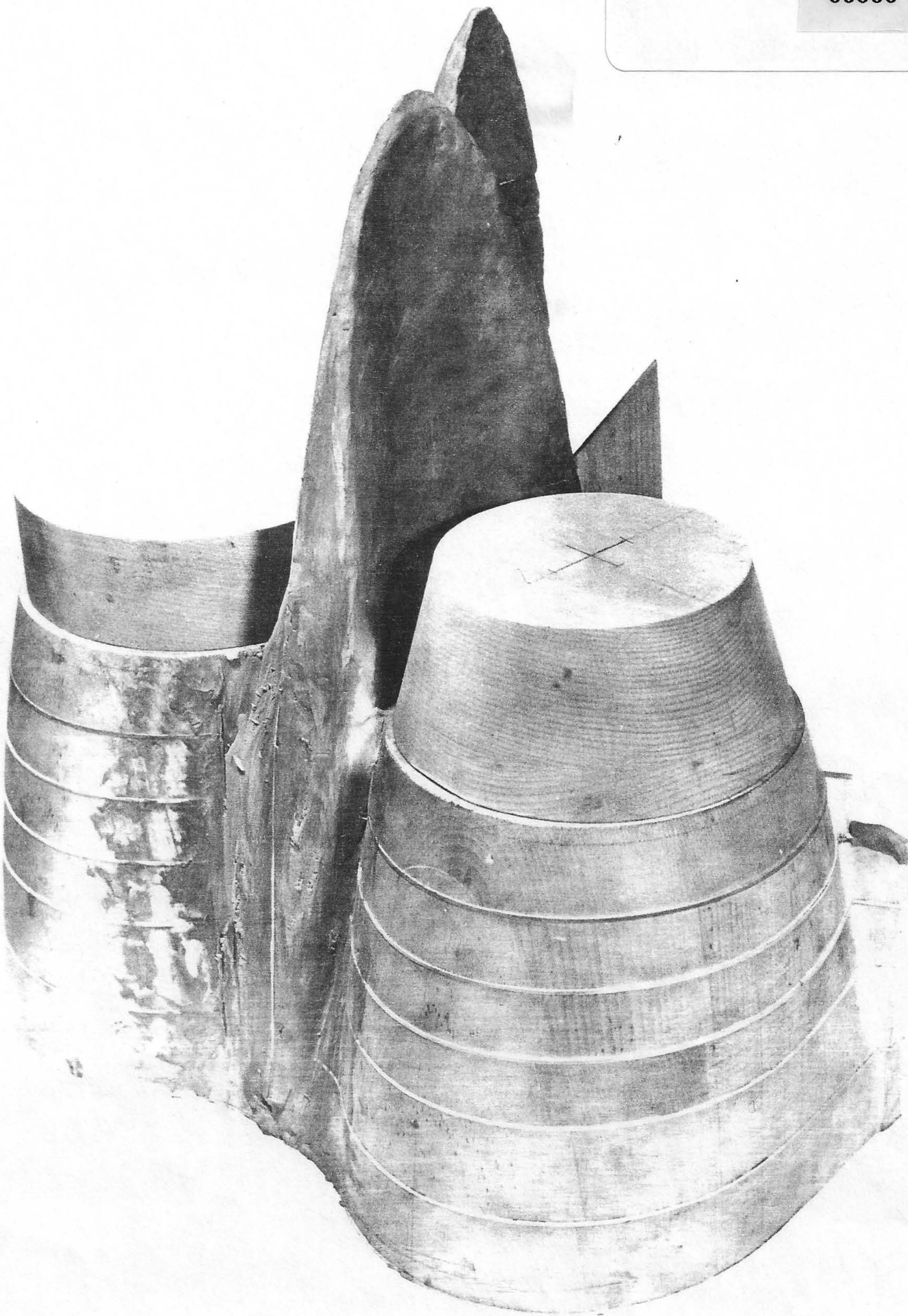

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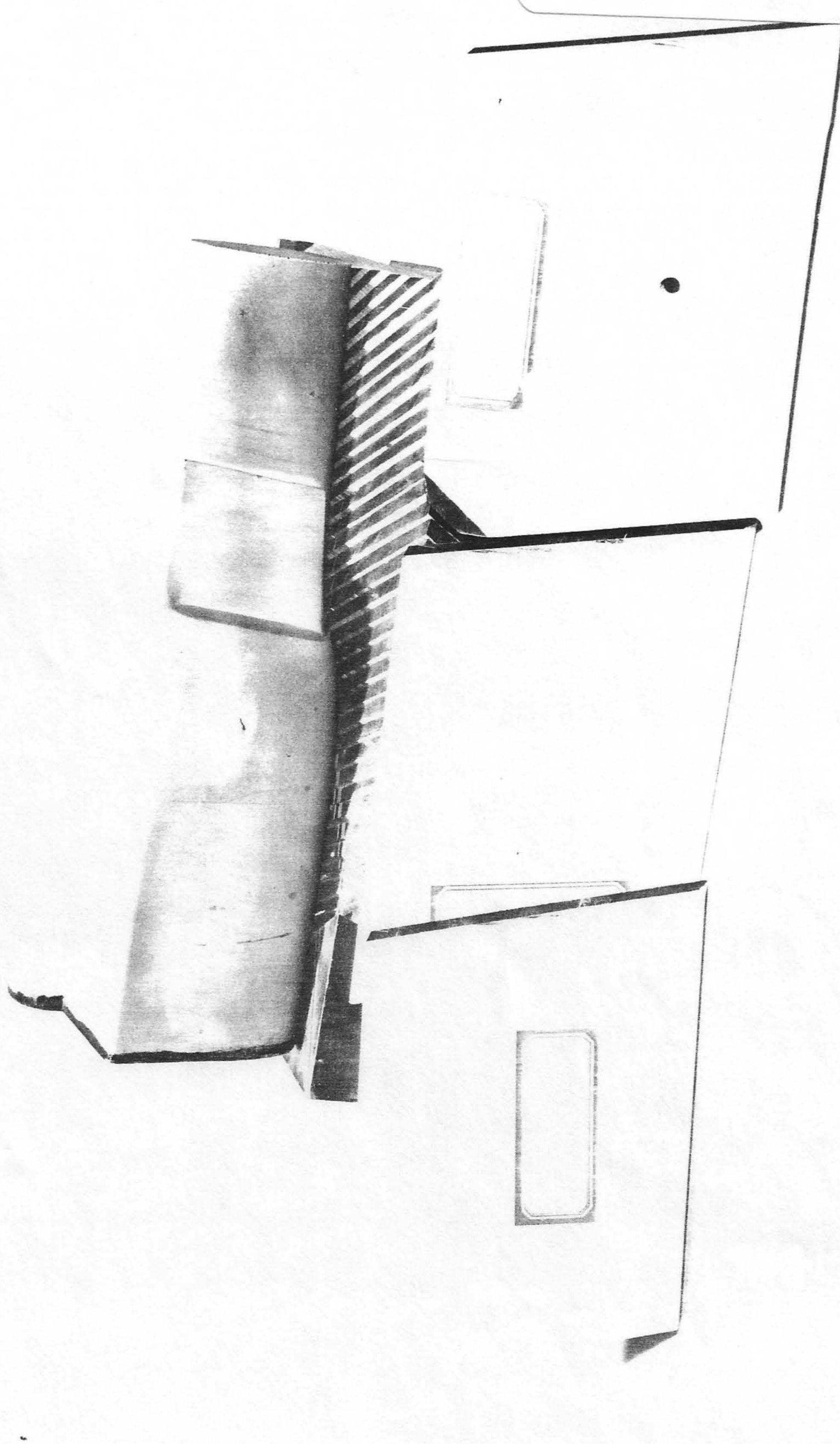

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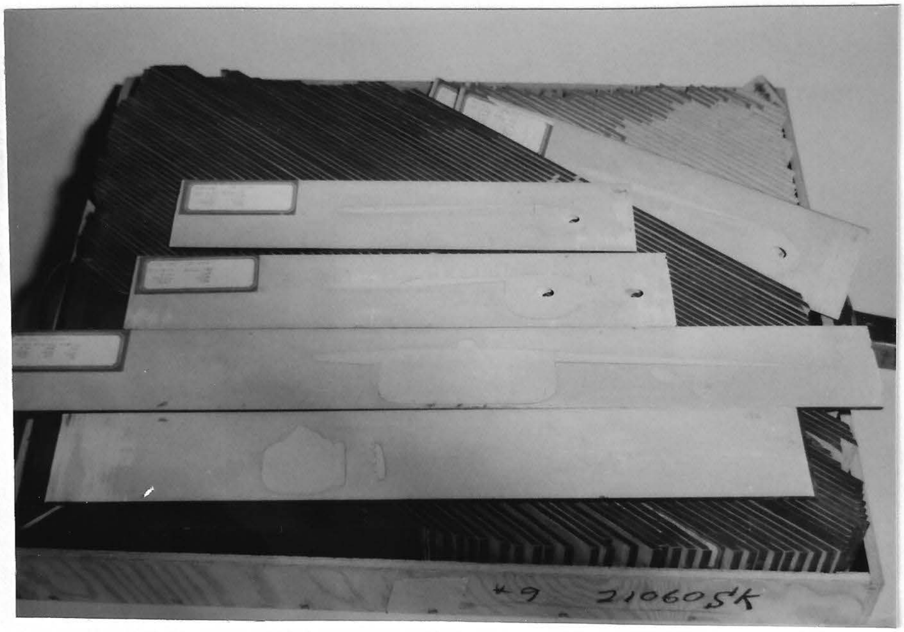


Fig Section cuts.
(Avro 58572)

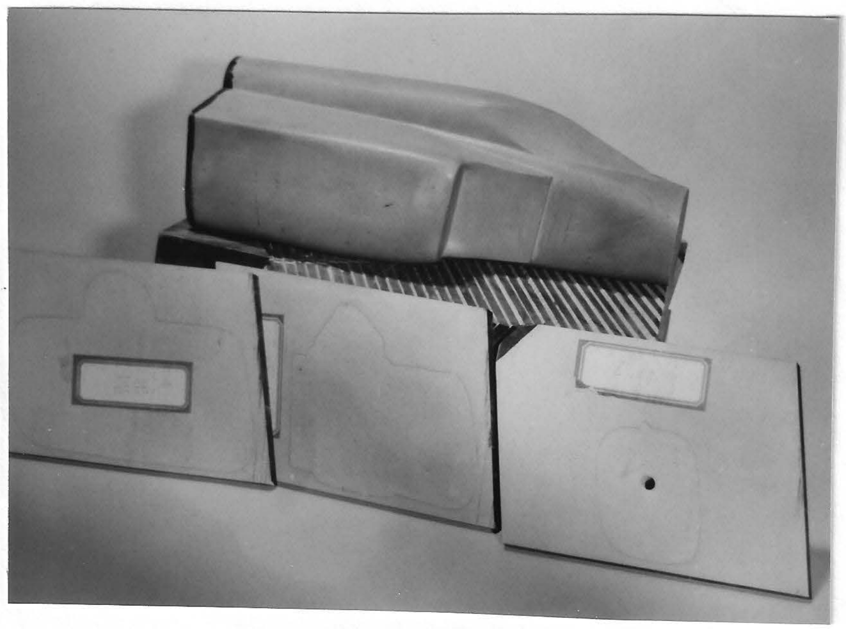


Fig 1/16 Scale model.
(Avro 58570)

Fig Boxed section cuts.
(Avro 58569)

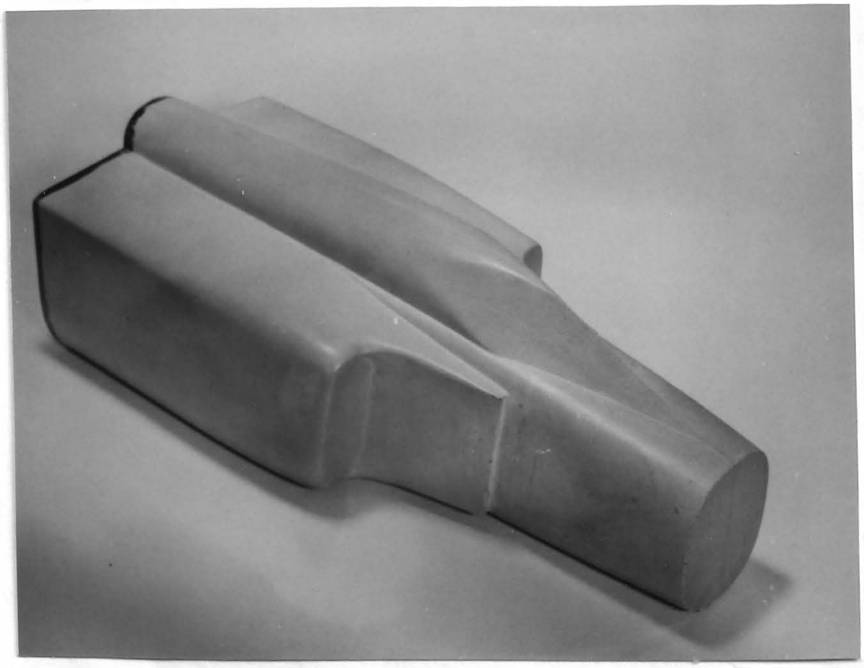


Fig 1/16 Scale model.
(Avro 58570)

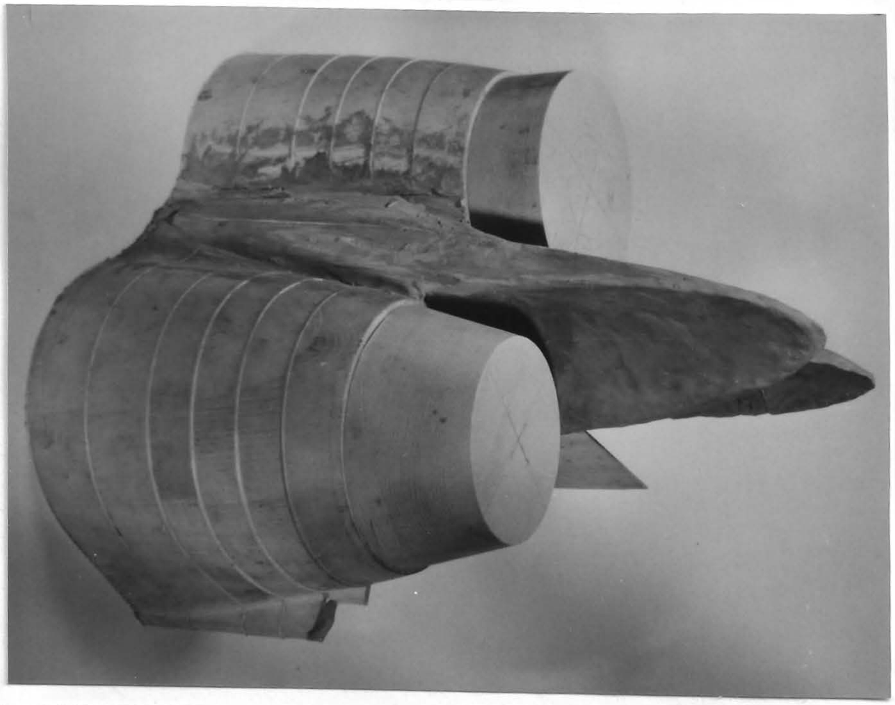
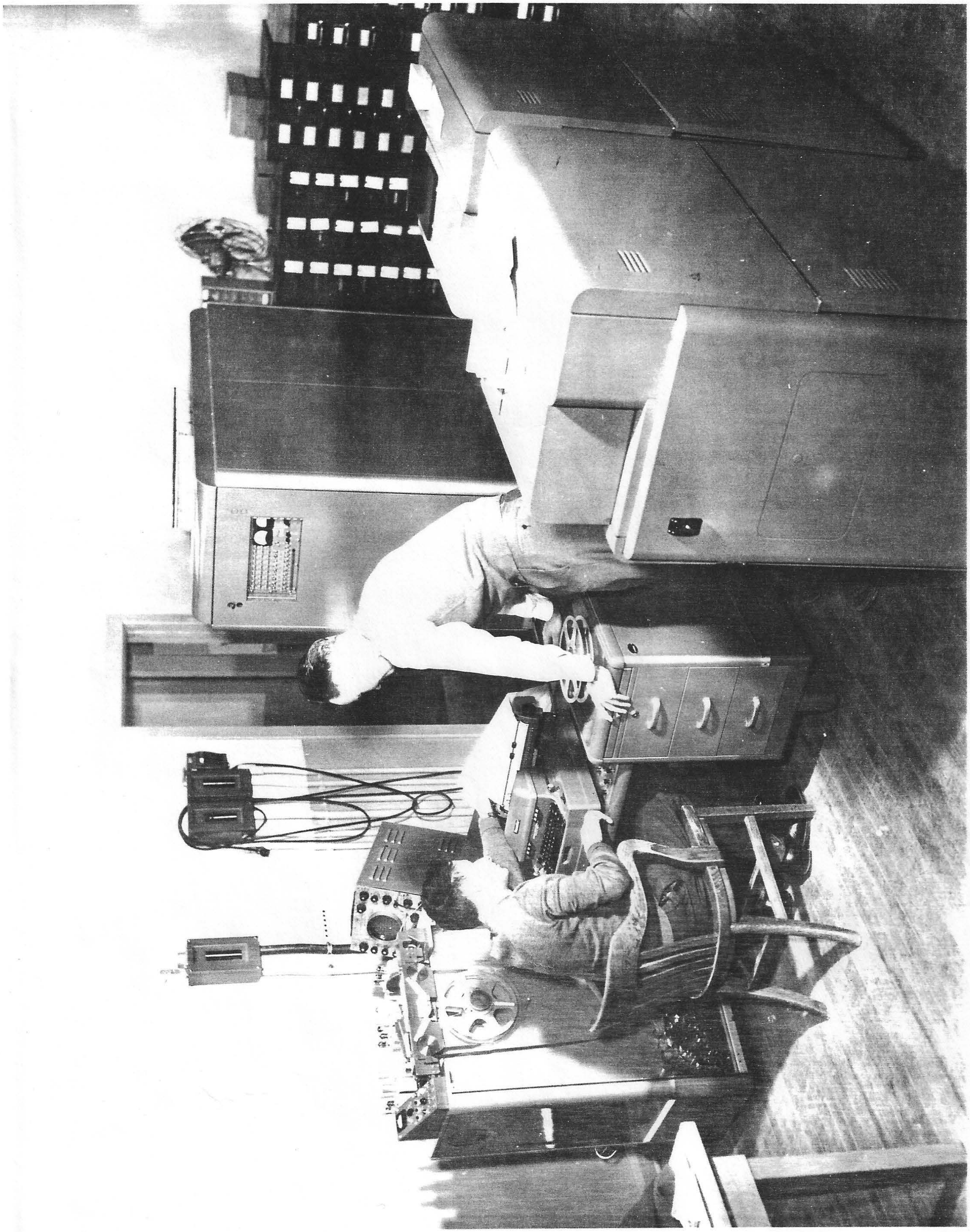


Fig Rear nacelle and stinger.
(Avro 58565)

Fig Cliff McIntosh measuring area
with a planimeter.
(Avro 58571)



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