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CF. 105

DIGITAL COMPUTATION OF RESPONSE USING
AN APPROXIMATION TO PITCH DAMPER SYSTEM

PREPARED BY M. V. Jenkins

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Notation

Symbols are in current Avro notation unless otherwise
stated in the tests.

Reference

Digital computation of response using an approximation to
lateral damper system.

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INTRODUCTION

This report includes a method of calculations aircraft longitudinal response by a step-by-step time solution. The 704 digital computer facilitates the determination of incremental response during extremely small time increments and hence the usual inaccuracy due to employing relatively large time increments is eliminated.

The report also includes a method of determining, damper engaged condition, effective aircraft derivatives which may alternatively be described as synthetic aerodynamic derivatives since damper system aerodynamic outputs are classified according to which particular aircraft response promoted the input and then summed with the natural aircraft derivative. A typical but simplified example of this, is an aircraft response q which promotes a proportional signal to the differential servo-producing a proportional elevator pitching moment and lift output in phase with q . ΔC_{m_q} and ΔC_{z_q} have been created and these may be added to the natural aircraft C_{m_q} and C_{z_q} respectively to form effective or synthetic aerodynamic derivatives \bar{C}_{m_q} and \bar{C}_{z_q} .

The method also includes determination of the control derivatives and this yields values of the natural aircraft derivatives subsequent to the evaluation of the effective derivatives.

It is thought that the framing of the damper engaged condition equations of motion into a notation analogous to the free aircraft equations of motion expressed in familiar aerodynamic notation facilitates a quick clear physical interpretation of the functions of the damper system as regards aircraft response.

The parallel servo output is proportional to the integration with respect to time of the error between normal acceleration command and response and this is additional to the output proportional to the integration with respect to time of q ; however, it is the former contribution to which the following remarks apply. As command normal acceleration is arbitrary, a numerical step-by-step time solution was preferred to a Laplace transformation, utilization of which, would necessitate an approximate transform of the arbitrary command, the approach to which would vary according to the circumstances.



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The principle of the step by step time solution of aircraft response to an arbitrary command signal is discussed in reference 1. It is considered unnecessary to discuss the obvious applicability of the method to longitudinal response to an arbitrary normal acceleration command.



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DISCUSSION

It is suggested in the report that due to recording frequency in some instances being fixed at 20 values per second, that the number of useful times a derivative may be experimentally determined is approximately 10. The derivative for any given flight case is a constant and the main reason for repititious determination is inaccuracy in physical measurement; the reason for the suggested restriction to 10 evaluations is the natural boundary of the rapid highly damped response to the new steady state demanded by a constant incremental command signal.

However, there may well be an advantage in introducing a mid-point linear interpolation digital computer sub-routine to effectively double the sampling frequency of the values of the response variables. For a given number of solutions, the additional records occurring immediately after the impulse when the response variables are varying the most will more than offset the inaccuracy of linear interpolation, especially in the case of the rotary derivatives.

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LET δ BE THE INCREMENTAL ELEVATOR MOVEMENT FROM THE INITIAL STEADY STATE CONDITION

δ OUTPUT DUE TO q

δ OUTPUT FROM THE PARALLEL SERVO DUE TO q IS ASSUMED OF THE FORM:

$$\delta_o = K_o \int q \cdot dt \quad \text{WHERE } K_o = K_o' + K_o''$$

$$K_o' = K_o(t=0) : K_o'' = - \frac{.67 \int \delta_o \cdot dt}{\int q \cdot dt}$$

δ OUTPUT FROM THE DIFFERENTIAL SERVO DUE TO q IS ASSUMED OF THE FORM:

$$\delta_3 = K_3 q \quad \text{WHERE } K_3 = K_3' + K_3''$$

$$K_3' = K_3(t=0) : K_3'' = - \frac{.67 \int \delta_3 \cdot dt}{q}$$

$$\therefore \text{TOTAL } \delta \text{ FROM } q \text{ IS } K_o \int q \cdot dt + K_3 q$$

FORCING NORMAL FORCE ALONG OZ AXIS DUE TO q IS:

$$\left\{ \frac{\Delta C_{z_q}}{4\mu_1} \cdot q + \frac{C_{z_\delta}}{2\gamma} \left[K_o \int q \cdot dt \right] \right\} V_m$$

$$\text{WHERE } \Delta C_{z_q} = C_{z_\delta} \cdot K_3 \cdot \frac{2V}{\bar{c}}$$



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Forcing pitching moment due to q is :

$$\left\{ \frac{\Delta C_{z_q} \cdot q}{4\mu_1} + \frac{C_{z_\delta}}{2\tau} \left[K_o \int q \cdot dt \right] \right\} V_m$$

Where $\Delta C_{z_q} = C_{z_\delta} \cdot K_3 \cdot \frac{2V}{\phi}$

Forcing pitching moment due to q is :

$$\left\{ \frac{\Delta C_{m_q} \cdot q}{4\tau K^2_y} + \frac{C_{m_\delta}}{\tau^2 \phi} \left[K_o \int q \cdot dt \right] \right\} mK^2_y$$

where $\Delta C_{m_q} = C_{m_\delta} \cdot K_3 \cdot \frac{2V}{\phi}$

δ output from the parallel servo due to n is assumed of the form:

$$\delta = K_1 \int n \cdot dt$$

δ output from the parallel servo due to incremental N command signal is of the form

$$\delta = -K'_1 \int N \cdot dt$$

where K_1, K'_1 are constant for any given flight case and are positive in value.

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δ OUTPUT FROM THE DIFFERENTIAL SERVO DUE TO n
IS ASSUMED OF THE FORM:

$$\delta = +K_2 n$$

δ OUTPUT FROM THE DIFFERENTIAL SERVO DUE TO INCREMENTAL
 N COMMAND SIGNAL IS ASSUMED OF THE FORM:

$$\delta = -K'_2 N$$

TOTAL δ OUTPUT FROM N, n

$$\delta = \int (K_1 n - K'_1 N) dt + K_2 n - K'_2 N$$

FORCING NORMAL FORCE ALONG OZ-AXIS DUE TO N, n IS :

$$\left\{ \frac{C_{z\delta}}{2\gamma} \left[\int (K_1 n - K'_1 N) dt + K_2 n - K'_2 N \right] V_m \right\}$$

FORCING PITCHING MOMENT DUE TO N, n IS :


$$\left\{ \frac{C_{m\delta}}{\gamma^2 h} \left[\int (K_1 n - K'_1 N) dt + K_2 n - K'_2 N \right] m K_y^2 \right\}$$

K_2, K'_2 ARE CONSTANTS FOR ANY GIVEN FLIGHT CASE
AND ARE POSITIVE IN VALUE

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$$n = \alpha \frac{C_{2\alpha}}{2\gamma F'} + \alpha \frac{C_{2\alpha}}{4\mu_1 F'} + q \frac{C_{2q}}{4\mu_1 F'} + \frac{\alpha \sin i}{m V_1 F'} \cdot \frac{\partial T}{\partial \alpha} + \frac{T i \cdot \sin i}{m V_1 F'} - \frac{\dot{q} \bar{X}}{V_1 F'} + \frac{C_{2\delta}}{2\gamma F'} \left[K_0 \int q \cdot dt + K_1 \int n dt - K_1' \int N dt - K_2 N \right]$$

WHERE i IS +VE IF



The diagram illustrates a horizontal line with points X and O. A vertical arrow labeled i points downwards from the line. A diagonal line below it has an arrow pointing left and is labeled "THRUST +VE".

$$\text{AND } F' = \left(-\frac{g}{V} - \frac{K_2 \cdot C_{z_8}}{2\tau} \right) : C'_{z_9} = C_{z_9} + \Delta_1 C_{z_9}$$

$$-\frac{ng}{V} = +\alpha_1 q_1 \sin \alpha_1 + \ddot{\alpha} \cdot \cos \alpha_1 + \Theta \frac{g}{V} \cdot \sin(H) \cos \phi_1 - q_1 \cos \alpha_1 - \frac{\bar{X} \dot{q}}{V}$$

$$\begin{aligned} \therefore -n \left[\frac{g}{V} + \frac{K_2 C_{z\delta}}{2\tau} \right] &= -\alpha \frac{\Delta C_{z\alpha}}{2\tau} - \dot{\alpha} \frac{\Delta C_{z\dot{\alpha}}}{4\mu_1} - q \frac{\Delta C_{zq}}{4\mu_1} \\ &- \alpha \frac{\sin i}{m V_1 F'} \cdot \frac{\partial T}{\partial \alpha} \cdot \frac{K_2 C_{z\delta}}{2\tau} + \dot{q} \frac{\bar{X}}{V F'} \cdot \frac{K_2 C_{z\delta}}{2\tau} \\ &- \frac{C_{z\delta}}{2\tau F'} \cdot \frac{K_2 C_{z\delta}}{2\tau} \left[\int (K_0 q + K_1 n - K'_1 N) dt - K'_2 N \right] \\ &- \frac{\sin i}{m V_1 F'} \cdot \frac{T_0}{2\tau} \cdot \frac{K_2 C_{z\delta}}{2\tau} + \alpha \cdot q_1 \cdot \sin \alpha_1 + \dot{\alpha} \cos \alpha_1 \\ &+ \Theta \frac{g}{V} \cdot \sin(\Theta) \cdot \cos \Phi_1 - q \cdot \cos \alpha_1 - \dot{q} \frac{\bar{X}}{V} \end{aligned}$$

$$\text{LET } \Delta C_{z_\alpha} = \frac{K_2 C_{z_\delta} \cdot C_{z_\alpha}}{2 \gamma F'} : \Delta C_{z_\beta} = \frac{K_2 C_{z_\delta} \cdot C_{z_\beta}}{2 \gamma F'} : \Delta C_{z_q} = \frac{K_2 C_{z_\delta} \cdot C'_{z_q}}{2 \gamma F'} \gamma$$

$$\begin{aligned}
 -\frac{n.C_{m\delta}.K_2}{\gamma^2 h} &= -\alpha \frac{\Delta C_{m\alpha}}{\gamma^2 h} - \frac{\alpha \Delta C_{m\dot{\alpha}} \bar{c}^2}{4 K_1^2 \gamma} - q \frac{\Delta C_{m\ddot{\alpha}} \bar{c}^2}{4 K_1^2 \gamma} \\
 &\quad - \frac{\alpha \sin i}{m V_1 F'} \frac{\partial T}{\partial \alpha} \frac{C_{m\delta}.K_2}{\gamma^2 h} + \frac{\dot{q} \bar{X}}{V F'} \frac{C_{m\delta}.K_2}{\gamma^2 h} \\
 &\quad - \frac{C_{z\delta}}{2 \gamma F'} \frac{C_{m\delta}.K_2}{\gamma^2 h} \left[\int (K_0 q + K_1 n - K_1' N) dt - K_2 N \right] \\
 &\quad - \frac{\sin i}{m V_1 F'} \frac{T_0.C_{m\delta}.K_2}{\gamma^2 h}
 \end{aligned}$$

$$\text{WHERE } \Delta C_{m\alpha} = \frac{K_2.C_{z\alpha}.C_{m\delta}}{2 \gamma F'} : \Delta C_{m\dot{\alpha}} = \frac{K_2.C_{m\delta}.C_{z\dot{\alpha}}}{2 \gamma F'} : \Delta C_{m\ddot{\alpha}} = \frac{K_2.C_{m\delta}.C_{z\ddot{\alpha}}}{2 \gamma F'}$$

THE FOLLOWING NOTATION IS NOW ADOPTED:

$$C_{\bar{z}\alpha} = C_{z\alpha} + \Delta C_{z\alpha} : C_{\bar{z}\dot{\alpha}} = C_{z\dot{\alpha}} + \Delta C_{z\dot{\alpha}} : C_{\bar{z}q} = C_{zq}' + \Delta_2 C_{zq}$$

$$C_{\bar{m}\alpha} = C_{m\alpha} + \Delta C_{m\alpha} : C_{\bar{m}\dot{\alpha}} = C_{m\dot{\alpha}} + \Delta C_{m\dot{\alpha}} : C_{\bar{m}q} = C_{m\dot{q}}' + \Delta_2 C_{m\dot{q}}$$

$$\text{WHERE } C_{m\dot{q}}' = C_{m\dot{q}} + \Delta_1 C_{m\dot{q}}$$

THE TWO LONGITUDINAL EQUATIONS OF MOTION EXCLUDING SPEED AND HEIGHT EFFECTS ARE PRESENTED ON THE FOLLOWING PAGE

APPROXIMATION TO ARROW I PITCH DAMPER SYSTEM

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$$-\alpha \left[\frac{C_{z\alpha}}{2T} + \frac{a \sin i}{m V_i} \frac{\partial T}{\partial \alpha} - q_i \sin \alpha_i \right] - \dot{\alpha} \left[\frac{C_{z\alpha}}{4\mu_i} - \cos \alpha_i \right]$$

$$+ \Theta \frac{g}{V} \sin \Theta \cos \phi - q \left[\cos \alpha_i + \frac{C_{z\beta}}{4\mu_i} \right] + \dot{q} \left(\frac{a}{V} \right) \frac{\bar{x}}{V} = a \left[\frac{C_{z\delta}}{2T} \left[\left(K_0 q + K_{1n} - K'_1 N \right) dt - K'_2 N \right] + \frac{\sin i T_i}{m V_i} \right] \dots \dots \dots \text{OZ EQUATION}$$

$$-\alpha \left[\frac{C_{m\alpha}}{T^2 F} + \frac{\partial T}{\partial \alpha} \left\{ \frac{e + \sin i \bar{e} b}{m K_y^2} \right\} \right] - \dot{\alpha} \frac{C_{m\alpha}}{4 K_y^2 T}$$

$$- q \frac{C_{m\beta}}{4 K_y^2 T} \bar{e}^2 + \dot{q} \left[1 + \frac{\bar{x} \bar{e} b}{K_y^2} \right] = a \frac{C_{m\delta}}{T^2 F} \left[\left(K_0 q + K_{1n} - K'_1 N \right) dt - K'_2 N \right] + T_i \left[\frac{e + \sin i \bar{e} b}{m K_y^2} \right] \dots \dots \dots \text{PITCHING EQUATION}$$

$$n = -\frac{V_i}{g} \left[\alpha q_i \sin \alpha_i + \dot{\alpha} \cos \alpha_i + \Theta \frac{g}{V} \sin \Theta \cos \phi - q \cos \alpha_i - \dot{q} \frac{\bar{x}}{V} \right]$$

KINEMATIC RELATIONSHIP

$$a = \left[1 + \frac{K_2 C_{z\delta}}{2 T F} \right]$$

$$b = \frac{K_2 C_{m\delta}}{2 T F}$$

$$F' = \left[-\frac{g}{V} - \frac{C_{z\delta} K_2}{2 T} \right]$$

$$C_{z\alpha} = a C_{z\alpha}$$

$$C_{z\alpha} = a C_{z\alpha}$$

$$C_{z\beta} = a C_{z\beta}'$$

$$C_{m\alpha} = C_{m\alpha} + b C_{z\alpha}$$

$$C_{m\alpha} = C_{m\alpha} + b C_{z\alpha}$$

$$C_{m\beta} = C_{m\beta}' + b C_{z\beta}'$$

$$C_{z\beta}' = C_{z\beta} + K_3 C_{z\delta} \frac{2V}{\bar{e}}$$

$$\delta_0 = K_0 \int q dt$$

$$K_0 = K'_0 + K''_0$$

$$C_{m\beta}' = C_{m\beta} + K_3 C_{m\delta} \frac{2V}{\bar{e}}$$

$$K'_0 = K_0 (t=0)$$

$$K''_0 = -\frac{67 \int \delta_0 dt}{\int q dt}$$

ALL FUNCTIONS OF K_0, K_3

$$\delta_3 = K_3 q$$

$$K_3 = K'_3 + K''_3$$

ARE FUNCTIONS OF TIME

$$K'_3 = K_3 (t=0)$$

$$K''_3 = -\frac{67}{q} \int \delta_3 dt$$

$$K_1 = \frac{\delta_1}{\int n dt}$$

$$K'_1 = \frac{\delta'_1}{-\int n dt}$$

$$K_2 = \frac{\delta_2}{n}$$

$$K'_2 = \frac{\delta'_2}{-N}$$

N - INCREMENTAL NORMAL ACCELERATION COMMAND

n - INCREMENTAL NORMAL ACCELERATION RESPONSE SENSED BY AN ACCELEROMETER DISPLACED \bar{x} FEET FORWARD OF CG

ALL OTHER SYMBOLS ARE IN CURRENT AVRO NOTATION

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INCREMENTAL ELEVATOR MOVEMENTS

WITH PITCH DAMPER ENGAGED

$$\delta = \int_{t=0}^{t=t} [K_1 n - K'_1 N + K_0 q] dt - K'_2 N + K_2 n + K_3 q$$

FOR DEFINITIONS OF $K_0, K_1, K'_1, K_2, K'_2, K_3$ SEE PAGE 9

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DETERMINATION OF DERIVATIVES WITH PITCH DAMPER ENGAGED

With the above in mind the OZ equation is more conveniently expressed as follows.

$$\begin{aligned}
 & -\alpha \left[\frac{C_{z\alpha}}{2\tau} + \frac{a \cdot \sin i}{m V_1} \cdot \frac{\partial T}{\partial \alpha} \right] - \frac{\ddot{\alpha} \cdot C_{z\ddot{\alpha}}}{4\mu_1} - \frac{q \cdot C_{zq}}{4\mu_1} + \dot{q} \cdot \frac{a \bar{X}}{V} - \frac{n g}{V} \\
 & = a \left[\frac{C_{z\delta}}{2\tau} \left[\int (K_0 q + K_1 n - K'_1 N) dt - K'_2 N \right] + \frac{\sin i \cdot T_1}{m V_1} \right]
 \end{aligned}$$

Let it be supposed that the above equation exactly expresses the side force equation and that physical measurements are 100% accurate; also that $\frac{\partial T}{\partial \alpha}$ is known and very small.

The unknowns in the equation are, C_{z0} , $C_{z\alpha}$, C_{zq} , $C_{z\delta}$

Now if the values of the variables α , $\dot{\alpha}$, q , \dot{q} , n , T_1 are sampled simultaneously at four different times, t_1 , t_2 , t_3 , t_4 and the values are substituted into the OZ equation to form eqn.

OZ_{t_1} , OZ_{t_2} , OZ_{t_3} , OZ_{t_4} : simultaneous solution of these equations will yield values of $C_{z\alpha}$, C_{zq} , $C_{z\delta}$

However in practice physical measurements are not 100% accurate. Hence some of the afore-mentioned derivatives would be evaluated with doubtful accuracy. This situation may be remedied by the formation of additional cases.

It is intended that in some instances flight recording frequency will be 20 values per second. Supposing the incremental normal command acceleration remains a constant, the response variables may well attain almost steady values within 0.5 seconds of the step input. Hence in many cases it is only useful to sample the values at N distinct times where $N = 0.5 / .05$. Hence the number of equations is restricted to Eqn. t_1 , Eqn. t_2 -----Eqn. t_6 where $(t_3 - t_1) = (t_3 - t_2) = (t_4 - t_3)$ etc. = 0.05 sec.

As time increases from the instance of the initial impulse command signal, the variation in the values of the response variables will become less.



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Hence if it is decided to form 10 sets of 4 equations, it is advantageous to utilise combinations of Eqn. t_1 ----Eqn. t_6 , to the maximum. Equation t_6 is used to form an additional four cases, although more are possible. Tentatively it is thought that increased accuracy will not be attained by more cases.

Hence the formation of the following selected cases on the next page:



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CASE 1 $t_1, t_2, t_3, t_4 \longrightarrow (C\bar{z}_{\alpha} C\bar{z}_{\alpha}; C\bar{z}_{\alpha}; C\bar{z}_{\alpha})_1$

2 $t_2, t_3, t_4, t_5, " (" " " ")_2$

3 $t_1, t_3, t_4, t_5, " (" " " ")_3$

4 $t_1, t_2, t_4, t_5, " (" " " ")_4$

5 $t_1, t_2, t_3, t_5, " (" " " ")_5$

6 $t_1, t_2, t_3, t_6, " (" " " ")_6$

7 $t_1, t_2, t_4, t_6, " (" " " ")_7$

8 $t_1, t_3, t_4, t_6, " (" " " ")_8$

9 $t_2, t_3, t_4, t_6, " (" " " ")_9$

10 $t_1, t_2, t_5, t_6, " (" " " ")_{10}$

$$C\bar{z}_{\alpha} = 1/n \left\{ (C\bar{z}_{\alpha})_1 + (C\bar{z}_{\alpha})_2 + \dots + (C\bar{z}_{\alpha})_n \right\}$$

$$C\bar{z}_{\alpha} = 1/n \left\{ (C\bar{z}_{\alpha})_1 + (C\bar{z}_{\alpha})_2 + \dots + (C\bar{z}_{\alpha})_n \right\}$$

etc. where $n \leq 10$ and n is the number of cases formed.



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The same process may be applied to the pitching moment equation.

Similarly

$$\bar{C}_{m_q} = 1/n \left\{ (\bar{C}_{m_\alpha})_1 + (\bar{C}_{m_\alpha})_2 + \dots + (\bar{C}_{m_\alpha})_n \right\}$$

$$\bar{C}_{m \cdot \alpha} = 1/n \left\{ (\bar{C}_{m \cdot \alpha})_1 + (\bar{C}_{m \cdot \alpha})_2 + \dots + (\bar{C}_{m \cdot \alpha})_n \right\}$$

etc. where $n \leq 10$ and n is the number of cases formed.

The solution of the two longitudinal equations of motion yields the values of C_{z_δ} , C_{m_δ} . From these values and the evaluation of the effective derivatives \bar{C}_{m_q} , $\bar{C}_{m \cdot \alpha}$ etc., the values of the free

aircraft derivatives C_{m_α} , $C_{m \cdot \alpha}$ may be determined. Now if the gains of the servo-mechanism system are altered, new values of the effective derivatives may be evaluated and the consequent response of the aircraft to an arbitrary command signal, can be calculated by the afore-mentioned step by step method of response determination.



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CONCLUSIONS

The following conclusions may be considered to be in addition to those that are obvious, appearing in the Introduction.

Development of the scope of the digital time response solution

Having embarked on analysis of time solutions of the response of the aircraft rather than restricting analysis to linear equations of motion with zero flying control forcing functions, it is no longer necessary to restrict the analysis to small motions, nor is it necessary for the longitudinal and lateral motions to be uncoupled. Caution must be applied if it is suspected that aircraft response frequency approaches that of the servo-mechanism system.

Advantages of digital solution

Once the response programmes have been proven chances of any error in the analysis are remote and time to solution using exact aerodynamic derivatives rather than "simulator" aerodynamic derivatives may well compare favourably with the Analogue Computer facility. In any event a very positive check is possible. Changes in servo-mechanism gains are also easily incorporated in the digital programme. Extrapolation to other flight conditions is an easy matter.

Importance of "Dummy Runs"

The theory underlying the digital analysis has been given in this report; however the suggestion of 10 cases is tentative and in the "Discussion" there is a suggestion for effectively biasing the evaluated average towards the earlier solutions when the response variables are varying the most. It is clear an attempt should be made to 'evaluate' from theoretical response records, derivatives of known value which generates the responses. This is far from a complete simulation of the method applied to flight results since the inevitable inaccuracy of physical measurement is not encountered. However, the exercise may well give guidance as to the character of the command signal which will confirm the value of a particular derivative.

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Futhermore although an attempt has been made to visualise practical difficulties, temporary practical snags might well be brought to light by a series of 'dummy runs', experience from which would ensure the smooth running of the programme.