AVRO AIRCRAFT LIMITED

MALTON • ONTARIO

TECHNICAL DEPARTMENT (Aircraft)

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MODIFICATION OF THE MASS FLOW CONTROLLER THEORY TO THE CONCEPT OF COMPLETE FLOW GUIDANCE BY THE ROTOR.

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AVRO AIRCRAFT LIMITED
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REPORT NO. 72/Systems 22/144

SHEET NO. 1

AIRCRAFT:		PREPARED BY	DATE
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INTRODUCTION

If the rotor is built such that the flow leaves the rotor with a relative velocity parallel to the blades (as viewed from the blades) then the flow may be considered to be completely guided by them. That is the flow leaves the rotor with the same angular velocity as the rotor. This results in a modification to the theory given in Report Number 72/Systems 22/137 and to several useful improvements in the characteristics of the device.

Theory

Assuming the normal force curve slop ($dC_N/d\alpha$) at approximately 2π and using the theory presented in Report No. 72/Systems 22/137, an inconsistency becomes apparent when a rotor of high solidity is used. For example; if $Nc = 12$ inches $C_L = .384$ $C_D = .03$ (i.e. $\alpha = 3.5^{\circ}$ based on $dC_L/d\alpha = 2\pi$) $r = 1.75"$ then the rotational outflow factor $b = 1.2657$.

Where c and r are the blade element chord & radius respectively.

N is the number of blades.

and C_L and C_D are the blade element lift and drag coefficient respectively. Now it is obvious that $b \not> 1.0$ since this would correspond to the outflow rotating faster than the rotor.

The reason for the above anomaly becomes apparent when the characteristics of aerofoils in cascade are examined. In "Aerodynamic Theory" by Durand section IV page 229 formula are quoted which give the behaviour of a cascade of flat plate aerofoils in potential flow.

If a cascade is considered where the chord lines are perpendicular to the line joining their leading edges, then the following formulae result.

$$\frac{1}{2\pi}\frac{dC_N}{d\alpha} = \frac{\frac{2s}{\pi c}\tanh\left(\frac{\pi}{2}\frac{c}{s}\right)}{1+\tanh\left(\frac{\pi c}{2 s}\right)} \qquad (1)$$

$$\frac{\delta}{\alpha} = \frac{2\tanh\left(\frac{\pi}{2}\frac{c}{s}\right)}{1+\tanh\left(\frac{\pi}{2}\frac{c}{s}\right)} \quad \text{———} \quad (2)$$

Where s is the spacing of the blades

α is the incidence of the undisturbed stream

δ is the angle through which the flow is deflected.

Now as $\frac{c}{s}$ increases $\tanh\left(\frac{\pi}{2}\frac{c}{s}\right) \rightarrow 1.0$

Therefore $$\frac{1}{2\pi}\frac{dC_N}{d\alpha} \rightarrow \frac{s}{\pi c} \quad \text{———} \quad (3.0)$$

and $$\frac{\delta}{\alpha} \rightarrow 1.0 \quad \text{———} \quad (4.0)$$

If the simple assumption is made that the flow passing through a cascade is deflected parallel to the aerofoils (i.e. $\frac{\delta}{\alpha} = 1.0$) then from elementary considerations of momentum a formula (consistent with (3) above) eventuates.

i.e. $$\frac{1}{2\pi}\frac{dC_N}{d\alpha} = \frac{1}{\pi}\frac{s}{c} \quad \text{———} \quad (5.0)$$

Now for application to this particular problem, the spacing between the blades, $s = \frac{2\pi r}{N}$

Which substituted in (1), (2) and (5) gives respectively,

$$\frac{1}{2\pi}\frac{dC_N}{d\alpha} = \frac{\frac{4r}{Nc}\tanh\left(\frac{Nc}{4r}\right)}{1+\tanh\left(\frac{Nc}{4r}\right)} \quad \text{———} \quad (6.0)$$

$$\frac{\delta}{\alpha} = \frac{2\tanh\left(\frac{Nc}{4r}\right)}{1+\tanh\left(\frac{Nc}{4r}\right)} \quad \text{———} \quad (7.0)$$

$$\frac{1}{2\pi}\frac{dC_N}{d\alpha} = \frac{2r}{Nc} \quad \text{———} \quad (8.0)$$

Equations (6) and (8) are shown plotted in Figure 1 and (7) in Figure 2.

From the above equations the following conclusions may be made,

(i) That $\frac{dC_N}{d\alpha} \ll 2\pi$ for all practical rotor configurations.

(2) That the blade spacing required to give nearly complete guidance $(\delta/\alpha \rightarrow 1{\cdot}0)$ is quite easily built into a practical rotor. (e.g. $\frac{Nc}{4r} = 2{\cdot}5$ gives $\frac{\delta}{\alpha} = {\cdot}993$)

(3) That due to the rapid decline in $dC_N/d\alpha$ with increasing solidity the normal force coefficients on the blades of high solidity rotors will be lower and their stalling much delayed. It would further seem likely that even with stalled blades of close spacing the flow must leave the rotor substantially parallel to the blades.

<u>The characteristics of a rotor achieving complete flow guidance</u>

In this case the angular velocity of the outflow is equal to that of the rotor so that from angular momentum considerations

Then $$\frac{dQ}{dr} = 2\pi\rho\omega V r^3.$$

Where ρ is the density downstream of the rotor,

ω is the angular velocity of the rotor (and therefore also of the outflow),

V is the axial velocity component of the flow downstream of the rotor.

r is the radius of the blade element,

Q is the torque on the rotor.

Assuming V and ρ to be independent of radius.

Then
$$Q = 2\pi \rho V \omega \int_{r_1}^{r_2} r^3 \, dr$$
$$= \frac{\pi}{2} \rho V \omega \left(r_2^4 - r_1^4\right)$$

Now
$$\rho V = \frac{m}{A} = \frac{m}{\pi\left(r_2^2 - r_1^2\right)}$$

Where A is the duct area

r_1 and r_2 are the root and tip radii respectively

Thus
$$Q = \frac{m\omega}{2}\left(r_2^2 + r_1^2\right)$$

Advantages of complete flow guidance

It may be concluded that, given complete guidance of the flow by the rotor the following advantages may be obtained.

(1) Low flows involving high relative incidences on the rotor are acceptable.

(11) The Mach Number of the incident flow is of no significance.

(111) The flow of solid particles (such as sand) could be measured.

Typical rotor design to achieve nearly complete guidance

Consider a rotor of 1" root radius and 2" tip radius.

Let sixteen blades have a chord of 1" each.

Then $\frac{Nc}{4r}$ = 4.0 at root

= 2.667 at mean radius

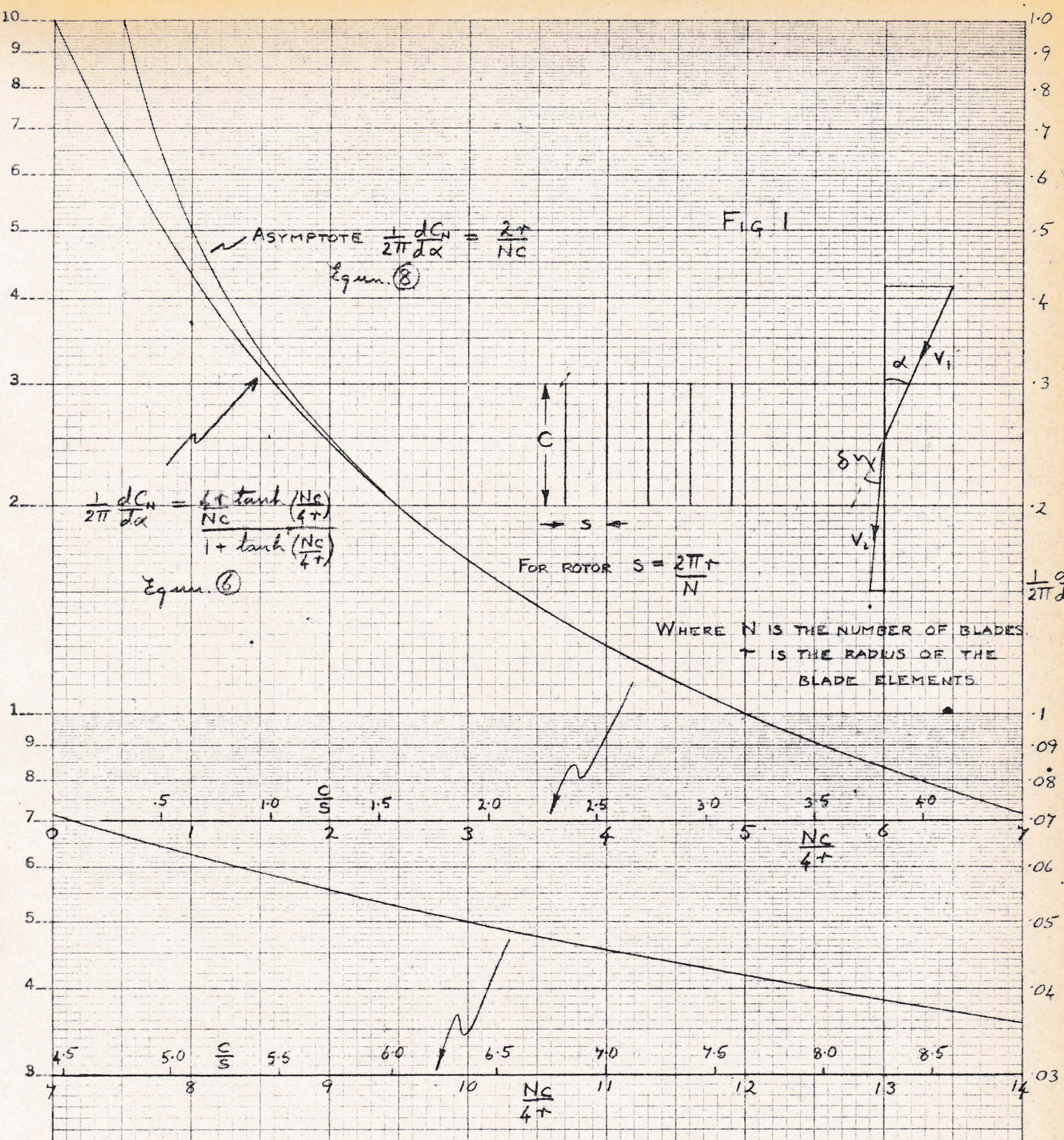
= 2.0 at tip

Thus $\frac{\delta}{\alpha}$ = .9997 at root

= .9952 at mean radius

= .9817 at tip

Which is obviously very close to complete guidance. The torque characteristics of such a rotor are shown in Figure 3.



Note: The asymptote assumes that the flow emerges parallel to the rotor blades. i.e. is completely guided by them.

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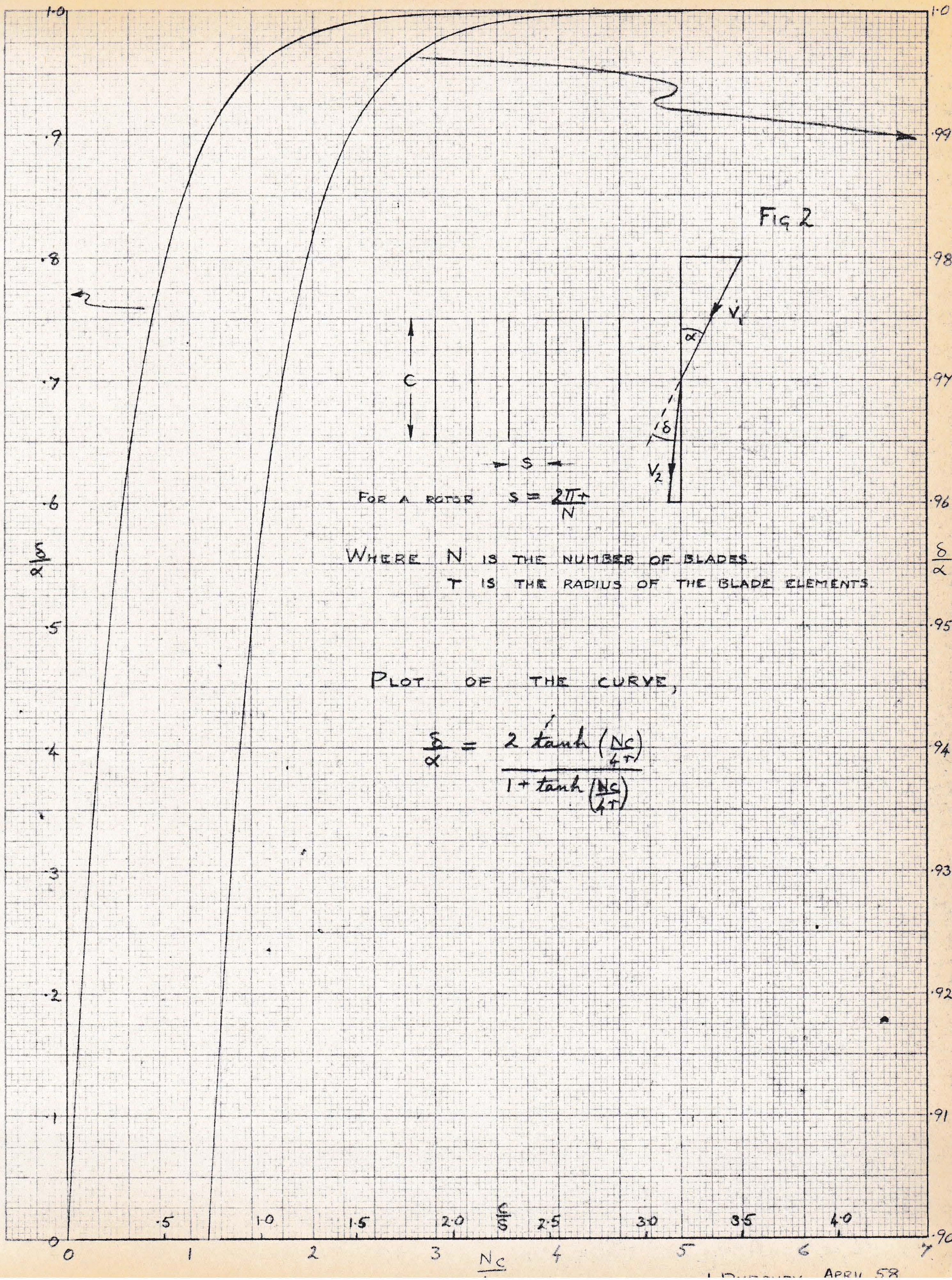

Fig 2

For a rotor $s = \frac{2\pi r}{N}$

Where N is the number of blades.
r is the radius of the blade elements.

Plot of the curve,

$$\frac{\delta}{\alpha} = \frac{2 \tanh\left(\frac{Nc}{4r}\right)}{1 + \tanh\left(\frac{Nc}{4r}\right)}$$

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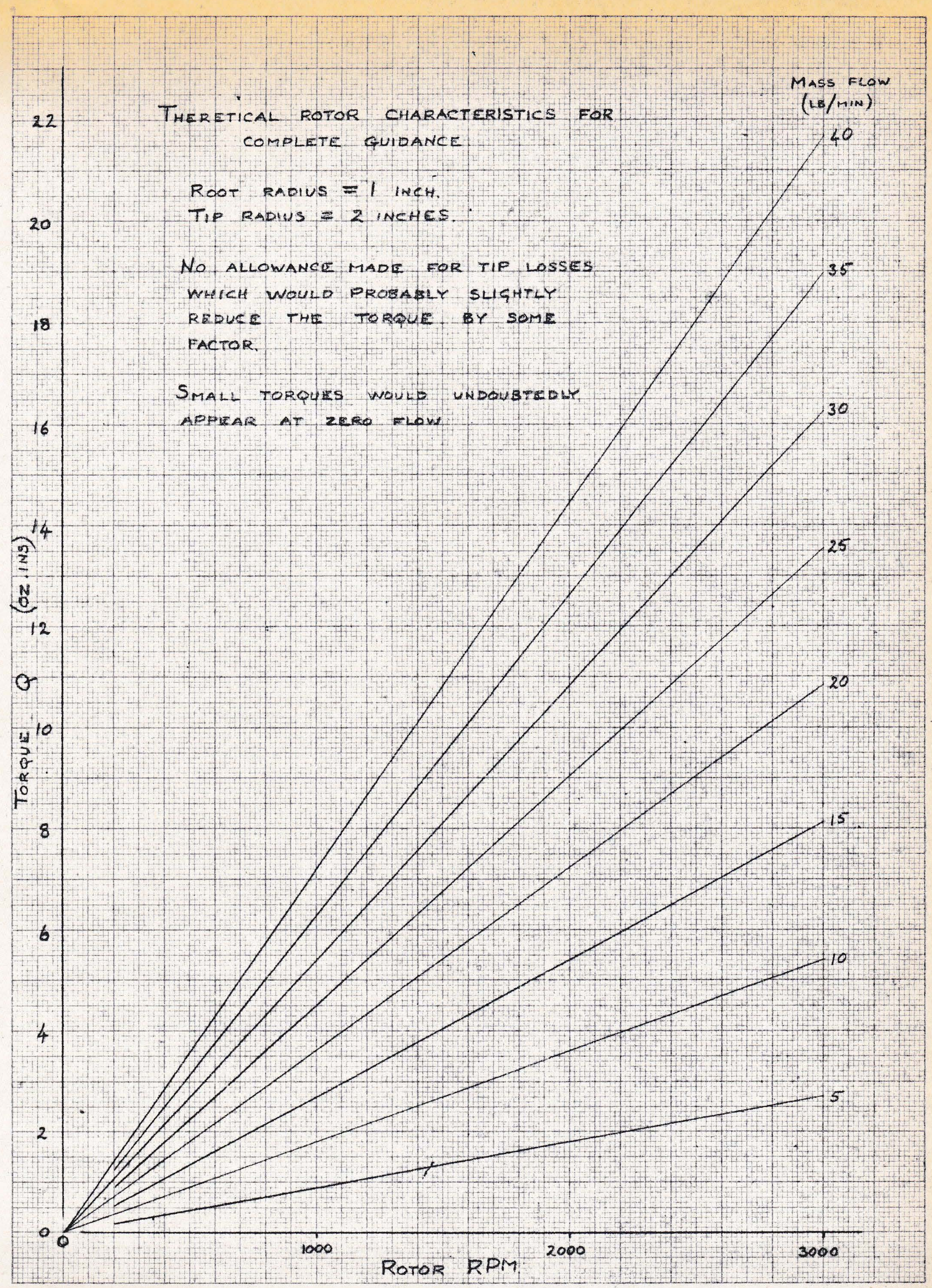

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