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The Arrow was one of the first M-2 fighters designed

SOME PERFORMANCE PROBLEMS ASSOCIATED WITH THE MACH 2 FIGHTER

by John Morris

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SUMMARY

This paper considers the choice of configuration for a fighter aircraft designed for a combat speed of M = 2.0 from the viewpoint of performance efficiency. Particular attention is given to the influence of drag due to lift and trim drag on the choice of wing planform and type of longitudinal control.

1. INTRODUCTION

Looking beyond the next generation of fighter aircraft it is obvious that requirements will exist in the reasonable future for fighters having operational combat speeds of M = 2.0 and above. The object of this paper is to show how performance requirements might influence the shape of the aircraft designed for cruise and combat at M = 2.0. In making the analysis no account has been taken of the variations in weight implicit in the different configurations considered. A generalised approach to weight variation is difficult as specific requirements for a given aircraft usually dominate.

For the purpose of this paper it is assumed that the aircraft will have combat altitude of 60,000 feet and a 2 'g' level turn capability at combat speed. It is also assumed that it will be capable of landing and taking-off conventionally and this restriction limits the wing leading edge sweep to about 65° and aspect ratios to greater than 1.5.

Fighter aircraft are usually designed to a specification issued by the appropriate Government body which lays down minimum requirements on range, maximum level speed, ceiling and thrust limited turn capability at a fixed speed and altitude. In the old days there was usually a conflict between the top speed and ceiling requirements because the optimum wing loading to satisfy the top speed requirement was much higher than that for ceiling - the designer had therefore to compromise. The situation is different today because top speeds will be limited by thermal rather than thrust-drag considerations, it is possible then to optimise the aircraft at the ceiling or for manoeuvrability, whichever is the more critical.

If the ceiling requirement is more difficult than that for manoeuvrability it obviously pays to choose the wing loading to give maximum lift-drag ratio at the ceiling. If the manoeuvrability requirement is the more difficult, then range will influence whether the lift-drag ratio be maximised for level flight, at the specified manoeuvring load factor in the turn or at some intermediate load factor. Under these conditions drag due to lift is very important, for example, if an aircraft with a flat wing (i.e. no camber or twist) designed for a 2 'g' level turn has the lift-drag ratio maximised in level flight then the drag due to lift at 2 'g' is four times the zero lift drag and if the lift-drag ratio is maximised at 2 'g' then the lift drag is equal to that at zero lift.

It will be shown that trim drag can also be very important and that for the Mach 2 fighter, drag-due-to-lift is more important than zero-lift-drag and serious consideration must be given to minimising trim-drag.

In the next section the problem of trim drag will be dealt with.

2. TRIM DRAG

To have positive longitudinal static stability the centre of gravity of the aircraft must be in front of the aerodynamic centre which means that for trimmed flight the controls must be deflected to re-distribute the air loads so that the centre of lift passes through the centre of gravity and brings the aircraft into balance. This re-distribution of load which takes place primarily on the controls also changes the drag and the difference in drag between controls deflected and undeflected at a given lift coefficient is called the trim drag. Methods for calculating the trim drag at supersonic speeds for tailless and tailled configurations are given in the Appendix to this paper.

The drag coefficient of an aircraft with a flat wing can be expressed as:-

$$C_D = C_{D_o} + K_w C_L^2 + C_{D_t}$$

For configurations of practical interest the trim drag coefficient for a tailless aircraft is given at Mach number greater than about 1.9 by the following approximate formula:-

$$C_{D_t} = \frac{1}{\frac{dC_L}{d\delta}} \left(\frac{h - h_o}{h_o - h_\delta} \right)^2 C_L^2$$

These two equations can be combined to give

$$C_D = C_{D_o} + K_{eff} C_L^2$$

where

$$K_{eff} = K_w + \frac{1}{\frac{dC_L}{d\delta}} \left(\frac{h - h_o}{h_o - h_\delta} \right)^2$$

The trend in tailless aircraft for operation at supersonic speeds seems to be towards the delta planform with aspect ratio $\simeq$ 2.0 and the effect of trim drag on the effective lift drag-rise factor K_{eff} will be illustrated by taking this planform as an example.

A typical variation of aerodynamic centre (h_o) with Mach number for an aspect ratio 2 delta is shown in fig. 1, and it will be noted that at supersonic speeds it is located at 50% of the mean chord. The location of the centre of elevator lift (h_δ) will, at supersonic speeds, be close to the centre of area of the control, - for practical configurations it will be around 90 - 95% of the mean chord. The following formula gives a reasonable approximation to the elevator lift slope at supersonic speeds:-

$$\frac{dC_L}{d_\delta} = \frac{3.5}{\sqrt{M^2 - 1}} \frac{S_f}{S}$$

The effective lift drag-rise factor for a tailless aircraft with an aspect ratio 2 delta wing can then be expressed (for $M > 1.9$) as:-

$$K_{eff} = K_w + 1.4\sqrt{M^2-1}\,\frac{S}{S_f}\,(h-0.5)^2$$

The two main parameters that influence trim drag at supersonic speeds are then, control area ratio and the location of the centre of gravity, which is what one would expect. Some examples which illustrate the effect of centre of gravity location and S_f/S on K_{eff} are shown in fig. 2. Also shown in this figure are typical values for a canard and an aircraft with a rear tail. We see that if the centre of gravity is located at 34% of the mean chord to give a static margin of 6% at subsonic speeds then the effective induced drag (i.e. drag due to lift plus trim drag) at $M = 2.0$ will be more than doubled with the part span control. Increasing the span of the control with the same centre of gravity position produces appreciable gains.

However, the reason for the high trim drag of the tailless aircraft in supersonic flight is the large increase in static margin resulting from the after movement of the aerodynamic centre. The obvious way to eliminate the problem is to move back the centre of gravity of the aircraft to maintain the same static margin as in subsonic flight. Since the proportion of total weight carried in the form of fuel will be high, it is possible to control this centre of gravity position by appropriate transfer of fuel. There are, of course, the practical problems associated with an installation of this type but in view of the very high incentive there is no doubt that these can be overcome and fig. 2 shows how the tailless aircraft with the controllable centre of gravity is comparable to the aircraft with a rear tail on the basis of trim drag. The effective lift drag rise factor of the canard is appreciably less than that of either of the two other configurations considered, and is in fact, less than the lift drag rise factor of the untrimmed aircraft wing, as in this case the trimming load is acting upwards and effectively reducing the load to be carried by the wing.

Although outside the particular scope of this paper it may well be asked why, in view of the foregoing, the delta planform has proved to be so popular for the present generation of supersonic fighters. Here again it is necessary to point out that trim drag is only one parameter in determining performance efficiency, and performance efficiency is only one aspect of the problem of designing an effective fighter aircraft. Practical considerations and weight variations can more than offset a result determined by considerations of performance efficiency alone.

3. DRAG DUE TO LIFT

Experiments show that the lift drag-rise factor for flat wings with supersonic leading edges is given to a close enough approximation for the purposes of this paper by the reciprocal of the lift slope i.e.

$$K_w = \frac{1}{\frac{dC_L}{d\alpha}}$$

A comparison of experimental values of the lift slope and these calculated by theory show good agreement.

At a Mach number of 2 the Mach cone is swept back at 60° to the normal to the flight path; we can therefore estimate the lift drag rise factor, using the theoretical lift slopes, for wings with leading edge sweep up to at least 60°.

The lift drag rise factor has been estimated over a range of aspect ratios from 1.5 to 4.0 and leading edge sweep up to 60° and is shown plotted in fig 3. The effect of taper ratio was found to be small and the curves in the figure can be taken as applying to taper ratios from zero to about one half. The lift drag-rise factor is almost constant over a wide range of sweep and aspect ratio; small gains might be achieved with wings of high aspect ratio and sweep but the very large wing weight penalties associated with the type of planform would discourage their use.

It would appear then that for flat wings the choice of wing planform will be dictated by considerations other than those of drag due to lift.

4. ZERO LIFT DRAG

It is difficult to generalise about the effect of wing planform on the zero lift drag at a Mach number of 2.0. We know from experiments on isolated wings that the zero lift drags do not agree very well with theory, particularly when the wing leading and trailing edges are near sonic. The wave drag of wing-body combinations can be calculated by methods such as the supersonic area rule but to the author's knowledge no comparison has been made between theory and experiment that would enable us to assess their accuracy at $M = 2.0$. We will content ourselves therefore with the following general observations based on a survey of experimental data on wing-body combinations.

1. The effect of sweepback for a given aspect and taper ratio is small; unswept wings having slightly less drag than the swept.
2. Taper ratio between 0 - 0.25 has no appreciable effect.
3. Increasing aspect ratio produces significant increase in drag.

It will be noted that the three conditions for minimum wing structure weight i.e. small sweepback, taper and aspect ratio satisfy the conditions for minimum zero lift drag coefficient.

Aircraft with small sweepback must have a tail for stability reasons and this will increase the minimum drag coefficient. The tailless aircraft incurs a drag penalty due to the sweepback necessary for stability. A comparison made by the author of a tailless and a tailled configuration with wings of the same aspect and taper ratio showed that the tailless aircraft had a zero lift drag coefficient that was 5% lower than that with a tail.

5. COMPARISON OF LIFT-DRAG RATIOS

Since we are considering the different configurations from the viewpoint of performance efficiency the maximum lift/drag ratio has been selected as the appropriate index by which to make the comparison. Figure 4 shows the comparison for the three configurations selected, namely, canard, tail aft and tailless. In the case of the configurations with tails the half chord sweep of the wing is zero and the tailless configuration has the delta planform. All have aspect ratios 2 wings with zero taper ratio.

The lift/drag ratio of the canard is about 6% better at a Mach number of 2.0 than the tailless with the centre of gravity located at 44% of the chord, and 8% better than the tail aft configuration. If the centre of gravity of the tailless configuration is not moved back at supersonic speeds its lift/drag ratio will be appreciably lower than those with a tail.

canard 28% better than Δ, cg ·34 part span elevator

It is interesting to note that from the viewpoint performance efficiency the configurations considered lie within an 8% spread.

6. WING LOADING

It is obvious that to minimise thrust and fuel consumption the lift coefficient for maximum lift/drag ratio should be made to occur between 1.0 'g' and the specified load factor at combat speed and altitude. It can easily be shown that in this case the wing loading is given by the following expression -

$$\frac{W}{S} = \left(\frac{C_{D_o}}{K_{eff}}\right)^{1/2} \frac{\gamma p M^2}{2 n_M}$$

where n_M is the load factor at which the lift/drag ratio is maximised and p is the ambient pressure. The variation of $^W/_S$ with n_M for M = 2.0 at 60,000 feet is given in fig 5; the values of the zero lift drag coefficient cover the likely range. The optimum wing loading should then be between 35 - 90 lbs/sq.ft.

The effect of wing loading on thrust required at 2.0 'g' is shown in fig 6, where the thrust required at a given wing loading divided by the thrust required for a wing loading of 85 lbs/sq.ft. is given as a function of wing loading. When compiling the data for fig 6 it was assumed that the fuselage shape and size remained fixed whilst the wing area was varied and allowances were made for variations in wing, fuel and engine weight. It was also assumed that the wing planform had zero half chord sweep, aspect ratio 2, zero taper ratio and 3% thickness chord ratio, but it is believed that a different planform would not produce significantly different results. We see that minimum thrust occurs at wing loadings between 40 - 55 lbs/sq.ft.

In considering the effect of range on optimum wing loading it has been necessary to select suitable values. Since we are discussing a very advanced aircraft with a cruising speed of Mach ≈ 2.0 it may well be that mission radii will be short, and values of 100 and 200 nautical miles have been selected. The fuel required for these missions is shown as a function of wing loading in figure 7. The variation of fuel required with wing loading is not large and a minima occurs at a wing loading of approximately 50 lbs/sq.ft.

The landing problem has some influence on the possible range of wing loading for different configurations. The maximum wing loading for the low aspect ratio tailless is not likely to exceed 45 lbs/sq.ft. due to limitations on ground angle.

The tail aft aircraft will, if flaps are used, have acceptable landing speeds and ground angles at wing loadings up to 90 lbs/sq.ft. and the canard could achieve similar loadings.

7. PERFORMANCE AT MACH NUMBERS LESS THAN 2.0

Up to this point we have concentrated on optimising the aircraft at a Mach number of 2.0 but performance at off-design conditions might influence the choice of wing planform. The parameter that has been chosen to illustrate the effect of wing planform at off-design Mach numbers is ceiling as this is most important to a high altitude fighter.

It is generally believed that aircraft with swept wings have superior performance to those with unswept wings at transonic speeds. A comparison was therefore made between an aircraft with an unswept wing and one swept at 40° (half chord sweep) both of aspect ratio 3, and the difference in ceiling at any Mach number below 2.0 was found to be small (see fig 8). The reason for this is that the aircraft with the lowest value of $\left(\frac{K_{eff}}{C_T - C_{D_o}}\right)^{\frac{1}{2}}$ at a given Mach number will have the highest ceiling, and at transonic speeds the thrust coefficient of the Mach 2 aircraft will be about six times as large as the zero lift drag coefficient. A 25% increase in C_{D_o} changes the ceiling parameter $\left(\frac{K_{eff}}{C_T - C_{D_o}}\right)^{\frac{1}{2}}$ by only 2 1/2%; a 25% increase of K_{eff} would change it by 11 1/2%.

The above example shows that the lift drag-rise factor is much more important than the zero lift drag coefficient in its effect on high altitude performance at off-design Mach numbers.

We know that at subsonic speeds the lift drag-rise factor can be reduced by increasing aspect ratio and a comparison has been made of the performance of aircraft with unswept wings of aspect ratio 2, 3 and 4 (see fig 9). Increasing the aspect ratio from 2 to 4 increases the ceiling by 6,500 ft. at a Mach number of 1.0. The thrust and weight penalties associated with increasing aspect ratio are large; the aircraft with the aspect ratio 4 wing would require about 20% more thrust and would be about 15% heavier than that with the aspect ratio 2 wing. If the thrust of the aspect ratio 2 aircraft were increased by 20% the ceiling would beincreased by 2,500 ft. at M = 2.0 and it s transonic and subsonic performance would be roughly equal to that with aspect ratio 3.

8. CONCLUSIONS

This investigation has shown that from the viewpoint of performance and efficiency there is little to choose between the canard, tail aft and tailless configurations for an aircraft with 2 'g' level turn capability at M = 2.0 at 60,000 feet. The highest efficiency is found in a canard with a wing loading of 55 lbs/sq.ft. It would have an unswept wing with low aspect ratio probably limited by landing considerations to above 1.5. The tail aft aircraft would be some 8% less efficient than the canard and the tailless aircraft, provided it has provision for moving the centre of gravity in flight would be 6% less efficient than the canard.

From this it may be concluded that other considerations resulting from the particular aircraft specification will almost certainly determine the configuration to be selected.

SYMBOLS

L	lift
D	drag
N	normal force
L_T	lift on tailplane
D_T	drag of tailplane
N_T	normal force on tailplane
C_D	$\frac{D}{q S}$
C_L	$\frac{L}{q S}$
C_N	$\frac{N}{q S}$
C_M	$\frac{M}{q S \bar{c}}$
C_{D_T}	$\frac{D_T}{q S_T}$
C_{L_T}	$\frac{L_T}{q S_T}$
C_{N_T}	$\frac{N_T}{q S_T}$
C_T	thrust coefficient $\frac{T}{q S}$
C_{D_t}	trim drag coefficient
C_{L_w}	lift coefficient of aircraft less tail
C_{D_o}	zero lift drag coefficient
$C_{D_{oT}}$	increment in a/c zero lift drag in terms of the tail area
C_M^{δ} =	$C_{M_o} + (h - h_o) C_L$
C_{M_o}	pitching moment at zero lift for aircraft less tail
ΔC_{D_c}	drag coefficient of control
ΔC_{L_c}	lift on control/q S
ΔC_{L_w}	lift on complete aircraft due to control deflection

SYMBOLS (Cont'd)

K_2	$= \dfrac{\Delta C_{L_c}}{\Delta C_{L_w}}$
K_w	lift drag-rise factor
K_{eff}	effective lift drag-rise factor
a_1	$\dfrac{\partial C_L}{\partial \alpha}$
a_2	$\dfrac{\partial C_L}{\partial \delta}$
a	$\dfrac{\partial C_L}{\partial \alpha}$ less tail
a_T	$\dfrac{\partial C_{L_T}}{\partial \alpha_T}$
$h\bar{c}$	centre of gravity location
h_o	aerodynamic centre
h_δ	centre of elevator lift
S_f	area of control
S_T	area of tail
S	area of wing
A	wing aspect ratio
$\bar{c}$	wing aerodynamic mean chord
ℓ_T	distance from 1/2 chord point of tail to c.g. of aircraft
ℓ_o	distance from 1/2 chord point of tail to h_o
M	Mach number
W	weight of aircraft
p	ambient pressure
n	load factor
e	induced drag factor

SYMBOLS (Cont'd)

α	wing incidence
α_T	incidence of tail relative to wing chord
ε	angle of downwash relative to wing chord
γ	ratio of specific heats
δ	control angle
q	$1/2 \rho V^2$

Corrections
O Michaelson
July /56.

APPENDIX

Approximate methods for estimating trim drag

1. Tailless Aircraft

It can easily be shown that the theoretical drag of a two dimensional airfoil at supersonic speeds can be expressed as

$$C_D = C_{D_o} + \frac{\partial C_D}{\partial \alpha^2} \alpha^2 + \frac{\partial C_D}{\partial (\delta + \alpha)^2} \delta^2 + \frac{\partial C_D}{\partial (\delta + \alpha)^2} 2\delta\alpha \qquad (1)$$

and the equation will have the same form in the three dimensional case.

The drag of the control surface of a two dimensional wing in an inviscid flow is equal to the component of the normal force on the control in the flight direction i. e.

$$\Delta C_{D_c} = \Delta C_{L_c} (\delta + \alpha) \cos\theta$$

$$\text{and } \Delta C_{D_c} = \frac{\partial C_L}{\partial \delta} (\delta + \alpha)^2 \cos\theta$$

$$\text{therefore } \frac{\partial C_D}{\partial (\delta + \alpha)^2} = \frac{\partial C_L}{\partial \delta} \cos\theta$$

In the three dimensional case

$$\Delta C_{D_c} = \frac{\Delta C_{L_c}}{\Delta C_{L_w}} \Delta C_{L_w} (\delta + \alpha) \cos\theta$$

where ΔC_{L_c} is the lift coefficient of the load on the control and ΔC_{L_w} is the lift coefficient of the load on the control plus any load induced on the wing by control deflection.

$$\therefore \frac{\partial C_D}{\partial (\delta + \alpha)^2} = K_2 \frac{\partial C_L}{\partial \delta} \cos\theta \qquad (2)$$

where $K_2 = \dfrac{\Delta C_{L_c}}{\Delta C_{L_w}}$

and K_2 can be estimated by linearised wing theory.

It is sometimes convenient to re-arrange eqn (1) by substituting for $\dfrac{\partial C_D}{\partial (\delta + \alpha)^2}$ from eqn (2) and for α from the lift equation

$$C_L = a_1 \alpha + a_2 \delta$$

we then have

$$C_D = C_{D_o} + \frac{C_L^2}{e\pi A} + \left(\frac{K_2}{a_2} + \frac{1}{e\pi A} - \frac{2K_2}{a_1}\right)(a_2\delta)^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right)2C_L a_2\delta \quad (3)$$

The first two terms in the above equation represent the zero lift drag coefficient and the conventional induced drag and what remains we define as the trimming drag coefficient C_{D_t}.

The pitching moment due to the controls at constant lift coefficient is $(h_o - h_\delta)\, a_2\delta = C_M{}^\delta$ and substituting for $a_2\delta$ in equation (3) gives

$$C_{D_t} = \left(\frac{K_2}{a_2} - \frac{2K_2}{a_1} + \frac{1}{e\pi A}\right)\left(\frac{C_M{}^\delta}{h_o - h_\delta}\right)^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right)\frac{C_M{}^\delta\, 2C_L}{(h_o - h_\delta)} \quad (4)$$

and in trimmed flight $-\,C_M{}^\delta = C_{M_o} + (h - h_o)C_L$
if $C_{M_o} - 0$ equation (4) becomes

$$C_{D_t} = \left(\frac{K_2}{a_2} - \frac{2K_2}{a_1} + \frac{1}{e\pi A}\right)\left(\frac{h - h_o}{h_o - h_\delta}\right)^2 C_L^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right)\left(\frac{h - h_o}{h_o - h_\delta}\right) C_L^2 \quad (5)$$

At Mach numbers greater than about 1.9

$$C_{D_t} \simeq \frac{K_2}{a_2}\left(\frac{h - h_o}{h_o - h_\delta}\right)^2 C_L^2$$

2. Tail Aft Aircraft

The aerodynamic incidence of the tail $= \alpha + \alpha_T - \varepsilon$ and its incidence relative to the flight path $= \alpha + \alpha_T$

If we assume the resultant force on the tail is normal to the chord and that $C_{N_T} \simeq C_{L_T}$ the drag coefficient of the tail is

Not necessarily, can assume tail induced drag as $C_{DiT} = K_T C_{LT}^2$

$$C_{D_T} = C_{D_{oT}} + C_{L_T}(\alpha + \alpha_T) \quad (1)$$

∴ Drag due tail in flight path direction: $C_{DT} = C_{DoT} + K_T C_{LT}^2 \cos\varepsilon + C_{LT}\sin\varepsilon$

The tail lift coefficient can be expressed as

$$C_{L_T} = a_T(\alpha + \alpha_T - \varepsilon)$$

But $K_T = \frac{1}{a_T}$ ∴ $C_{DT} = C_{DoT} + C_{LT}\left(\frac{C_{LT}}{a_T} + \varepsilon\right)$

and substituting for $\alpha + \alpha_T$ in eqn (1) we get

No!; $\alpha + \alpha_T = \frac{C_{LT}}{a_T} + \varepsilon$

$$C_{D_T} = C_{D_{oT}} + C_{L_T}\left(\frac{C_{L_T}}{a_T} - \varepsilon\right) = C_{D_{oT}} + C_{L_T}\left(\frac{C_{L_T}}{a_T} - \frac{d\varepsilon}{d\alpha}\alpha\right) \quad (2)$$

The lift coefficient of the aircraft less tail $C_{L_w} = a\alpha$ and substituting for α in eqn (2) gives

$$C_{D_T} = C_{D_{oT}} + C_{L_T}\left(\frac{C_{L_T}}{a_T} - \frac{d\varepsilon}{d\alpha}\frac{C_{L_w}}{a}\right)$$

The drag coefficient of the aircraft less tail

$$C_D = C_{D_o} + K_w C_{L_w}^2$$

and the drag coefficient of the complete aircraft is then

$$C_D = C_{D_o} + C_{D_{oT}} \frac{S_T}{S} + K_w C_{L_w}^2 + C_{L_T} \left(\frac{C_{L_T}}{a_T} - \frac{d\varepsilon}{d\alpha} \frac{C_{L_w}}{a} \right) \frac{S_T}{S} \quad \text{---(3)}$$

The lift coefficient of the complete aircraft

$$C_L = C_{L_w} + C_{L_T} \frac{S_T}{S} \quad \text{---(4)}$$

and substituting for C_{L_w} in eqn (3) and re-arranging gives

$$C_D = C_{D_o} + C_{D_{oT}} \frac{S_T}{S} + K_w C_L^2 + C_{L_T}^2 \left(\frac{S_T}{S} \right)^2 \left(\frac{1}{a_T} \frac{S}{S_T} + K_w + \frac{d\varepsilon}{d\alpha} \frac{1}{a} \right)$$

$$- C_{L_T} \frac{S_T}{S} \left(2K_w C_L + \frac{d\varepsilon}{d\alpha} \frac{C_L}{a} \right) \quad \text{---(5)}$$

The pitching moment coefficient of the complete aircraft

$$C_M = C_{M_o} + (h - h_o) C_{L_w} - \frac{\ell_T}{\bar{c}} C_{L_T} \frac{S_T}{S}$$

and substituting for C_{L_w} from eqn (4) and re-arranging gives

$$C_M = C_{M_o} + (h - h_o) C_L - \frac{\ell_o}{\bar{c}} \frac{S_T}{S} C_{L_T}$$

In trimmed flight $C_{L_T} \frac{S_T}{S} = \frac{\bar{c}}{\ell_o} \left(C_{M_o} + (h - h_o) C_L \right) \equiv \frac{\bar{c}}{\ell_o} \overline{C_M}$

and if we substitute for $C_{L_T} \frac{S_T}{S}$ in eqn (5) and re-arrange then

$$C_D = C_{D_o} + C_{D_{oT}} \frac{S_T}{S} + \left(\frac{\bar{c}}{\ell_o} \overline{C_M} \right)^2 \left(\frac{1}{a_T} \frac{S}{S_T} + K_w + \frac{d\varepsilon}{d\alpha} \frac{1}{a} \right) + K_w C_L^2$$

$$- \overline{C_M} \frac{\bar{c}}{\ell_o} \left(2K_w C_L + \frac{d\varepsilon}{d\alpha} \frac{C_L}{a} \right) \quad \text{---(6)}$$

if $C_{M_o} = 0$ then

$$C_D = C_{D_o} + C_{D_{oT}} \frac{S_T}{S} + \left[X^2 \left(\frac{1}{a_T} \frac{S}{S_T} + K_w + \frac{d\varepsilon}{d\alpha} \frac{1}{a} \right) - X \left(2K_w + \frac{d\varepsilon}{d\alpha} \frac{1}{a} \right) + K_w \right] C_L^2$$

where $X = (h - h_o) \frac{\bar{c}}{\ell_o}$

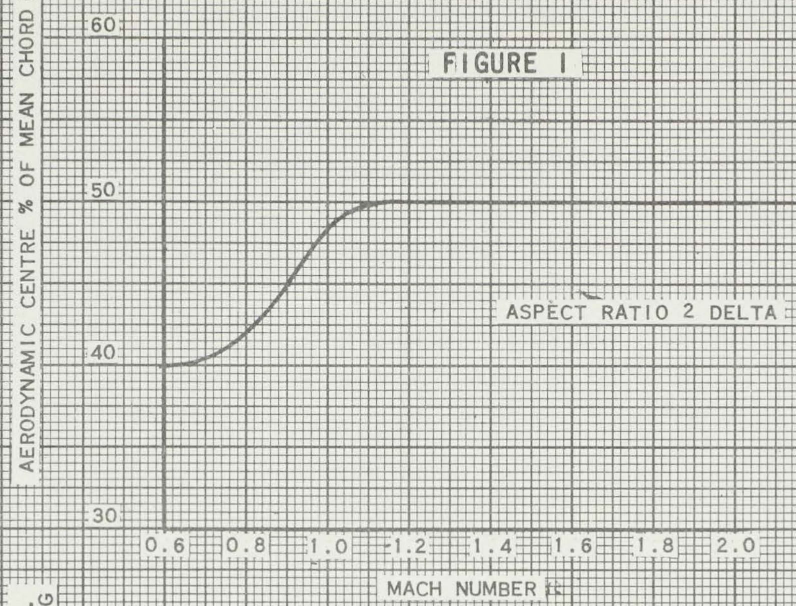

FIGURE I
AERODYNAMIC CENTRE % OF MEAN CHORD
60
50
40
30
ASPECT RATIO 2 DELTA
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
MACH NUMBER

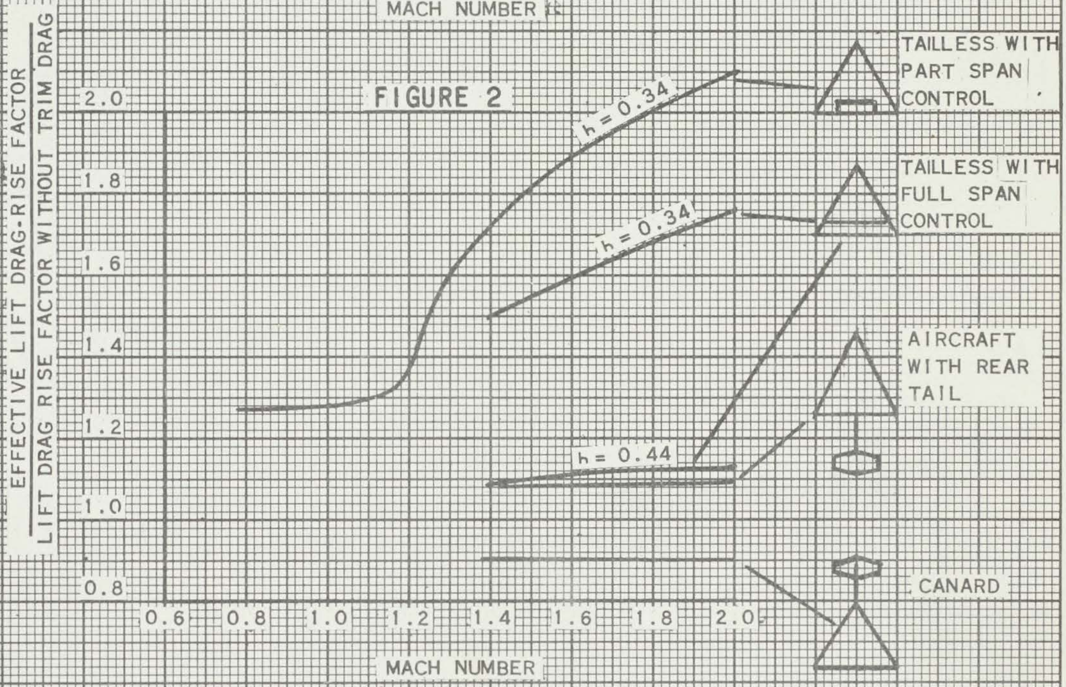

FIGURE 2
EFFECTIVE LIFT DRAG-RISE FACTOR
LIFT DRAG RISE FACTOR WITHOUT TRIM DRAG
2.0
1.8
1.6
1.4
1.2
1.0
0.8
h = 0.34
h = 0.34
h = 0.44
TAILLESS WITH PART SPAN CONTROL
TAILLESS WITH FULL SPAN CONTROL
AIRCRAFT WITH REAR TAIL
CANARD
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
MACH NUMBER