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PRELIMINARY NOTE ON A METHOD OF ESTIMATING TRIM DRAG AT SUPERSONIC SPEEDS

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It can easily be shown that the theoretical drag, at supersonic speeds, of a two dimensional airfoil with a control surface is as follows:

$$C_D = C_{D_0} + \frac{\partial C_D}{\partial \alpha^2}\alpha^2 + \frac{\partial C_D}{\partial(\delta+\alpha)^2}\delta^2 + \frac{\partial C_D}{\partial(\delta+\alpha)^2}2\delta\alpha \quad \text{———} (1)$$

Obviously the equation will have a similar form in the three dimensional case.

The first thing to establish is that the equation has the right form when compared with experimental results. The data of RM A52L04 has been used to make this comparison and is presented in Fig. 1 and 2. It can be seen that the equation compares very well with the experimental data up to $\alpha = 12^\circ$ and $\delta = -15^\circ$; above $\delta = -15^\circ$, the experimental drags are lower than the equation would predict.

The object of this note is to devise a method of estimating $\frac{\partial C_D}{\partial(\delta+\alpha)^2}$.

In the theoretical two dimensional case, the drag of the control surface is equal to the component of the normal force on the control in the flight direction, i.e.

$$\Delta C_{D_c} = \Delta C_{L_c}(\delta+\alpha)\cos\theta \text{ to the first order}$$

where ΔC_{L_c} is the lift on the control divided by $\frac{1}{2}\rho V^2 c$ & θ is the control leading edge sweep.

and $$\Delta C_{D_c} = \frac{\partial C_L}{\partial \delta}(\delta+\alpha)^2\cos\theta = a_2(\delta+\alpha)^2\cos\theta$$

$$\frac{\partial C_D}{\partial(\delta+\alpha)^2} = a_2\cos\theta$$

In the three dimension case -

$$\Delta C_{D_c} = \frac{\Delta C_{L_c}}{\Delta C_{L_W}}\Delta C_{L_W}(\delta+\alpha)\cos\theta$$

Where ΔC_{L_W} is the lift coefficient on the aircraft due to the control.

Therefore - $$\frac{\partial C_D}{\partial(\delta+\alpha)^2} = K_2 a_2 \cos\theta$$

Where $$K_2 = \frac{\Delta C_{L_c}}{\Delta C_{L_W}}$$

The ratio of experimental to theoretical K_2 vs Mach number has been plotted in Fig. 3. From these results, it would appear that the theoretical K_2's agree very well with those obtained from experiments.

The estimated K_2's for the C-105 are shown in Fig. 4 and the experimental K_2's from the Cornell tests are also plotted.

It is sometimes convenient to re-arrange equ. (1), substituting for α from the lift equation.

$$C_L = a_1 \alpha + a_2 \delta$$

We then have -
$$C_D = C_{D_0} + \frac{C_L^2}{e\pi A} + \left(\frac{K_2}{a_2} + \frac{1}{e\pi A} - \frac{2K_2}{a_1}\right)(a_2\delta)^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right) 2 C_L a_2 \delta \quad \text{---} (2)$$

The first two terms in the above equation represent the minimum drag coefficients and the conventional induced drag and what remains are defined as the trimming drag C_D^{δ}.

The pitching moment due to controls at constant $C_L = (h_0 - h_\delta) a_2 \delta \equiv C_M^{\delta}$

and:
$$C_D^{\delta} = \left(\frac{K_2}{a_2} - \frac{2K_2}{a_1} + \frac{1}{e\pi A}\right)\left(\frac{C_M^{\delta}}{(h_0 - h_\delta)}\right)^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right)\frac{C_M^{\delta}\, 2C_L}{(h_0 - h_\delta)} \quad \text{---} (3)$$

and in trimmed flight $-C_M^{\delta} = C_{M_0} + (h - h_0) C_L$

If we include the facts that the minimum drag does not occur at $C_L = 0$ and that C_L is more generally $C_L = a_1(\alpha - \alpha_0) + a_2\delta$ then the drag equation becomes

$$C_D = C_{D_{MIN}} + \frac{(C_L - C_{L_{C_{D MIN}}})^2}{e\pi A} + \left(\frac{K_2}{a_2} - \frac{2K_2}{a_1} + \frac{1}{e\pi A}\right)\left(\frac{C_M^{\delta}}{h_0 - h_\delta}\right)^2 + \left(\frac{K_2}{a_1} - \frac{1}{e\pi A}\right)\frac{C_M^{\delta}\, 2(C_L - C_{L_{C_{D MIN}}})}{(h_0 - h_\delta)}$$

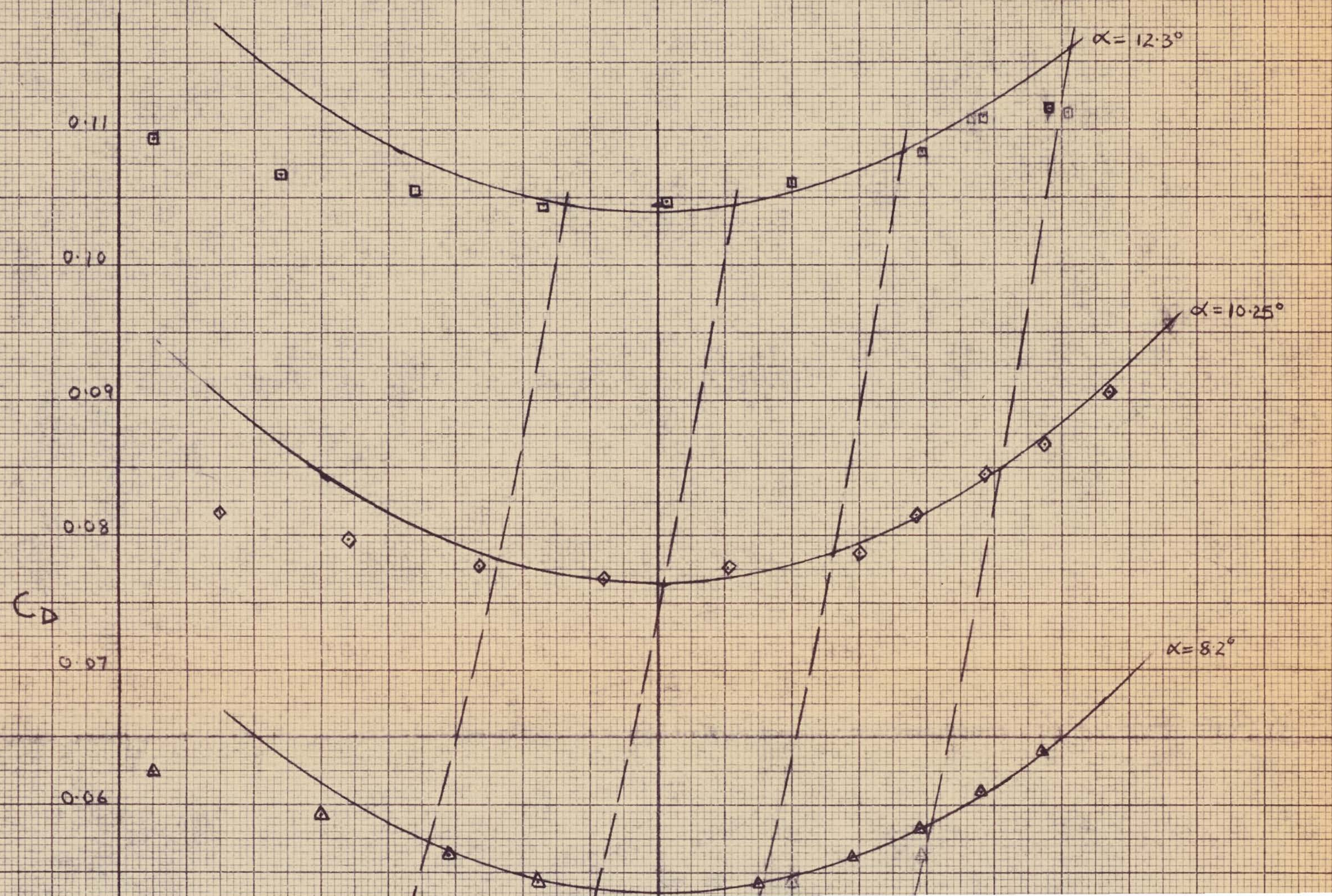
The drag coefficient vs C_L curve for the C-105 at M = 1.5 has been evaluated using the above equation and is presented in Fig. 5; the C_D vs C_L curve using the method outlined in C-105 Performance Report No. 1 is also shown.

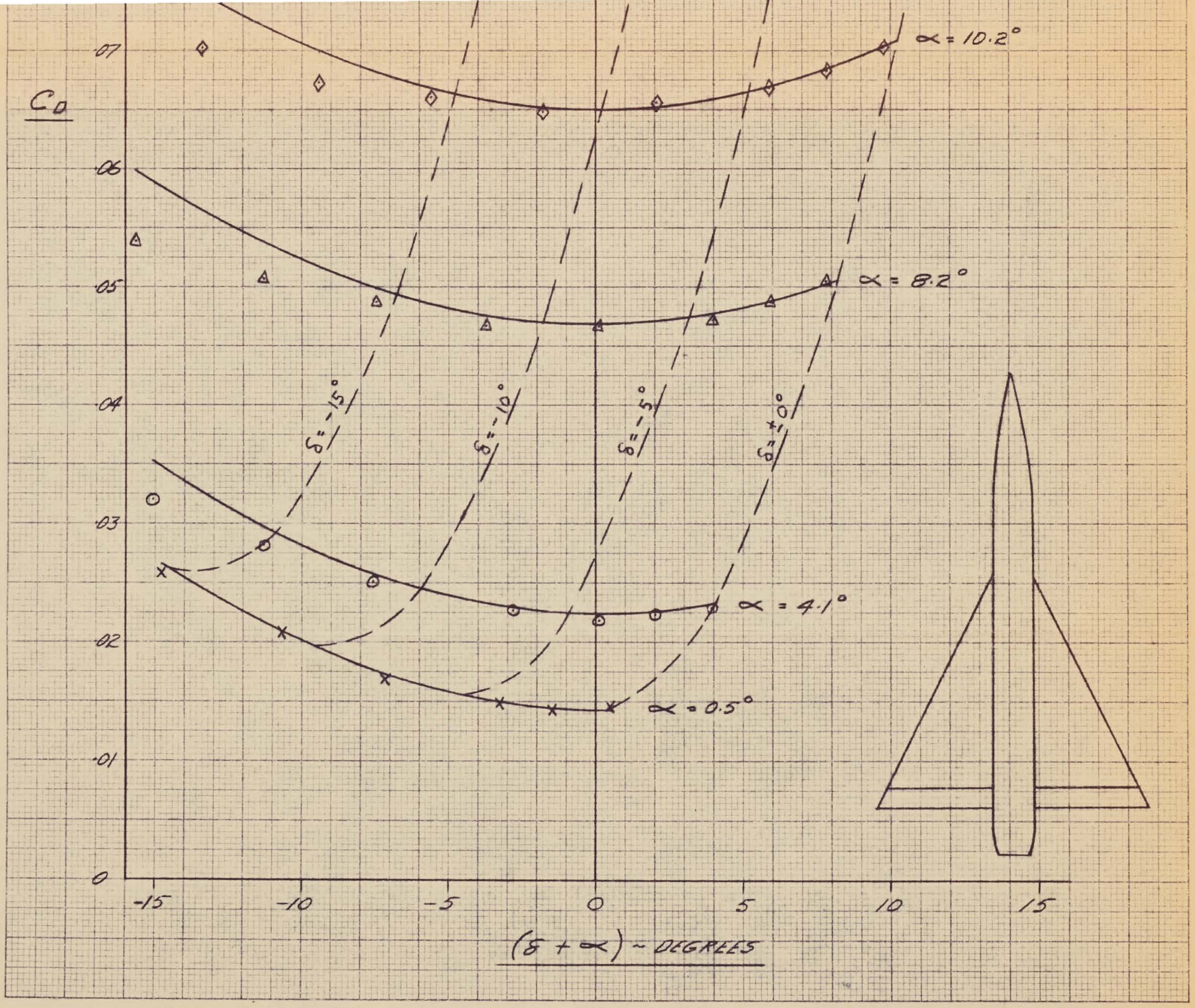
Drag of Delta Wing at M = 1·5

Experimental Points From A52L04

Curves From Formula $C_D = C_{D_0} + \left(\frac{\partial C_D}{\partial \alpha^2}\right)_{\delta=0} \alpha^2 + \left(\frac{\partial C_D}{\partial \delta^2}\right)_{\alpha=0} \delta^2 + \left(\frac{\partial C_D}{\partial \delta^2}\right)_{\alpha=0} 2\delta\alpha$

C_{D_0}, $\left(\frac{\partial C_D}{\partial \alpha^2}\right)_{\delta=0}$, $\left(\frac{\partial C_D}{\partial \delta^2}\right)_{\alpha=0}$ were obtained from Experimental Data





C_D
·07
·06
·05
·04
·03
·02
·01
0
-15
-10
-5
0
5
10
15
(δ + α) ~ DEGREES
α = 10.2°
α = 8.2°
α = 4.1°
α = 0.5°
δ = -15°
δ = -10°
δ = -5°
δ = +0°

Fig 2

C105

VARIATION OF K_2 WITH MACH N°

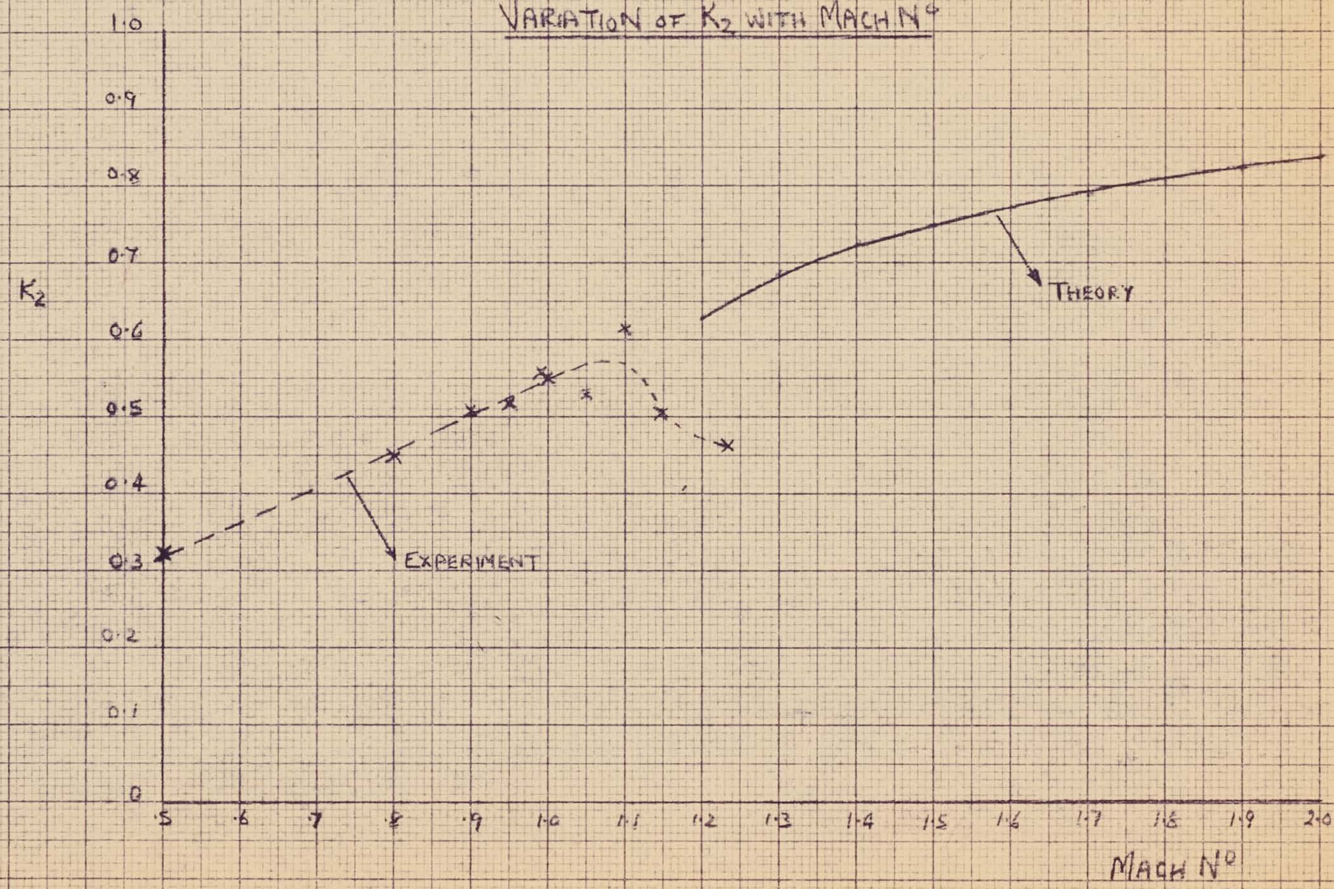


FIG 4

Fig 5

C105

ESTIMATED C_D vs C_L AT M = 1·5 (h = 0·29)