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REVIEW OF
NOTE ON STABILITY AND CONTROL CHARACTERISTICS OF A. V. ROE CANADA PROJECT Y"

A. V. Roe Canada Limited
Malton, Ont.

March 24, 1953.
Copy No. 1
To - Dr. J. J. Green

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REVIEW OF

NOTE ON STABILITY AND CONTROL CHARACTERISTICS OF A. V. ROE CANADA PROJECT Y"

Quoting from this note: "Two .... problems .... have been introduced by the proposal to use a large gyroscope for direct stabilization of the aircraft:

1. Some means must be sought for damping the high frequency oscillations in roll and pitch.

2. The different type of response of the aircraft under different flight conditions means that the design of an adequate control system will be difficult if not impossible."

1. SUMMARY REVIEW

The inertia values used in this note were guessed at an early stage. A recent detail evaluation produced very different values. Furthermore, coupled with this fact, two serious errors in the note invalidate its conclusions.

1. If it were right, the first criticism - that the nutation is undamped - is the more serious in our view, because this oscillation is a purely gyroscopic phenomenon. But, and please note that this is a serious and inexcusable blunder, the curves of Fig. 2(a) are not even the right answer to equations (6) and (7) (themselves incorrect) of the report; using the values given. For these equations show that the period of the nutation is about $2\frac{1}{2}$ cycles/sec. and that it damps to $\frac{1}{2}$ amplitude in about $\frac{3}{4}$ sec. This is simple arithmetic and you don't need a Philbric computer to work it out. Indeed, the inaccuracy of the computer for this calculation may be responsible for the blunder.

2. We are mystified by the N.A.E.'s excessive alarm about handling characteristics, although we realize it is partly engendered by their wrong results.

It appears that the value of K_3/ρ assumed is about double what it should be. $K_3/\rho = 2200\ h\sigma M^2$ implies a lift curve slope of nearly 5.0 per radian whereas later in the report a value of 2.0 per radian is mentioned and with this we agree. Hence the N.A.E. who say they have assumed a C.P. range of 25% - which is excessive in our opinion - have in fact effectively assumed 60% for these purposes, which is ridiculous. See Figures 2(b) and 4(b) in their corrected form at the end of this review.

Consider the subsonic case 4(b) which has shocked the author of this note so much: the pilot intends a pitch up; there is a big negative static margin so when he pulls the stick back and the aircraft starts to pitch, it gets its extra lift well forward of the C.G. and the moment thus created induces a roll; the pilot opposes the roll by moving the stick sideways thus putting the flaps down and restoring the trim; the immediate correction of the induced roll is natural and obvious as we have said (all this happens quite slowly), and having learned to fly the aeroplane the pilot would automatically move the stick along the right path. Then if necessary the stick can be trimmed back into the middle at the new speed and attitude. Supersonically, the pilot finds there is little change of trim with speed or attitude.

March 24, 1953.

1. SUMMARY REVIEW (continued)

So much the better. Again, in accelerating through the transonic region the aircraft would not tend to tuck under, but to roll. The pilot would be perfectly aware of his change of trim and response. This behaviour cannot with propriety be called "erratic" and it is impertinent to deduce from such a cursory examination that "the aircraft as described in the Company's brochure would be uncontrollable in flight".

2. DETAIL REVIEW

The nutation (the word is in the O.E.D.) was referred to in the "Proposal for a Gas-Turbine Propelled All-Wing Aircraft of Circular Plan-Form" - April, 1952 (" .... there may be some form of nutatory motion ...." (p.9. 3rd para.).

It was not possible to investigate the stability thoroughly in the limited time available to produce a preliminary statement on the Project (July-Sept., 1952) and indeed there is still a great deal to be done), and the probable effects of having a powerful gyroscope built in were presented in simple terms. Subsequent to Dr. Mack's work at the beginning of October, which highlighted the possible importance of the nutation, general equations of motion were set up and the nutation mode was especially carefully studied. Using our guessed values for the relative inertias, which subsequently turned out to be badly wrong, Dr. Mack's equations had shown that the nutation mode might be serious if undamped. The full equations show that in general the nutation mode is satisfactorily damped and this we understand has now been confirmed by a further investigation on the REAC simulator.

Equations (6) and (7) of the N.A.E. note are incorrect since they assume straight level motion of the aircraft C.G., which is impossible with changing incidence at constant speed. Hence the damping derivative Z_ω has been omitted (this is elaborated later in this review). However, taking the equations as they stand and writing λ for d/dt the complementary function can be written

$$\begin{vmatrix} \lambda^2 + \frac{K_1}{\alpha}\lambda & \frac{\Omega}{\alpha}\lambda \\ -\frac{\Omega}{\beta}\lambda & \lambda^2 + \frac{K_2}{\beta}\lambda + \frac{K_3}{\beta} \end{vmatrix} = 0$$

This yields the cubic

$$\lambda^3 + \left(\frac{K_1}{\alpha} + \frac{K_2}{\beta}\right)\lambda^2 + \left(\frac{K_3}{\beta} + \frac{K_1K_2}{\alpha\beta} + \frac{\Omega^2}{\alpha\beta}\right)\lambda + \frac{K_1K_3}{\alpha\beta} = 0$$

Taking $\frac{K_1}{\alpha} = \frac{K_2}{\beta} = 1$ as suggested is to say that the damping in pitch and roll is 4,000 ft.lb./rad. per sec. This is reasonable for M = 2 at 50,000 ft. but is excessive for M = 0.5 at 36,000 ft. for which we can only imagine it has also been used since it has not been presented in co-efficient form.

March 24, 1953.

2. DETAIL REVIEW (continued)

At M = 2 at 50,000 ft. then with h = + 0.05

$$K_3/\beta = 67$$

Hence the equation becomes

$$\lambda^3 + 2\lambda^2 + 264\lambda + 67 = 0$$

One root of this equation is $\lambda = -0.254$ and the remaining quadratic is

$$\lambda^2 + 1.846\,\lambda + 263.5 = 0$$

from which $\lambda = .923 \pm 16.2\,i$

i.e. the period is .39 secs. ($2\frac{1}{2}$ cycles per second) and the time to damp to altitude is 0.75 secs.

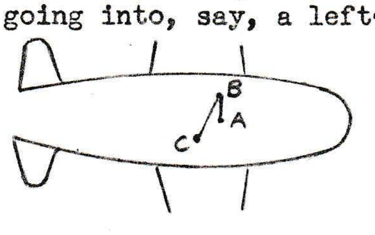
This is not what has been drawn on Figure 2(a).

It can readily be verified that the period and damping are not altered significantly by omitting K_3/β. If the most recent estimate of rotor inertia is used we find the period is about .14 sec. (7 cycles per sec.) and the time to damp to $\frac{1}{2}$ amplitude about $\frac{1}{2}$ sec.

Thus the damping is similar to that evaluated for the S.L. M = 0.75 case, as would be expected since the dynamic pressure is of the same order.

The possibility of exciting the nutation statically where the only damping considered is due to the flow of air through the engine has been examined and found to be negligible.

From the original paper about this layout - "Proposal for a Gas-Turbine Engined All-Wing Aircraft of Circular Plan-Form" on p.8 we quote: "Power operation (of controls) is envisaged with a trim control to place the stick central, at any trimmed condition. The large nose-up moment due to the C.P. and C.G. positions at subsonic speed is balanced by a downward trim on the elevator part of the halo". The change in response characteristic with Mach No. has been appreciated ab initio. Even with the quite large "cross-back" angles which may result in some flight conditions we consider it quite acceptable since the response is slow. In general what matters is the immediate response of the aircraft to a control movement and not the aircraft's subsequent behaviour. Pull the stick back and the aircraft starts to pitch; if a roll developes move the stick against the roll and the roll checks. To illustrate the point that the controls have to be moved all over the place to regulate an aircraft's behaviour subsequent to an initial response consider a conventional aircraft going into, say, a left-hand turn:



March 24, 1953.

2. DETAIL REVIEW (continued)

1. Start a roll by stick movement from A to B plus some left rudder.
2. Then move the stick back to C, to stay in a steady turn.

The control rolling moment is in the "opposite direction" in order to trim to zero rate of roll and the rearward stick movement is required to prevent the nose dropping.

It is just as relevant to say that the conventional aircraft has rolled in the "opposite direction" to the trimmed rolling moment as it is to say the same, more or less, of the V.T.O. aircraft. In fact it is not relevant at all. Indeed the ultimate response to a control step function is frequently academic, as when the response to a control step function on the rudder of a normal aircraft is considered, for this will, after a lapse of time, send the aircraft into a steep spiral dive.

It is clear that the induced roll will follow fairly hard on the heels of the required pitch and although for instance, Fig. 4(b) of the N.A.E. note should be considerably modified as shown, nevertheless the induced roll angle can be large at high subsonic speeds at low altitude with a large negative static margin. We feel this will probably be quite acceptable. However, it must be realized that this evaluation makes assumptions as to C.P. position which are unconfirmed by test and that C.P. position is excessively imponderable on this aircraft at subsonic speed as has been pointed out in a recent memorandum. Furthermore, it should not be taken for granted that equal control stick movements, fore and aft or side to side will give equal applied moments on the aeroplane. A compromise is possible to give better overall harmonization if the roll control is made relatively more sensitive: e.g. if the full range of elevator up and down is obtained with a fairly small control movement then to get pure pitch in the worst case the movement of the stick will be more fore and aft-wise than sideways.

The philosophy behind this design is to make the handling good at supersonic speed. It need only be "acceptable" at subsonic speed. If it can be made acceptable then is the proper time to consider fitting an auto-pilot to relieve the pilot of fatigue, should any considerable subsonic duration be required.

It tends to be forgotten that we are dealing with a basically stable configuration. The fastest divergence so far found takes about <u>one minute</u> to double amplitude and is, therefore, negligible whereas the damping of the oscillatory modes has been found quite satisfactory.

The value of K_3/β may be arrived at as follows:

March 24, 1953.

2. DETAIL REVIEW (continued)

Pitching moment due to increase of $\theta_2 = \frac{\delta C_L}{\delta \theta_2} \theta_2 \times qS \times h\bar{c}$

From equation (7) This pitching moment $= K_3 \theta_2 C'$

Hence $\frac{K_3}{\beta} = \frac{C_L}{\theta_2} \cdot \frac{S\bar{c}}{\beta C'} \cdot qh$

Above the tropopause $q = 1110 \sigma M^2$ lb./sq. ft.

For the aircraft the values assumed in the N.A.E. note are
$S = 500$ sq. ft. $\bar{c} \doteq 22$ ft. $\beta = 6.0$ $C' = 4000$ slug ft.^{2}
and it is also stated that the lift curve slope in 2.0 per rad.

$$\therefore \quad \frac{K_3}{\beta} = 2.0 \times \frac{500 \times 22}{6 \times 4000} \times 1110 \quad h\sigma M^2 = 1000\ h\sigma M^2$$

The value in the N.A.E. note is 2200 h M^2. Moreover, a C.P. range of 25% has been assumed. Effectively, therefore, they have assumed a C.P. shift of 12 ft. on this little aircraft, which is absurd.

3. MODIFIED EQUATIONS AND RESULTS

We have pointed out that the equations set up by the N.A.E. are unrealistic in that they do not, as is admitted, include effects due to deviation of the aircraft C.G. from constant speed straight line motion. In considering response to control as distinct from the nutation oscillation we do not think this is an acceptable approximation.

If interest is concentrated on the behaviour of the aircraft within a few seconds of a disturbance being initiated it will, however, be legitimate to consider

$$u = \psi = v = 0$$

Then the equations of motion based on gyro axes are reduced to:

$$\begin{bmatrix} m\lambda - Z_w & -(mU_o + Z_{\dot{\theta}}) + mg\sin\theta_o & — \\ — & \omega I_z \lambda & A\lambda^2 - L_{\dot{\phi}}\lambda \\ -M_w & B\lambda^2 - m_{\dot{\theta}}\lambda & -\omega I_z \lambda \end{bmatrix} \begin{bmatrix} w \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} Z(t) \\ L(t) \\ M(t) \end{bmatrix} \qquad (1)$$

From previous work we know that the term $mg\sin\dot{\theta}_o$ has little influence on the motion and it will, therefore, be neglected.

March 24, 1953.

3. MODIFIED EQUATIONS AND RESULTS (continued)

Using Laplace transforms the subsidiary equation of (1) becomes

$$\begin{bmatrix} mp - Z_w & -(mU_o + Z_{\dot\theta}) & - \\ - & \omega I_z & Ap - L_{\dot\phi} \\ -M_w & Bp - M_{\dot\theta} & -\omega I_z \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{\theta}_p \\ \bar{\phi}_p \end{bmatrix} = \begin{bmatrix} \bar{Z}_p + mw_o - (mU_o + Z_{\dot\theta})\,\theta_o \\ \bar{L}_p + \omega I_z \theta_o + A(p\phi_o + \dot\phi_o) - L_{\dot\phi}\,\phi_o \\ \bar{M}_p + B(p\theta_o + \dot\theta_o) - M_{\dot\theta}\theta_o - \omega I_z \phi_o \end{bmatrix} \quad (2)$$

The complementary function is equal to

$$p^3 - \left(\frac{L_{\dot\phi}}{A} + \frac{M_{\dot\theta}}{B} + \frac{Z_w}{m}\right) p^2 + \left(\frac{\omega^2 I_z^2}{AB} + \frac{L_{\dot\phi} M_{\dot\theta}}{AB} + \frac{L_{\dot\phi}}{A}\frac{Z_w}{m}\right.$$

$$\left. + \frac{M_{\dot\theta}}{B}\frac{Z_w}{m} - U_o \frac{M_w}{B} - \frac{M_w}{B}\frac{Z_{\dot\theta}}{m}\right) p - \frac{\omega^2 I_z^2}{AB}\frac{Z_w}{m}$$

$$- \frac{L_{\dot\phi}}{A}\frac{M_{\dot\theta}}{B}\frac{Z_w}{m} + U_o . \frac{L_{\dot\phi}}{A} . \frac{M_w}{B} + \frac{L_{\dot\phi}}{A} . \frac{M_w}{B} . \frac{Z_{\dot\theta}}{m}$$

which will in general yield one real root and a complex pair. If the engine is considered to be stationary $\omega I_z = 0$ and it is easily seen that the longitudinal and lateral modes are uncoupled, the simplified lateral mode $(A_p - L_{\dot\phi})$ being a factor of the remaining expression.

In the N.A.E. note step functions in pitching and rolling moment are applied, neglecting normal forces due to control deflection. For this case

$$w_o = \theta_o = \theta_o = \phi_o = \phi_o = 0$$

$$\bar{Z}_p = 0 \quad \bar{L}_p = \frac{L_o}{p} \quad \bar{M}_p = \frac{M_o}{p}$$

The expression for $\bar\theta$ and $\bar\phi$ become

$$\bar\theta = \frac{\left(p - \frac{Z_w}{M}\right)\left[\frac{\omega I_z L_o}{AB} + \left(p - \frac{L_{\dot\phi}}{A}\right)\frac{M_o}{B}\right]}{p^2 (p + \alpha)(p^2 + \beta p + \gamma)}$$

$$\bar\phi = \frac{\left[p^2 - \left(\frac{M_{\dot\theta}}{B} + \frac{Z_w}{m}\right)p - \frac{U_o M_w}{B} + \frac{M_{\dot\theta}}{B} . \frac{Z_w}{m} - \frac{M_w}{B} . \frac{Z_{\dot\theta}}{m}\right]\frac{L_o}{A} - \left(p - \frac{Z_w}{m}\right)\frac{\omega I_z M_o}{AB}}{p^2 (p + \alpha)(p^2 + \beta p + \gamma)} \; .$$

Where the denominators represent the roots of the complementary function,

With $\omega I_z = 0$

$$\bar\theta = \frac{\left(p - \frac{Z_w}{M}\right)\frac{M_o}{B}}{p^2 (p^2 + \beta p + \gamma)} \qquad \bar\phi = \frac{L_o/A}{p^2 (p + \alpha)}$$

March 24, 1953.

3. MODIFIED EQUATIONS AND RESULTS (continued)

The results of these equations are contrasted with the N.A.E. results in the attached graphs.

In the first place the solution for the motion of the aircraft in response to a 10,000 ft.-lb. step function is given in Fig. 1(a) which shows the N.A.E. curve also. The corrected response is quite different, showing a continual increase of θ with time and a much more reasonable damping of the short period oscillation for a conventional aircraft with a 5% stable static margin.

Fig. 2(a) shows the damping of the nutation, which is contrasted with the N.A.E. note.

Fig. 2(b) shows three sets of curves.

1. As plotted in the N.A.E. note.

2. As modified by us, using the N.A.E. assumptions and also substituting a value of K_3/β corresponding to the assumed lift curve slope of 2.0 per radian, i.e. the answer the N.A.E. should have got.

3. Using the correct equations, the derivatives estimated by Dr. Bull in his advance note, and our recent inertia values i.e. making the following assumptions:

I_z = 8000 slug-ft.^{2}
A = 12000 " "
B = 17000 " "
m = 825 slugs
$\bar{c}$ = 20 ft.
S = 420 sq. ft.

$\frac{Z_w}{m} = -.2863$ $\quad \frac{M_w}{B} = -.01390$ $\quad \frac{L_{\dot{\phi}}}{A} = -.3656$

$\frac{Z_{\dot{\theta}}}{m} = -.07159$ $\quad \frac{L_0}{A} = .8333$ $\quad \frac{M_{\dot{\theta}}}{B} = -.8437$

(3) above, is very different from (1). The nutation is hardly noticeable on the scale of the plotting and has been omitted though it is there; and the induced roll lags well behind the required pitch. If the applied moment were increased 5 times for instance, then after 1 sec.(quite a long time) the pitch angle would be 4° and the induced roll angle if uncorrected by the pilot would have reached only $1\frac{1}{2}^{\circ}$. This assumes a 5% stable margin whereas the design may place this margin near zero, with much the same response as is shown in the N.A.E. Fig. 3, but without the nutation.

Fig. 4(b) shows two similar sets of curves (corresponding to (1) and (3) above) but here the induced roll is even milder than in Fig. 2(b).

We conclude that the N.A.E. note has painted a completely false picture of the likely stability and control characteristics of the V.T.O. aircraft; and finish with the following observations:

March 24, 1953.

3. MODIFIED EQUATIONS AND RESULTS (continued)

1. With regard to the control angle deflections required to produce a given rate of pitch or roll: the V.T.O. aircraft due to its gyroscope will in general require larger control angles than a conventional aircraft, in spite of assistance from thrust forces. It may be argued that quite large control angles and stick movements will be required on conventional aircraft at very high dynamic pressures due to aeroelastic distortion of the structure and large supersonic static margins but this can hardly be cited as an advantage.

2. The corollary of induced roll occuring in response to a pitching demand if uncorrected by the pilot, where the aircraft has a negative static margin, is increased rate of roll in response to rolling demand. Considering this case in more detail: the pilot moves the stick laterally applying a pitching moment: the aircraft starts to roll and the damping in roll induces pitch which, because of the negative margin reinforces the pitching moment he has applied: this is easily seen to be the case whether the applied moment induces pitch nose-up or nose-down i.e. whether a roll to right or left is desired. It appears that the one thing not required is a positive ("stable") margin.

3. The assumption that an applied moment acts instantaneously as a step function is, of course, unrealistic. Furthermore, it is unduly pessimistic for the purpose of evaluating the nutation amplitude; for where the time actually required to apply the moment is appreciably long compared with the period of the nutation the amplitude is much reduced. A report is in course of preparation in which a reasonable forcing functions have been considered with reference to the excitation of nutations and we conclude that for this aeroplane even in the static case (where damping is likely to be a minimum) the maximum amplitude is very small (of the order of $\pm 1/20^{\circ}$).

4. CRITIQUE

1. The note is technically very imperfect, as has been shown.

2. We cannot understand why we were not consulted about the two points which were so lengthily criticized. To say the least we could have supplied some corrected inertia figures.

3. The continual reference to applied pitching and rolling moments instead of pilot's control actions is muddling. At first glance for instance the reader may easily be confused by the statement "when a rolling moment is applied the aircraft goes into a roll in the opposite direction" - not realizing that the pilot would be moving the stick backwards or forwards and that he would in practice prevent the "roll in the opposite direction" from developing.

4. In discussions with Dr. Mack on the nutation question naturally all possible forms of damping were discussed. The N.A.E. lifts various somewhat impractical embryonic suggestions from Dr. Mack's report and gives them prominence without reference to us for the importance we attach to them. We find this objectionable.

5. In view of the above, and of the fact that it involves very sweeping criticism of other people's work we do not think the disparaging tone of the N.A.E. note can be excused.

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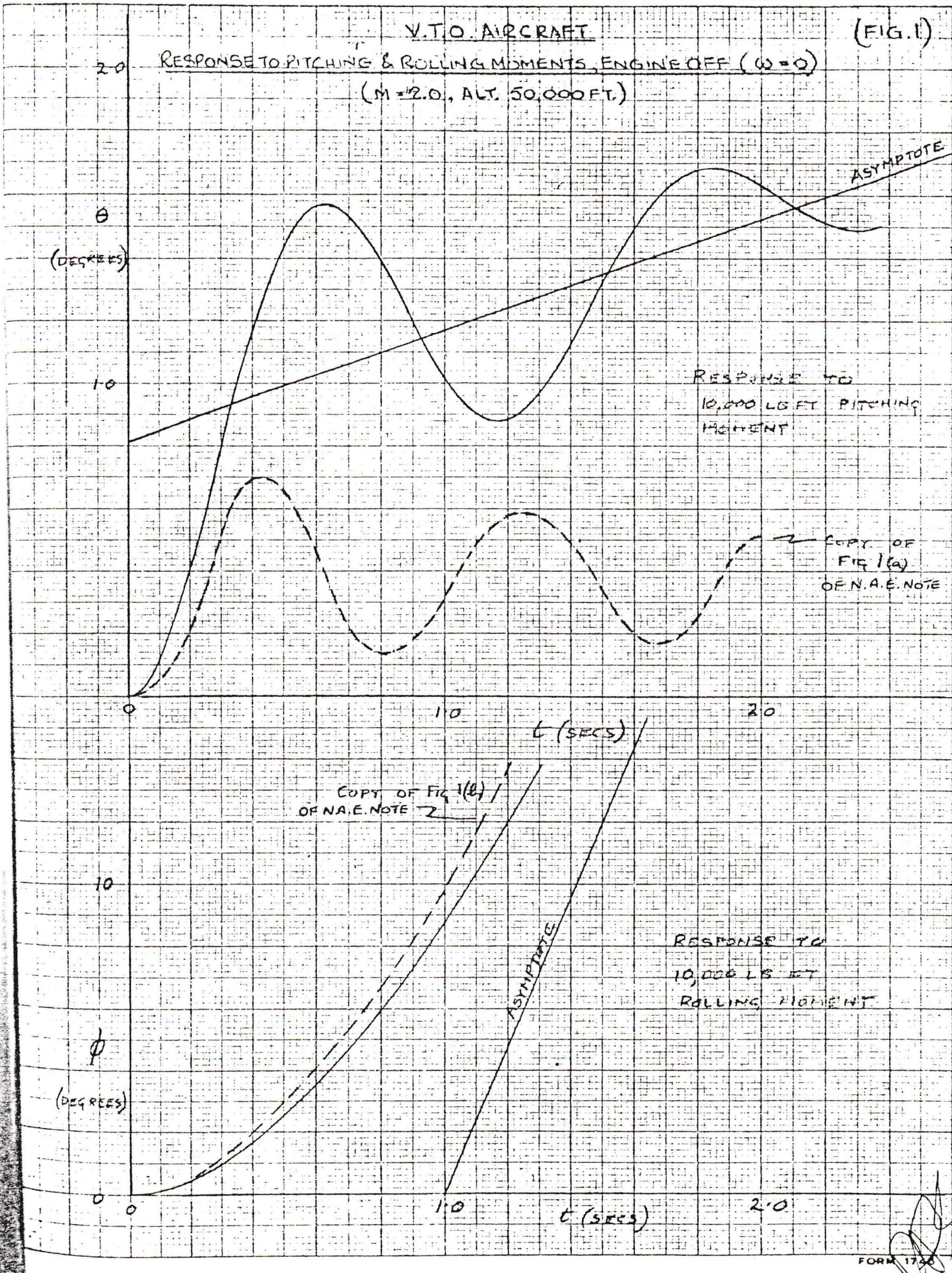


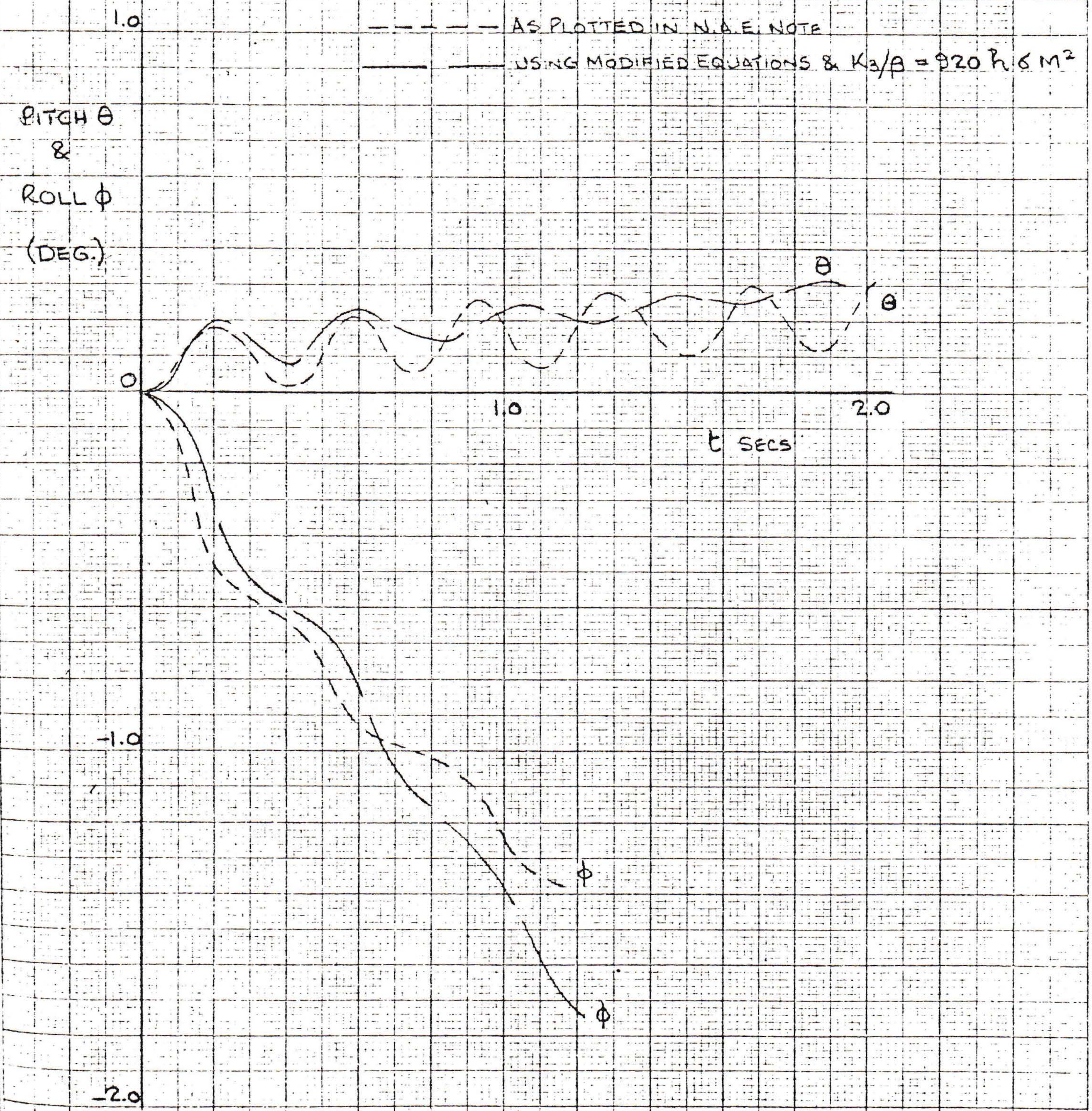
FORM 1750

FIG. 2 (a)

V.T.O. AIRCRAFT

RESPONSE TO APPLIED PITCHING MOMENT OF 10,000 LB. FT.

(M = 2.0, ALT. 50,000 FT.)



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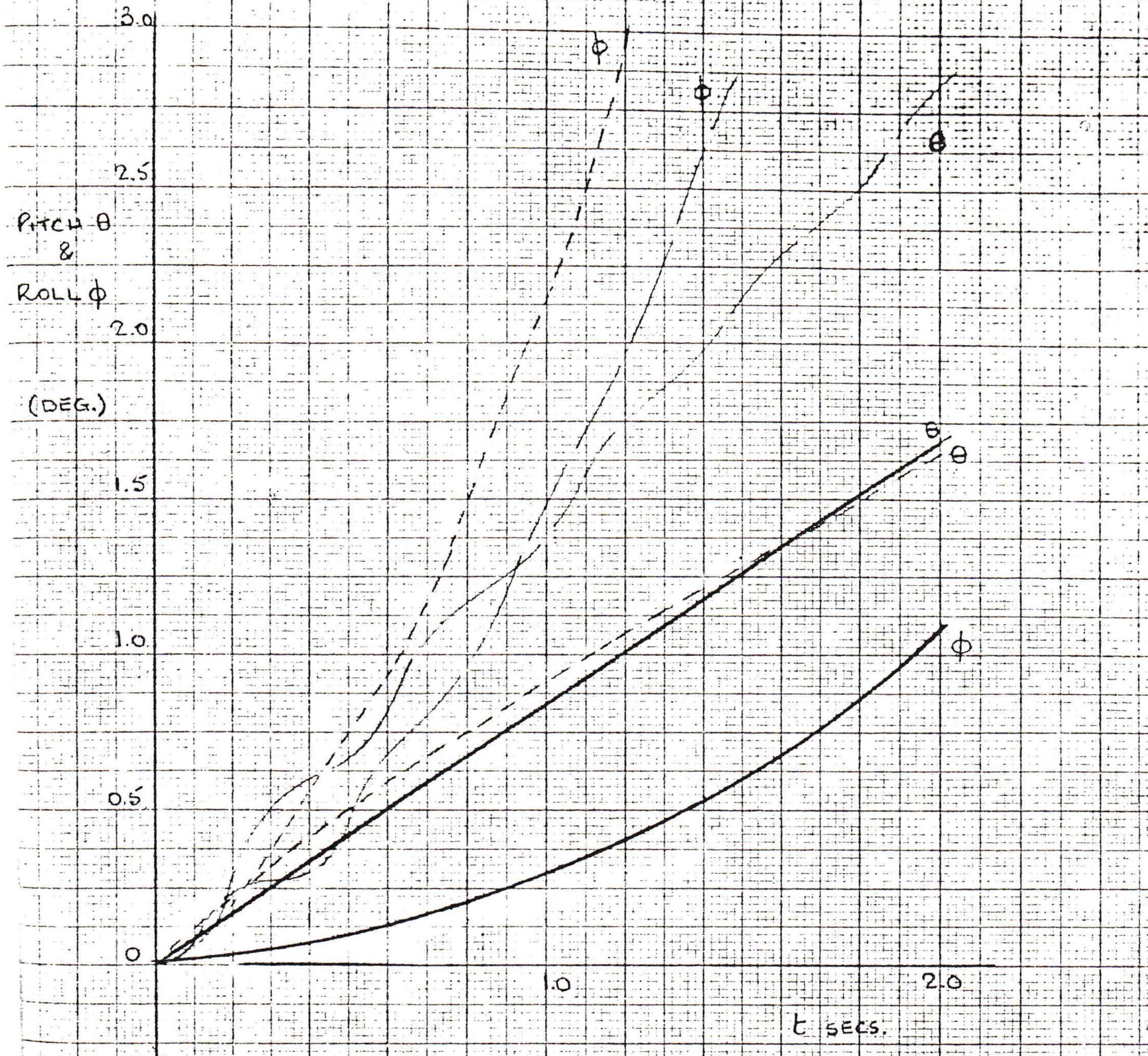
V.T.O. AIRCRAFT

FIG. 2 (b)

RESPONSE TO AFT STICK MOVEMENT (APPLIED ROLLING MOMENT = 10,000 LB. FT)

(M = 2.0 , ALT. 50,000 FT.)



- - - - - - - - - - - AS PLOTTED IN N.A.E. NOTE

______ ______ USING MODIFIED EQUATIONS AND $K_3/\beta = 920\ h\ \sigma\ M^2$

━━━━━━━━ USING MODIFIED EQUATIONS, DERIVATIVES FROM C.A.R.D.E. NOTE AND LATEST INERTIA VALUES.

PERIOD OF NUTATION = .1331 SECS

MAX. NUTATION AMPLITUDE IN θ = ± .018°

" " " φ = ± .021°

TIME TO HALVE AMPLITUDE = 1.147 SECS.

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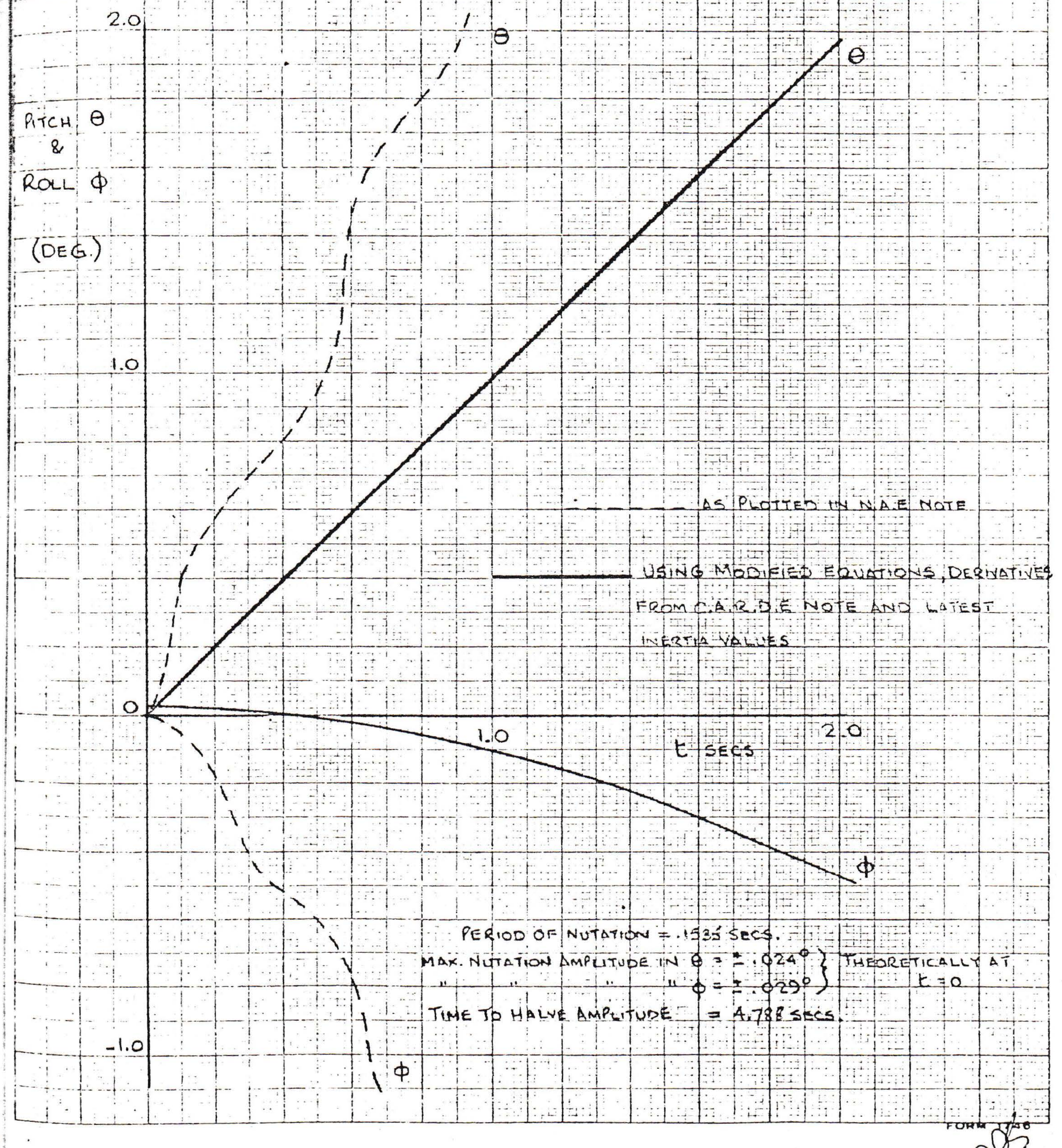
FIG. 4(b)

V.T.O. AIRCRAFT

RESPONSE TO AFT STICK MOVEMENT (APPLIED ROLLING MOMENT = 10,000 LB. FT.)

(M = 0.5, ALT. 36,000 FT.)



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