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CONTROL SURFACE HINGE MOMENT DERIVATIVES FROM FLIGHT TESTS

INTRODUCTION AND ASSUMPTIONS

Some hinge moment derivatives may be obtained by steady state tests, and provided that the information is accurate enough, further derivatives can be obtained from oscillation tests.

The <u>STEADY STATE</u> tests should be as follows.

Elevator

Horizontal banked circles at steady speed with zero sideslip, or pull ups. The former alternative is the most practical, and the equations in the form presented are equally applicable to spiral dives at constant $V_{E.A.S.}$ or mach number provided that the effects of longitudinal acceleration are ignored.

The variables are, α , δ_A, q, r, and p, & to obtain the derivatives with respect to all of these parameters and in addition C_{H_o} would require at least 6 different normal accelerations. However the effects of p & r will be small and consequently will be assumed zero. q is also likely to be small unless high normal accelerations are used, but its effect should be included if only to improve the accuracy of the other derivatives.

A minimum of 4 different normal accelerations is $\therefore$ required for each flight case.

Both sides of the elevator should be analysed separately so that a mean can be taken to eliminate any stray lateral effects present.

Aileron

1 "g" barrel rolls.

The parameters influencing the hinge moment will be α , δ_A and p. α will be a constant for a given flight case.

It would appear $\therefore$ that 4 different rates of roll will be required for each flight case.

AVRO AIRCRAFT LIMITED MALTON – ONTARIO TECHNICAL DEPARTMENT	REPORT NO. 71/F.A.R./5 SHEET NO. 2	
AIRCRAFT	PREPARED BY: V. Baddeley	DATE: Oct. 1957.
	CHECKED BY:	DATE:

Aileron .. continued..

However by alternatley adding and subtracting the contributions of each aileron the number of r.o.r's can be reduced to 2. This implies the assumption that C_{H_O} & C_{H_α} are independant of δ_A.

Rudder

Steady Sideslipe

The parameters influencing the hinge moment will be β, δ_R and α. To solve for the derivativee with respect to the above variablee and C_{H_O} will .·. require at least 4 different sideslip angles.

It ehould be noted that the effect of α ie really non linear but it is thought that a linear approximation will be of some value.

It is clear from the above that to solve for the steady etate derivatives for each control eurface at least 10 different manoeuvres are needed for each flight case.

The OSCILLATION should be carried out from steady level flight for simplicity's sake, and will provide approximate valuee for an additional 2 derivatives per control.

SUMMARY

The following derivatives can be obtained

Control Surface	Steady State Tests	Oscillation Tests
Elevator	C_{H_o}, C_{H_α}, $C_{H_{\delta_E}}$, C_{H_q}	C_{H_q}, $C_{H_{\dot\alpha}}$
Ailerons	C_{H_o}, C_{H_α}, $C_{H_{\delta_A}}$, C_{H_p}	C_{H_β}, $C_{H_{\dot p}}$
Rudder	C_{H_o}, C_{H_α}, $C_{H_{\delta_R}}$, C_{H_β}	$C_{H_{\dot\beta}}$, C_{H_r}

NOTATION

In the oscillation section the notation of Ref. 4 is used, but in the steady state section since all velocities and angles are steady state values, the suffix 1 is omitted.

In addition

C_H Non dimensional hinge moment coefficient
hinge moment/ $q\, S_S\, c_S$

C_H (C) Hinge moment coefficient applied by the aircraft control system, corrected for inertia and gravity effects

C_H (A) Hinge moment coefficient applied by the aircraft control system.

C_{H_α} $\partial C_H / \partial \alpha$

C_{H_q} $\partial C_H / \partial \frac{q \bar{c}}{2V_1}$

$C_{H_{\dot{\alpha}}}$ $\partial C_H / \partial \frac{\dot{\alpha} \bar{c}}{2V_1}$

$C_{H_{\delta_S}}$ $\partial C_H / \partial \delta_S$

C_{H_β} $\partial C_H / \partial \beta$

C_{H_p} $\partial C_H / \partial \frac{p b}{2V_1}$

$C_{H_{\dot{p}}}$ $\partial C_H / \partial \frac{\dot{p} b T}{2V_1}$

C_{H_r} $\partial C_H / \partial \frac{r b}{2V_1}$

$C_{H_{\dot{\beta}}}$ $\partial C_H / \partial \frac{\dot{\beta} b}{2V_1}$

C_{H_o} Hinge moment coefficient for all angles, velocities (except forward speed) and accelerations zero.

NOTATION

S_S	control surface area
c_S	control surface mean chord
$\bar{c}$	aircraft mean aerodynamic chord
n_n	normal acceleration
n_l	lateral acceleration
$\bar{q}$	$\frac{\rho V_1^2}{2}$
Γ	wing anhedral angle
χ_S	sweepback of control surface hinge line
δ_S	control surface deflection
τ	$m_s/\rho S_S V_1$
ρ	air density
r'	rate of yaw in horizontal plane
l_S	distance of control surface cg to hinge line
$k_{E_Y}^2$, $k_{E_{XY}}$	defined in Ref. 1
$k_{A_Y}^2$, $k_{A_{XY}}$	defined in Ref. 1
k_{R}^{2} [illegible] $k_{R_{X}}$ [illegible]	defined in Ref. 1
x_{0_R}	defined in Ref. 1
x_{0_A} z_{0_A}	defined in Ref. 1
x_{0_E} z_{0_E}	defined in Ref. 1

AVRO AIRCRAFT LIMITED MALTON – ONTARIO TECHNICAL DEPARTMENT		REPORT NO. 71/F.A.R./5 SHEET NO. 5	
		PREPARED BY: V. Baddeley	DATE: Oct. 1957.
AIRCRAFT		CHECKED BY:	DATE:

Notation ... continued...

ξ general disturbance angle, velocity, or acceleration of the oscillation.

$|\xi_0|$ amplitude of the envelope of the oscillation at some mean time.

$\angle\xi$ phase lead of ξ, relative to α in the longitudinal oscillation, β in the lateral oscillation.

Other parameters are defined in the section to which they are appropriate.

Note that the foot, slug second, radian system should be used throughout.

SUFFICES

A	Aileron
E	Elevator
R	Rudder
S	Control Surface
PT	Port
ST	Starboard

SIGN CONVENTION

The sign convention for control surface deflections and hinge moments are defined in Ref. 1.

REFERENCES

1. 71/F.A.R./4 — Control surface mass contribution to hinge moment - V. Baddeley

2. 71/STAB/5 — Digital computer determination of lateral derivatives from oscillatory flight tests - M. Jenkins

3. 71/STAB/6 — Digital computer determination of longitudinal derivatives from oscillatory flight test - M. Jenkins

4. P/STAB/132 — Dynamic equations relative to body axes - M. Jenkins

STEADY STATE TESTS

THE HINGE MOMENT, CORRECTED FOR INERTIA & GRAVITY EFFECTS BY THE EQUATIONS OF REF. I SHOULD FIRST BE OBTAINED. IN THIS SECTION SINCE ALL VELOCITIES & ANGLES ARE STEADY STATE, NO SUFFICES WILL BE USED TO DENOTE THIS.

ELEVATOR

$$0 = C_H(C) + C_{H_0} + \alpha C_{H\alpha} + \delta_E C_{H\delta_E} + \frac{q\bar{c}}{2V} C_{Hq}$$

Now $\quad r' = \frac{n_n g}{V} \sin\phi$

$$q = r' \cos\Theta \sin\phi = \frac{n_n g}{V} \cdot \cos\Theta \sin^2\phi = Q \cdot \frac{2V}{\bar{c}}$$

$$\therefore -C_H(C) = C_{H_0} + \alpha C_{H\alpha} + \delta_E C_{H\delta_E} + Q C_{Hq}$$

4 DIFFERENT CASES SUFFICES 1 TO 4:

$$C_{H_0} = \left. \begin{vmatrix} -C_H(C)_1 & \alpha_1 & \delta_{E_1} & Q_1 \\ \vdots & \vdots & \vdots & \vdots \\ -C_H(C)_4 & \alpha_4 & \delta_{E_4} & Q_4 \end{vmatrix} \middle/ \Delta \right. = \nabla(C_{H_0})/\Delta$$

SIMILARLY

$$C_{H\alpha} = \nabla(C_{H\alpha})/\Delta \; ; \; C_{H\delta_E} = \nabla(C_{H\delta_E})/\Delta \; ; \; C_{Hq} = \nabla(C_{Hq})/\Delta$$

WHERE

$$\begin{aligned} \Delta = {} & \alpha_2(\delta_{E_3} \cdot Q_4 - \delta_{E_4} \cdot Q_3) - \alpha_3(\delta_{E_2} \cdot Q_4 - \delta_{E_4} \cdot Q_2) + \alpha_4(\delta_{E_2} \cdot Q_3 - \delta_{E_3} \cdot Q_2) \\ & - \alpha_1(\delta_{E_3} \cdot Q_4 - \delta_{E_4} \cdot Q_3) + \alpha_3(\delta_{E_1} \cdot Q_4 - \delta_{E_4} \cdot Q_1) - \alpha_4(\delta_{E_1} Q_3 - \delta_{E_3} Q_1) \\ & + \alpha_1(\delta_{E_2} \cdot Q_4 - \delta_{E_4} \cdot Q_2) - \alpha_2(\delta_{E_1} \cdot Q_4 - \delta_{E_4} \cdot Q_1) + \alpha_4(\delta_{E_1} \cdot Q_2 - \delta_{E_2} \cdot Q_1) \\ & - \alpha_1(\delta_{E_2} \cdot Q_3 - \delta_{E_3} \cdot Q_2) + \alpha_2(\delta_{E_1} \cdot Q_3 - \delta_{E_3} \cdot Q_1) - \alpha_3(\delta_{E_1} \cdot Q_2 - \delta_{E_2} Q_1) \end{aligned}$$

$$\begin{aligned} \nabla(C_{H_0}) = {} & -C_H(C)_1\{\alpha_2(\delta_{E_3} \cdot Q_4 - \delta_{E_4} \cdot Q_3) - \alpha_3(\delta_{E_2} \cdot Q_4 - \delta_{E_4} \cdot Q_2) + \alpha_4(\delta_{E_2} \cdot Q_3 - \delta_{E_3} \cdot Q_2)\} \\ & + C_H(C)_2\{\alpha_1(\delta_{E_3} \cdot Q_4 - \delta_{E_4} Q_3) - \alpha_3(\delta_{E_1} \cdot Q_4 - \delta_{E_4} \cdot Q_1) + \alpha_4(\delta_{E_1} \cdot Q_3 - \delta_{E_3} \cdot Q_1)\} \\ & - C_H(C)_3\{\alpha_1(\delta_{E_2} Q_4 - \delta_{E_4} \cdot Q_2) - \alpha_2(\delta_{E_1} \cdot Q_4 - \delta_{E_4} Q_1) + \alpha_4(\delta_{E_1} \cdot Q_2 - \delta_{E_2} \cdot Q_1)\} \\ & + C_H(C)_4\{\alpha_1(\delta_{E_2} \cdot Q_3 - \delta_{E_3} \cdot Q_2) - \alpha_2(\delta_{E_1} \cdot Q_3 - \delta_{E_3} Q_1) + \alpha_3(\delta_{E_1} Q_2 - \delta_{E_2} \cdot Q_1)\} \end{aligned}$$

$$
\begin{aligned}
\nabla(C_{H_\alpha}) = & -C_H(C)_2(\delta_{E_3}\cdot Q_4 - \delta_{E_4}\cdot Q_3) + C_H(C)_3(\delta_{E_2}Q_4 - \delta_{E_4}\cdot Q_2) - C_H(C)_4(\delta_{E_2}\cdot Q_3 - \delta_{E_3}\cdot Q_2) \\
& + C_H(C)_1(\delta_{E_3}\cdot Q_4 - \delta_{E_4}\cdot Q_3) - C_H(C)_3(\delta_{E_1}\cdot Q_4 - \delta_{E_4}\cdot Q_1) + C_H(C)_4(\delta_{E}\cdot Q_3 - \delta_{E_3}\cdot Q_1) \\
& - C_H(C)_1(\delta_{E_2}\cdot Q_4 - \delta_{E_4}\cdot Q_2) + C_H(C)_2(\delta_{E_1}Q_4 - \delta_{E_4}\cdot Q_1) - C_H(C)_4(\delta_{E_1}Q_2 - \delta_{E_2}\cdot Q_1) \\
& + C_H(C)_1(\delta_{E_2}\cdot Q_3 - \delta_{E_3}\cdot Q_2) - C_H(C)_2(\delta_{E_1}\cdot Q_3 - \delta_{E_3}\,Q_1) + C_H(C)_3(\delta_{E_1}\cdot Q_2 - \delta_{E_2}\cdot Q_1)
\end{aligned}
$$

$$
\begin{aligned}
\nabla(C_{H_{\delta_E}}) = & \ \alpha_2(C_H(C)_4\cdot Q_3 - C_H(C)_3\cdot Q_4) - \alpha_3(C_H(C)_4\cdot Q_2 - C_H(C)_2 Q_4) + \alpha_4(C_H(C)_3\cdot Q_2 - C_H(C)_2\cdot Q_3) \\
& - \alpha_1(C_H(C)_4\cdot Q_3 - C_H(C)_3\cdot Q_4) + \alpha_3(C_H(C)_4\cdot Q_1 - C_H(C)_1\cdot Q_4) + \alpha_4(C_H(C)_3\,Q_1 - C_H(C)_1\cdot Q_3) \\
& + \alpha_1(C_H(C)_4\cdot Q_2 - C_H(C)_2\cdot Q_4) - \alpha_2(C_H(C)_4\cdot Q_1 - C_H(C)_1\cdot Q_4) + \alpha_4(C_H(C)_2\cdot Q_1 - C_H(C)_1\cdot Q_2) \\
& - \alpha_1(C_H(C)_3\cdot Q_2 - C_H(C)_2\cdot Q_3) + \alpha_2(C_H(C)_3\cdot Q_1 - C_H(C)_1\cdot Q_3) - \alpha_3(C_H(C)_2\cdot Q_1 - C_H(C)_1\cdot Q_2)
\end{aligned}
$$

$$
\begin{aligned}
\nabla(C_{H_q}) = & \ \alpha_2(\delta_{E_4}\cdot C_H(C)_3 - \delta_{E_3}\cdot C_H(C)_4) - \alpha_3(\delta_{E_4}\cdot C_H(C)_2 - \delta_{E_2}\cdot C_H(C)_4) + \alpha_4(\delta_{E_3}\cdot C_H(C)_2 - \delta_{E_2}\cdot C_H(C)_3) \\
& - \alpha_1(\delta_{E_4}\cdot C_H(C)_3 - \delta_{E_3}\cdot C_H(C)_4) + \alpha_3(\delta_{E_4}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_4) - \alpha_4(\delta_{E_3}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_3) \\
& + \alpha_1(\delta_{E_4}\cdot C_H(C)_2 - \delta_{E_2}\cdot C_H(C)_4) - \alpha_2(\delta_{E_4}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_4) + \alpha_4(\delta_{E_2}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_2) \\
& - \alpha_1(\delta_{E_3}\cdot C_H(C)_2 - \delta_{E_2}\cdot C_H(C)_3) + \alpha_2(\delta_{E_3}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_3) - \alpha_3(\delta_{E_2}\cdot C_H(C)_1 - \delta_{E_1}\cdot C_H(C)_2)
\end{aligned}
$$

RUDDER

$$0 = C_H(C) + C_{H_0} + \alpha C_H + \delta_R C_{H_{\delta_R}} + \beta C_{H_\beta}$$

AGAIN 4 DIFFERENT CASES

IT WILL BE NOTICED THAT THE ABOVE EXPRESSION IS ALMOST IDENTICAL WITH THAT FOR THE ELEVATORS & IF

δ_R IS READ FOR δ_E
β " " " Q
C_{H_β} " " " C_{H_q}

THE EQUATIONS FOR THE ELEVATOR DERIVATIVES CAN BE USED FOR THE RUDDER

AILERONS

$$0 = C_H(C) + C_{H_0} + \alpha C_{H\alpha} + \delta_A C_{H\delta_A} + \frac{pb}{2V} C_{Hp}$$

IF PORT & STBD HINGE MOMENTS ARE SUMMED

$$0 = C_H(C)_{PT} + C_H(C)_{ST} + C_{H\delta_A}(\delta_{A_{PT}} + \delta_{A_{ST}}) + \frac{pb}{V} C_{Hp}$$

PROVIDED THAT THE AILERONS ARE A MIRROR IMAGE OF EACH OTHER IN THE PLANE OF SYMMETRY

NOW SUBTRACTING THE HINGE MOMENTS

$$0 = C_H(C)_{PT} - C_H(C)_{ST} + 2(C_{H_0} + \alpha C_{H\alpha}) + C_{H\delta_A}(\delta_{A_{PT}} -$$

2 DIFFERENT RATES O ROLL; SUFFICES 1 & 2

$$C_{H\delta_A} = \frac{\left[(C_H(C)_{PT} + C_H(\,.\,)_{ST})_2\, P_1 - (C_H(C)_{PT} + C_H(C)_{ST})_1 \cdot P_2\right]}{\left[(\delta_{A_{PT}} + \delta_{A_{ST}})_1\, P_2 - (\delta_{A_{PT}} + \delta_{A_{ST}})_2 \cdot P_1\right]}$$

$$\frac{b}{V} C_{Hp} = \frac{\left[(\delta_{A_{PT}} + \delta_{A_{ST}})_2 (C_H(C)_{PT} + C_H(C)_{ST})_1 - (\delta_{A_{PT}} + \delta_{A_{ST}})_1 (C_H(C)_{PT} + C_H(C)_{ST})_2\right]}{\left[(\delta_{A_{PT}} + \delta_{A_{ST}})_1 \cdot P_2 - (\delta_{A_{PT}} + \delta_{A_{ST}})_2 \cdot P_1\right]}$$

THE VALUE OF $C_{H\delta_A}$ CALCULATED ABOVE WILL NOW BE USED IN THE ESTIMATION OF C_{H_0} & $C_{H\alpha}$

$$0 = \frac{C_H(C)_{PT} - C_H(C)_{ST} + C_{H\delta_A}(\delta_{A_{PT}} - \delta_{A_{ST}})}{2} + C_{H_0} + \alpha C_{H\alpha}$$

$$= R + C_{H_0} + \alpha C_{H\alpha}$$

$$C_{H_0} = (R_2 . \alpha_1 - R_1 . \alpha_2) / (\alpha_2 - \alpha_1)$$

$$C_{H\alpha} = (R_1 - R_2) / (\alpha_2 - \alpha_1)$$

IT SHOU D BE NOTED THAT THE VALUES OF C_{H_0} & $C_{H\alpha}$ APPLY TO THE PORT AILERON. THE STBD' AILERON VALUES ARE NUMERICALLY THE SAME BUT THE SIGN MUST BE REVERSED.

OSCILLATION TESTS

ALL ANGLES, VELOCITIES, ACCELERATIONS, & HINGE MOMENTS ARE DISTURBANCE VALUES FROM THE STEADY STATE UNLESS A SUFFIX 1 IS ATTACHED, IN WHICH CASE THEY ARE STEADY STATE VALUES.

THE TESTS SHOULD BE MADE BY DISTURBANCE FROM STEADY LEVEL FLIGHT, OTHERWISE THE FOLLOWING EQUATIONS DO NOT HOLD

ELEVATOR

SINCE THE ESTIMATE OF C_{Hq} FROM THE STEADY STATE IS NOT EXPECTED TO BE RELIABLE IT WILL BE RE-ESTIMATED HERE.

$$0 = C_H(C) + \alpha C_{H\alpha} + \frac{q\bar{c}}{2V_1} C_{Hq} + \frac{\dot{\alpha}\bar{c}}{2V_1} C_{H\dot{\alpha}}$$

$$\dot{\alpha} - q = -(n_n + \theta \sin \Theta)\frac{g}{V_1}$$

$$\therefore 0 = C_H(C) + \alpha C_{H\alpha} + \frac{q\bar{c}}{2V_1}(C_{Hq} + C_{H\dot{\alpha}}) - n_n \frac{\bar{c}g}{2V_1^2} C_{H\dot{\alpha}} - \theta\left(\sin\Theta . \frac{g\bar{c}}{2V_1^2} . C_{H\dot{\alpha}}\right)$$

NOW FROM REF 1

$$C_H(C) = C_H(A) + \frac{m_E V_1}{\bar{q} S_E C_E}\left(I_1 \frac{g}{V_1}(n_n + \theta \sin\Theta) - I_2 . \alpha_1 . q + I_3 \frac{\dot{q}}{V_1} - \theta . W_1 . \frac{g}{V_1}\right)$$

THE RESULTING EQN. FOR THE HINGE MOMENT DERIVATIVES CAN BE EXPRESSED THUS:

$$0 = C_H(A) + \alpha C_{H\alpha} + n_n k_1(k_2 - C_{H\dot{\alpha}}) + q . k_3(k_4 + C_{Hq} + C_{H\dot{\alpha}}) + \dot{q}\, k_5 . k_6 - \theta k_1(k_7 + \sin\Theta(C_{H\dot{\alpha}} - k_2))$$

THE AMPLITUDE & PHASING OF THE OSCILLATION SHOULD BE OBTAINED BY THE METHOD INDICATED N REF. 3. THE PHASING IS MEASURED W.R.T. α. AFTER EQUATING REAL & IMAGINARY PARTS OF THE ABOVE & SOLVING THE RESULTING EQNS.

$$C_{H\dot{\alpha}} = (c\, b_2 - c_2 b_1)/(a_1 b_2 - a_2 b_1)$$

$$C_{Hq} = (a_1 c_2 - a_2 c_1)/(a_1 b_2 - a\ b_1)$$

WHERE

$$-c_1 = |C_H(A)_0| \sin \angle C_H(A) + k_1 . k_2 |n_{n_0}| \sin \angle n_n + k_3 . k_4 |q_0| \sin \angle q + k_5\, k_6 . |\dot{q}_0| \sin \angle \dot{q}$$
$$- k_1(k_7 - k_2 \sin\Theta)|\theta_0| \sin \angle \theta$$

$$-C_2 = |C_H(A)_0| \cos\underline{\lvert C_H(A)} + C_{H\alpha}|\alpha_0| + k_1 k_2 |n_{n_0}| \cos\underline{\lvert n_n} + k_3 . k_4 |q_0| \cos\underline{\lvert q}$$

$$+ k_5 . k_6 |\dot{q}_0| \cos\underline{\lvert \dot{q}} - k_1 (k_7 - k_2 \sin\Theta) |\theta_0| \cos\underline{\lvert \theta}$$

$$a_1 = k_3 |q_0| \sin\underline{\lvert q} - k_1 \left(|n_{n_0}| \sin\underline{\lvert n_n} + |\theta_0| \sin\underline{\lvert \theta} . \sin\Theta \right)$$

$$a_2 = k_3 |q_0| \cos\underline{\lvert q} - k_1 \left(|n_{n_0}| \cos\underline{\lvert n_n} + |\theta_0| \cos\underline{\lvert \theta} . \sin\Theta \right)$$

$$b_1 = k_3 |q_0| \sin\underline{\lvert q}$$

$$b_2 = k_3 |q_0| \cos\underline{\lvert q}$$

$$k_1 = \frac{g\bar{c}}{2V_1^2} \; ; \; k_2 = \frac{4 m_E I_1}{\rho S_E c_E \bar{c}} \; ; \; k_3 = \frac{\bar{c}}{2V_1} \; ; \; k_4 = -\frac{4 m_E I_2 \alpha_1}{\rho S_E c_E \bar{c}} ; \; k_5 = \frac{\bar{c}\,\tau}{2V_1}$$

$$k_6 = \frac{4 I_3}{c_E \bar{c}} \; ; \; k_7 = \frac{4 m_E W_1}{\rho S_E c_E \bar{c}} \; ; \; \tau = \frac{m_E}{\rho S_E V_1}$$

$$I_1 = (\cos\Gamma . \cos\delta_E + \sin\chi_E . \sin\Gamma . \sin\delta_E) \ell_E$$

$$I_2 = \cos\chi_E . \sin\delta_E \, \ell_E$$

$$I_3 = -\Big\{ I_1 x_{0_E} + I_2 z_{0_E} + \cos\chi_E . \cos\Gamma . k_{E_Y}^2$$

$$+ (\sin\chi_E . \cos\Gamma \cos\delta_E + \sin\Gamma \sin\delta_E) k_{E_{XY}} \Big\}$$

$$W_1 = \left\{ (\sin\chi_E . \sin\Gamma . \sin\Theta + \cos\chi_E . \cos\Theta) \sin\delta_E + \cos\Gamma . \sin\Theta . \cos\delta_E \right\} \ell_E$$

ALL THE ABOVE EQNS. APPLY EQUALLY WELL TO BOTH SIDES OF THE ELEVATOR

AILERONS

$$0 = C_H(C) + \beta C_{H\beta} + \frac{pb}{2V_1} C_{Hp} + \frac{\rho b \tau}{2V_1} C_{H\dot{p}}$$

STBD AILERON

$$C_H(C) = C_H(A) + \frac{m_A}{\bar{q} S_A c_A}\left(-g(n_e + \phi \cos\Theta)I_1 + \dot{p} I_2 + \dot{r} I_3 + \phi g W_1\right)$$

&

$$0 = C_H(A) + \beta C_{H\beta} + n_e k_1 k_2 + p k_3 C_{Hp} + \dot{p} k_4 (k_5 + C_{H\dot{p}}) + \phi k_1 (k_6 + k_7) + \dot{r}\, k_4 . k_8$$

THE AMPLITUDE & PHASING OF THE OSCILLATION SHOULD BE OBTAINED BY THE METHOD OF REF 2. THE PHASING IS MEASURED W.R.T β. AFTER EQUATING REAL & IMAGINARY PARTS OF THE ABOVE & SOLVING THE RESULTING EQNS.

$$C_{Hp} = c_1 / b_1$$

$$C_{H\beta} = c_2/a_2 - b_2 c_1/a_2 b_1$$

WHERE

$$-c_1 = |C_H(A)_0| \sin \underline{C_H(A)} + k_1 k_2 |n_{e_0}| \sin \underline{n_e} + k_3 C_{Hp} |p_0| \sin \underline{p} + k_4 . k_5 |\dot{p}_0| \sin \underline{\dot{p}}$$
$$+ k_1(k_6 + k_7) |\phi_0| \sin \underline{\phi} + k_4 . k_8 |\dot{r}_0| \sin \underline{\dot{r}}$$

$$-c_2 = |C_H(A)_0| \cos \underline{C_H(A)} + k_1 k_2 |n_{e_0}| \cos \underline{n_e} + k_3 C_{Hp} |p_0| \cos \underline{p} + k_4 . k_5 |\dot{p}_0| \cos \underline{\dot{p}}$$
$$+ k_1(k_6 + k_7) |\phi_0| \cos \underline{\phi} + k_4 . k_8 |\dot{r}_0| \cos \underline{\dot{r}}$$

$$a_1 = 0 \quad ; \quad b_1 = k_4 |\dot{p}_0| \sin \underline{\dot{p}}$$

$$a_2 = |\beta_0| \quad ; \quad b_2 = k_4 |\dot{p}_0| \cos \underline{\dot{p}}$$

$$k_1 = \frac{gb}{2V_1^2} \; ; \; k_2 = \frac{-4 m_A I_1}{\rho S_A c_A b} \; ; \; k_3 = \frac{b}{2V_1} \; ; \; k_4 = \frac{bT}{2V_1} \; ; \; k_5 = \frac{4 I_2}{c_A b} \; ; \; k_6 = \frac{4 m_A W_1}{\rho S_A c_A b}$$

$$k_7 = \frac{-4 m_A \cos\Theta . I_1}{\rho S_A c_A b} \; ; \; k_8 = \frac{4 I_3}{c_A b} \; ; \; \tau = \frac{m_A}{\rho S_A V_1}$$

$$I_1 = (\sin\chi_A . \cos\Gamma . \sin\delta_A - \sin\Gamma . \cos\delta_A) \ell_A$$

$$I_2 = \left(z_{0_A} I_1 + \sin\chi_A k_{AY}^2 - \cos\chi_A . \cos\delta_A k_{A_{XY}}\right)$$

$$I_3 = \left(x_{0_A} I_1 - \cos\chi_A . \sin\Gamma . k_{AY}^2 + (\cos\Gamma . \sin\delta_A - \sin\chi_A . \sin\Gamma . \cos\delta_A) k_{A_{XY}}\right)$$

$$W_1 = (\sin\chi_A.\cos\Gamma.\cos\Theta.\sin\delta_A - \sin\Gamma\cos\Theta.\cos\delta_A)\ell_A$$

PORT AILERON

WITH THE FOLLOWING VALUES FOR I & W THE EQNS. FOR $C_{H\dot{p}}$ & $C_{H\beta}$ ARE THE SAME AS FOR THE STARBOARD AILERON.

$$I_1 = -(\sin\chi_A.\cos\Gamma.\sin\delta_A + \sin\Gamma.\cos\delta_A)\ell_A$$

$$I_2 = z_{0_A} I_1 + \sin\chi_A k_{A_y}^2 - \cos\chi_A.\cos\delta_A.k_{A_{xy}}$$

$$I_3 = x_{0_A} I_1 - \cos\chi_A.\sin\Gamma.k_{A_y}^2 - (\cos\Gamma.\sin\delta_A + \sin\chi_A.\sin\Gamma.\cos\delta_A)k_{A_{xy}}$$

$$W_1 = -(\sin\chi_A.\cos\Gamma.\cos\Theta.\sin\delta_A + \sin\Gamma.\cos\Theta\cos\delta_A)\ell_A$$

RUDDER

$$0 = C_H(C) + \beta C_{H\beta} + \frac{rb}{2V_1}\left(C_{H_r} - C_{H\dot{\beta}}\right) + \left(\frac{g}{V_1}\left(n_\ell + \phi\cos\Theta\right) + p\alpha_1\right)\frac{b}{2V_1} C_{H\dot{\beta}}$$

$$C_H(C) = C_H(A) + \frac{m_R}{qS_R c_R}\left(g(n_\ell + \phi\cos\Theta)I_1 - \dot{r} I_2 - \dot{p} I_3 - g\phi W_1\right)$$

&

$$0 = C_H(A) + \beta C_{H\beta} + n_\ell k_1\left(k_2 + C_{H\dot{\beta}}\right) + p\,k_3\alpha_1 C_{H\dot{\beta}} + \dot{p}\,k_4 k_5 + r k_3\left(C_{H_r} - C_{H\dot{\beta}}\right) + \dot{r} k_4 k_6$$
$$+ \phi k_1\left(k_2\cos\Theta + k_7 + C_{H\dot{\beta}}\cos\Theta\right)$$

THE AMPLITUDE & PHASING SHOULD BE OBTAINED BY THE METHOD OF REF. 2. THE PHASING IS MEASURED W.R.T. β. AFTER EQUATING IMAGINARY & REAL PARTS OF THE ABOVE & SOLVING THE RESULTING EQUATIONS:

$$C_{H\dot{\beta}} = (c_1 b_2 - c_2 b_1)/(a_1 b_2 - a_2 b_1)$$

$$C_{H_r} = (a_1 c_2 - a_2 c_1)/(a_1 b_2 - a_2 b_1)$$

WHERE

$$-c_1 = |C_H(A)_0|\sin\underline{\angle C_H(A)} + k_1 k_2 |n_{\ell_0}|\sin\underline{\angle n_\ell} + k_4 . k_5 |\dot{p}_0|\sin\underline{\angle \dot{p}} + k_4 . k_6 |\dot{r}_0|\sin\underline{\angle \dot{r}}$$
$$+ k_1(k_2\cos\Theta + k_7)|\phi_0|\sin\underline{\angle\phi}$$

$$-c_2 = |C_H(A)_0|\cos\underline{\angle C_H(A)} + C_{H\beta}|\beta_0| + k_1 k_2 |n_{\ell_0}|\cos\underline{\angle n_\ell} + k_4 k_5 |\dot{p}_0|\cos\underline{\angle \dot{p}}$$
$$+ k_4 k_6 |\dot{r}_0|\cos\underline{\angle \dot{r}} + k_1(k_2\cos\Theta + k_7)|\phi_0|\cos\underline{\angle\phi}$$

$$a_1 = k_1|n_{\ell_0}|\sin\underline{\angle n_\ell} + k_3\alpha_1|p_0|\sin\underline{\angle p} - k_3|r_0|\sin\underline{\angle r} + k_1\cos\Theta|\phi_0|\sin\underline{\angle\phi}$$

$$a_2 = k_1|n_{\ell_0}|\cos\underline{\angle n_\ell} + k_3\alpha_1|p_0|\cos\underline{\angle p} - k_3|r_0|\cos\underline{\angle r} + k_1\cos\Theta|\phi_0|\cos\underline{\angle\phi}$$

$$b_1 = k_3|r_0|\sin\underline{\angle r} \;;\; b_2 = k_3|r_0|\cos\underline{\angle r}$$

$$k_1 = gb/2V_1^2 \;;\; k_2 = 4I_1 m_R/\rho S_R c_R b \;;\; k_3 = b/2V_1 \;;\; k_4 = b\tau/2V_1$$

$$k_5 = -4I_3/c_R b \;;\; k_6 = -4I_2/c_R b \;;\; k_7 = -4m_R W_1/\rho S_R c_R b$$

$$I_1 = \cos\delta_R . \ell_R$$

$$I_2 = \cos X_R k_{R_Z}^2 + \cos\delta_R(x_{O_R} . \ell_R + \sin X_R . k_{R_{XZ}})$$

$$I_3 = \sin X_R k_{R_Z}^2 - \cos\delta_R . \cos X_R . k_{R_{XZ}}$$

$$W_1 = \cos\Theta . \cos\delta_R . \ell_R$$

NOTE THAT IN USING $\dot{p}$, $\dot{r}$, p & r, THE CORRECTED VALUES NOT THE TRACE VALUES SHOULD BE USED. THE CORRECTED VALUES CAN BE OBTAINED BY THE METHOD INDICATED IN REF. 2.