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ARROW WEAPON SYSTEM

CO-ORDINATING CONTRACTOR

REPORT NO. 10

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THE MATHEMATICAL MODEL OF THE ARROW 2 WEAPON SYSTEM

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THE MATHEMATICAL MODEL OF THE ARROW 2 WEAPON SYSTEM

FOREWORD

A/AWS has stated that a requirement exists for a mathematical model of the operation of the Arrow 2, for the following purposes:-

1. To determine the theoretical potential of the Weapon System,
2. To indicate suspect areas and influence the flight test program accordingly,
3. To form a basis for evaluation of the Weapon System.

The following terms of reference for an immediate study program have been laid down:-

a. Single interceptor versus single target will be considered, multiple interception studies being considered too complex for the initial program,
b. The model will commence at target acquisition by the ground environment,
c. The model will terminate at the completion of the missile phase,
d. Both clear and ECM environments will be considered, the greater emphasis being placed on ECM,
e. The results of the Arrow Weapon System demonstration and the predictions of the model shall be shown to be in statistical agreement,
f. The 1961 weapon system shall have priority over studies of the ultimate system.

This document sets out a program of digital and analog simulations to meet these requirements.

It may be noted in respect of item (d) that ECM is expected to be the normal environment by 1961. It is appreciated that the specification to which Astra I was designed did not state this but merely required the provision of

INTRODUCTION

The program proposed here relates to the Arrow 2 weapon system which comprises:-

Arrow 2 airframe

~~Astra I airborne radar and integrated electronic system~~ (out)

Iroquois Series 2 engines

~~Sparrow 2 Mk. 1~~ or Genie Armament

A ground environment, which should initially be representative of the conditions that may be expected at Cold Lake for the test program.

Early phases of the program will consider the version of the airborne system that will enter service in 1961, and by postulating a ground environment that is capable of being reproduced or simulated at Cold Lake, direct comparison can be made between model and test results. The work will be extended later to cover the fully developed Arrow 2 system and a ground environment that is more representative of the actual operational system. The program will not include assessment of major developments of the system, although it will be possible to adapt the model to cope with such developments. At any time that an assessment of this kind appears desirable, A/AWS will initiate a request to the contractors, who will then discuss it between themselves and present cost estimates. Where necessary, specific contracts will be raised to cover these additional studies.

answers.

It is proposed that the program be extended to include a real-time simulation of the entire system which will integrate the essential features of the other studies into a single model. This will enable human operators to be introduced into the complete system and will also allow the investigation of problems, such as the effect on system capability under ECM conditions of extending GCI guidance into the A.I. phase. This study is complementary to the rest of the program and is shown as such in the table. Its main purpose is to check the results of the evaluation carried out using the component studies; it is not an alternative approach.

The following advantages accrue from adopting the "block" approach to the mathematical model.

(i) Use may be made of programs which are primarily intended for systems development. In several cases these programs are either completely written or nearly so, and it is hoped that several others may be derived from these by relatively simple modifications.

(ii) Considerably increased flexibility is obtained, due to the greater ease of making changes to the model.

(iii) Since parts of the model are already in existence results will be available with the minimum delay.

(iv) Economy of operation is achieved by running parts of the program only, when specific features of the system are to be studied. Further economy stems from the ability to combine the results of several modes of operation of the system in one phase with those of several modes in another, without

ARROW MID-COURSE GUIDANCE STUDIES

1. Automatic Operation Under Close Control

This study will utilize a model, at present being programmed at Avro for digital computation, in which the GCI radar is simulated by random sampling techniques, and the midcourse guidance calculated according to the procedures used by SAGE. In the present model the target tracking processes are based on clear environment.

For the initial phase of the program, in which the validity of the model is to be demonstrated by correlation with the results of the integrated flight test program, it will be necessary to replace the ground environment in the model by "Test Environments" which are capable of being reproduced at Cold Lake. These environments will be postulated by Avro, after consultation with the RCAF, and should be such as to require the minimum of modifications to the present model, while still being realistic in terms of the capability of the facilities of Cold Lake. However, it is appreciated that extensive revisions to the present model may be necessary to introduce the environment corresponding to the ECM situation.

Once the validity of the model has been established by direct comparison with flight tests, it can be put to its intended use. This is to determine the capability of the weapon system against actual enemy targets, under operational conditions. For this phase of the program the postulated test ground environments will be replaced by operational environments which correctly represent the procedures expected to be used in all modes of operation. It should be noted that this may not necessarily be SAGE.

The study of the test system will constitute the first part of study no. 2 and will be started as soon as full information is available.

Output:- (a) Distribution of target and interceptor tracking errors - position and velocity.

(b) Distributions of the interceptor placement errors relative to the target.

(c) Distributions of the interceptor heading errors relative to the desired approach to the target.

(d) Probability of successful attack.

(e) Analysis of various interceptor and ground environment quantities of interest in systems development studies.

2. Manual Operation Under Close Control

(i) Interim System (1961)

In study no. 1 the interceptor is assumed to be steered automatically by the AFCS coupler, acting on heading commands received via data link. Approximations are made to simulate the properties of the automatic control system.

The 1961 system will have neither data link nor automatic steering, and other automatic features will be absent from the airborne system and from both test and operational ground environments. The heading commands generated by the ground computer will be relayed to the pilot verbally by the intercept director. The pilot will then attempt to correct the heading by control stick steering. The approximations used in No. 1 for the response of the aircraft to heading commands will not apply here,

3. Midcourse Guidance - Broadcast Control

In this mode of operation the guidance commands are calculated in the air by means of the navigation computer. The ground environment supplies data on the position, heading and velocity of the target. The interceptor's own position coordinates relative to a common frame of reference are subtracted and the course required for collision with the target is computed. Complete details required for writing a model will be made available; it is anticipated that some parts of programs No. 1 and 2 can be carried over, and that those parts which do not apply directly can be used after fairly simple modifications.

Steering in this mode may be either manual or automatic, and both must therefore be considered.

The output of the study will be similar to No. 1.

A.I. PHASE STUDIES

4. Digital Simulation of Automatic and Manual Operation

It is anticipated that both automatic and manual modes of operation will be studied by means of a digital computer program which has been written and programmed at Avro and will shortly be ready for use. This program applies directly only to the automatic mode, since it is based on the rigidly defined steering properties of the AFCS coupler. However, it is thought that by making a study of the responses of a human pilot under widely representative conditions, it should be possible to derive an "equivalent" coupler, and in this way the program may be adapted for studying manually steered attacks. This study of human response characteristics will be described below under study no. 10.

The approximations implicit in the study, insofar as they affect inputs required from other parts of the program, are enumerated below.

(a) System Response

A very simple approximation has been used for this which will be the output of studies not included in the program.

(b) Radar Ranges

Several contours are input to the program to define various aspects of radar performance. These may be functions of aspect angle, closing rate, altitude differential, radar scan pattern and ECM type, where present. The use of these ranges in the A.I. phase program implies that exact ranges for detection, lock-on, etc., can be defined. In fact, these events have varying cumulative probabilities of occurrence with range, and the ranges quoted are defined by a specific choice of probability.

The outputs of studies nos. 7 and 8 are input to this study as the criteria for the success of an attack. When the computer program is used for a 'multiple run', for each course difference it will find the limiting aspect angles on a given detection contour from which successful attack is possible.

The detection contour is automatically varied up to the maximum A.I. range and down to the minimum missile range, so that the limiting aspect angles form a marginal conversion boundary. The 'multiple run' facility will produce these boundaries for initial course differences all round the clock in one computer run.

The given detection contour corresponds to a certain cumulative probability of detection, as already stated, and a probability of successful conversion from this detection contour may be defined as the probability of placement by the midcourse guidance in the region between the limiting aspect angles, the success region. Detection contours corresponding to other cumulative probabilities may be placed on the marginal conversion boundary to give further conversion probabilities from the midcourse study, and by associating each conversion probability with the cumulative probability of detection having occured at that contour, the conversion probability for one approach lane is found, independent of where target detection occurs.

(c) Manual Override at Limits of A.I. Scan

Perfect tracking of the target by the A.I. antenna is assumed, until the maximum antenna look angles or angular rates in either plane of freedom are exceeded, when the A.I. radar loses the target. It is assumed that

These may be input to the midcourse guidance studies, nos. 1, 2, and 3.

(b) Various items of interceptor performance as required for system development studies.

5. Analog Simulation of Automatic and Manual Operation in Clear Environment

In this study system noise, radome aberrations, random computer errors and target scintillation effects will be introduced into a model in which the dynamic motion of the aircraft and the feedback characteristics of the electronic system are represented and inter-related.

The purpose of this study is to check the effects of these quantities, which, particularly in respect of their interrelation, are more suitably examined by analog rather than digital methods. The simulation will be used to study the variation in the "success-domains" determined by the digital program No. 4, due to these effects. Part of study no. 6 below, will serve the same purpose for the ECM environment.

Programs No. 4 and 5 and are considered to be complementary and not a duplication of effort. The determination of success domains is a process of trial and error in which the initial relative geometry of interceptor and target is varied systematically to find the limiting conditions from which an attack may be successfully initiated. To carry out this process for all the cases that will be required, in terms of target type and altitude, evasive manoeuvre, ECM, interceptor speed and altitude, initial course difference, type of attack, etc., will be a major undertaking. To attempt to do it by means of a program in which random variations are permitted, would increase the magnitude of the task many fold, due to the necessity for statistical sampling.

A detailed proposal has been made by RCA for a study requiring 120 man-months of effort. This will cover all expected types of ECM. The output of the study that is required for use in other parts of this program is given below. Since the primary purpose of the study was for system development, additional studies beyond the scope of the original proposal will be required.

In later phases of the program this study will also be used to provide a check on the results of the digital evaluation of the A.I. phase under ECM conditions, as indicated in Study No. 5.

Output:- (a) Conditions under which both range and angular information may be obtained.

(b) Accuracy of information under all conditions, for all types of ECM likely to be encountered.

(c) When ECM is used continuously from long range

(i) Detection range and delay before lock-on, where this is possible, as functions of aspect angle, altitude differential, closing rate and scan pattern, for all targets and all ECM types. The detection ranges should correspond to some agreed cumulative probability of detection.

(ii) The optimum procedure for application of CCM facilities and the delay involved in selecting the best operational mode. Clearly this delay will be a function of the type of ECM encountered.

(iii) The degree to which the aircraft can, or should, be steered manually, by joint pilot - Obs/A.I. action, during the period from detection to either lock-on or selection of the best CCM mode.

MISSILE STUDIES

7. Evaluation of Sparrow 2 Performance Capabilities

Analog and digital methods will be used to determine the capability of the missile against expected types of target. These studies will include the effects of:-

Missile dynamics

Aerodynamic and hydraulic limitations on range

Seeker characteristics

Radome aberrations

Target characteristics

Target scintillation and glint

Missile computer errors

System noise

Transient effects of the launch conditions

K-Band ECM

Both co-altitude and altitude-difference cases will be considered, and for the latter, studies should include cases where steering errors exist in azimuth and elevation planes both individually and simultaneously. The complete flight envelope of the Arrow should be covered, from sea-level to 65,000 ft. and from high subsonic speeds to M = 2.0. However, the initial studies should concentrate on the high altitude, supersonic launch capability of the 1961 system.

8. Evaluation of Genie (MB-1) Capability

The Genie rocket has been selected as the alternative armament to the Sparrow 2 missile, and a study will therefore be carried out as part of this program to evaluate the capability of this weapon.

It will first be necessary to obtain full information on the various versions of Genie from the manufacturers and other sources. Digital computer methods will then be used to establish the effects on the flight characteristics and miss distance of:-

Projectile aerodynamics

Thrust variation

Launch dispersion

Target altitude and altitude differential

Evasive manoeuvres

Both co-altitude and altitude difference cases will studied, covering the complete flight envelope of the Arrow. Again the initial emphasis should be on the high altitude supersonic launch capability of the 1961 system. The study will determine, in conjunction with lethality studies, which do not lie within the scope of this program, the accuracy requirements for the launch conditions in order that an acceptable kill probability is obtained.

Output:- (a) Miss distance and kill probability as a function of aspect angle, angular firing error, launch range, closing rate, target altitude and evasive manoeuvres, and altitude separation between interceptor and target.

(b) Acceptable tolerances on launch range and angular firing error.

OTHER STUDIES

10. Study of Aircrew Response

This study, which should be planned in consultation with specialists in human engineering, is needed to determine the manner in which a pilot may be expected to steer the aircraft under the following conditions:-

a) When he is simply required to turn to a specified heading, with no knowledge of the geometry of the attack.

b) When he is required to turn to a specified heading, knowing the range or time remaining to weapon launch,

c) When he is required to steer according to the fire control steering dot presentation, with no knowledge of range or time to go.

d) When he steers according to the fire control presentation, with full information on range and time to go.

e) When he steers according to verbal instructions from the Obs/A.I. with no other information.

Of these situations, (a) relates to study No. 2
(b) to No. 3
(c) to No. 4 under ECM conditions
(d) to No. 4 under clear conditions, and
(e) to No. 4 under ECM conditions, prior to selection of the optimum mode.

The study should also determine the combined behaviour of the aircrew in the situation, corresponding to broadcast mode operation, where the Obs/A.I. receives verbal information about the target, which he inserts manually into the navigation computer, the output of the computer being either displayed to the pilot or transmitted verbally by the Obs./A.I.

11. EVALUATION OF ENTIRE WEAPON SYSTEM

The overall evaluation of the system using the models described above is given here as a separate study, although in actual fact it does not exist as such. In reality the links between the various phases of the mission are written into the models themselves and are accomplished in one of two ways.

a) From studies nos. 7 and 8 the allowable launch zones for the armament are determined and these are input to study no. 4 the digital analysis of the A.I. phase. A postulated A.I. detection contour, corresponding to the type of target, ECM conditions, etc. is also input to this study. For each initial target-interceptor course difference, study no. 4 then determines, by a process of trial and error, the limiting aspect angles at which detection may occur for conversion to a launch position to be possible. The output is then given in the form of "success zones" in the plane of course difference - aspect angle. If the values of these two quantities at A.I. detection place the interceptor within such a zone, it can then successfully convert and launch its armament.

The success zones are then input, together with the corresponding A.I. detection contours, into the appropriate midcourse guidance study No. 1,2 or 3. These studies trace the complete history of target and interceptor during the period that the interceptor is subject to ground control, and determine the point at which A.I. detection may be expected to occur. At this point the true values of course difference and aspect angle are evaluated, and by comparison with the success zone the success or failure of the run is determined. Random variations are allowed to occur from run to run, and statistical analysis of a large number of runs for the same initial conditions gives the overall probability of the success or failure of each of them in each run. It is also considered preferable that the studies should be linked by actual numbers, as in the first method, rather than by probabilities, as in the second.

COMPLEMENTARY STUDY

12. Real-Time Simulation of the Complete System

In this study a cockpit simulator in conjunction with analog computation is used to simulate the aircraft and is linked via analog-digital and digital-analog conversion equipment to a digital computer which simulates the target path and the ground environment. The program for this will draw upon several of the preceding studies, although considerable revision of the ground environment simulations will be required to enable them to operate in real time. Wherever possible simplifications based on the results of the earlier studies will be introduced.

By simulating the complete system in this way it will be possible to examine the problems associated with operating a complex airborne system in conjunction with a complex ground environment in the proper time scale and sequence of events and in controlled laboratory conditions. This will enable consideration to be given to several problems which cannot be studied by means of the "block" approach. Of these the most important is the additional system capability that may be made available under ECM conditions by continuing the ground control into the A.I. phase. Even in the situation where the ground radars are jammed it is possible that the ground controller has better information on the target than is supplied by the A.I., this being particularly true when deception jamming is used against the A.I. Under such conditions the overall positioning probability may be improved by the ground controller continuing to give information to the interceptor after A.I. contact has been made. A similar problem is that of the interceptor attempting to attack a quiet target in the presence of jamming decoys. To study such problems it is essential to place both aircrew simulation, it represents a most important complement to the other studies in the program.

It is appreciated that there will be items of capital expenditure involved in setting up this model; in particular the purchase of analog-digital and digital-analog conversion equipment will be required. The contractors are requested to present cost estimates for such equipment in order that contracts may be raised.

CONCLUSION

An outline has been presented of a program for developing the mathematical model as a synthesis of a group of studies that will also be used by the contractors for systems development. These studies are for the most part either in existence at present or planned for initiation in the near future, and it is considered that when linked together by proper matching of outputs and inputs, as indicated above, they should be suitable for the purposes of evaluation required of the mathematical model.

This program, and the division of responsibilities shown in Table I, have been discussed with Avro, RCA and Canadair. In the course of discussions it has become apparent that:-

a) Considerable information is required, principally from RCAF sources, to enable the simulated environment at Cold Lake to be formulated in such a manner as to be representative of the 1961 operational environment.

b) Improved communications between the United States and Canada are needed so that this work may be pursued in an adequate manner. Specifically, the requirement exists for contractors' representatives to be authorized to carry classified documents across the border for discussion and transmission.

c) Certain items of capital expenditure will be required to enable the program to be carried out in its entirety.

APPENDIX A

INFORMATION REQUIRED BY THE CONTRACTORS IN SUPPORT OF THE MATHEMATICAL MODEL

The contractors have submitted a request for information on the topics listed below, to assist them in setting up the mathematical model. Where this information is available, arrangements will be made for it to be supplied by the appropriate government agencies.

GROUND ENVIRONMENT

1) GCI Radar

(a) Details of the radar network envisaged for 1961 in respect of:-

Location of stations.

Type of installation and capability (i.e. early warning, tracking, height - finding etc.) at each station.

Quality of radar data (distributions of random and systematic error, blip/scan ratio, noise).

(b) Information as above on the proposed extensions and developments of the network, with the dates into service of such modifications.

2) Data Processing

(a) If SAGE is not operational when the Arrow enters service, the following information is required for the alternative ground environment:-

Operational procedures for detection and identification of enemy aircraft, interceptor assignment and scramble. Procedures for radar data processing, transmission and integration. Interrelation with U.S. facilities.

Application rate, in terms of the separation between aircraft required for radar resolution.

2) Target Characteristics

Details of the performance of enemy aircraft to be intercepted by the Arrow.

Should include speed, altitude, manoeuvreability and expected confusion and evasion Manoeuvres.

Size and radar cross section of expected threats.

Probable form of enemy attack, including number of aircraft, type of formation, direction of attack.

Complete characteristics of stand-off bombs.

3) Arrow Armament

Sparrow 2. Details of the types of warhead and fuze to be used, with the associated lethality characteristics against the expected targets.
Outline of policy on Sparrow development programs.
What plans exist for armament other than Sparrow 2.
Is there an operational requirement for Genie. If so, what characteristics should be used in systems studies with this weapon.

ECM

For both 1961 and developed versions of broadcast control and for SAGE, information is required on:-

The types of ECM to be expected and their effects on the capability

TABLE I

SUMMARY OF PROGRAM

STUDY NUMBER	SUBJECT	TYPE OF COMPUTER	CONTRACTOR
1	Automatic Operation in Midcourse (Close Control)	Digital	Avro
2	Manual Operation in Midcourse (Close Control)	Digital	Avro
3	Midcourse Operation (Broadcast Control)	Digital	Avro
4	A.I. Phase Automatic and Manual	Digital	Avro
5	A.I. Phase, Automatic and Manual Clear Environment	Analog	RCA
6	A.I. Phase, Automatic and Manual. ECM Environment	Analog	RCA
7	Evaluation of Sparrow 2 Capability	(Digital (Analog	Canadair
8	Evaluation of Genie Capability	Digital	Avro
9	Evaluation of Warhead Lethality		CARDE
10	Simulation of Crew Response.	Analog	Avro RCA
11	The overall evaluation of the entire weapon system is obtained by taking studies 1 to 10 together in appropriate combinations.		
	Complementary Study		
12	Simulation of the Complete System in real-time	Combined analog-Digital Simulation	Avro