



3.2 Material Properties - Metals

Tables of specifications, heat treatment and strength properties for the standard materials to be used in this company are given in the following pages. The use of these tables is explained in the notes below.

When a material not shown in the following tables is required, the Standard Stock Material List should first be consulted. If the material is not shown there in the desired size, then Material Control should be advised at once so that the material may be ordered as far in advance of shop requirements as possible.

3.2.1 Explanation and Notes on Use of Tables

3.2.1.1 Commercial Alloy Column - The commercial designation is included for reference purposes only and is intended to aid in locating the specification required. It is not to be included on drawings. The designations used are as follows:

Aluminum Alloys -

- (a) Upper line - the Alcoa (Aluminum Company of America) designation.
- (b) Lower line, in brackets - the Alcan (Aluminum Company of Canada) designation.

Magnesium Alloys - The American Magnesium Company (AMC), Dow Chemical Company and/or the American Society for Testing Materials (ASTM) designations are given in the case of magnesium.

Carbon Steels - The SAE designation number.

Corrosion-resisting Steels - The American Iron and Steel Institute (AISI) number is given for these steels.

3.2.1.2 Specification, Condition and Heat Treatment Columns - Information on the correct methods of call-up of material specifications and conditions, and on the heat treatment block to be used on drawings of steel and nickel alloy parts, will be found in Sect III, 3.18. In the case of steel and nickel alloys, the type of final heat treatment and the physical properties required will be taken from the Heat Treated by User and Physical Properties columns in the steel tables in this section.

The Condition column refers to the final condition for all metals other than steel and nickel alloys.

3.2.1.3 Physical Properties Columns - All physical properties listed are minimum values, unless otherwise stated. Properties listed in brackets are typical minimum values not covered by specification or ANC-5.

Abbreviations:

- T.S. Ultimate tensile strength
- Y.S. Tensile yield strength at 0.2% set, unless otherwise stated.
- 1.5 Y.S. To be used in place of T.S. for stress calculations whenever it is less than T.S.
- S.S. Shear strength
- B.S. Bearing strength. The value listed is the smaller of the ultimate bearing strength or 1.5 times the yield bearing strength. The value is for $e/D=2$, where e = edge distance measured from hole centre line, and D = hole diameter. For smaller edge distances, see ANC-5.

3.2.1.4 Critical Size Column - The critical size is the maximum thickness (diameter) of steel that can be uniformly hardened to a required strength. This critical size varies with each composition of steel and is a criterion of the hardenability of the steel. While the critical size values as shown in the steel tables are based on circular section, they can be considered as applying equally to non-circular sections without serious error.

Critical size values are not applied directly to any particular part. They affect the choice of material however, in that they establish the maximum size beyond which the "Ruling Section" cannot go without reduction in strength.

In all cases where the application of critical size is in doubt, the Metallurgical Department should be consulted.

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AIRCRAFT ENGINEERING MANUAL
VOL. 1 (DESIGN)

Sect II
Sub-Sect 3.2 (p.1)
Materials - Aluminum

3.2.2 Aluminum Alloys

3.2.2.1 Bars, Rods, Shapes and Wire

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Wire	2SH14 (2S 1/2H)	QQ-A-411 Cond. 1/2H	1/2 Hard	Up to .374	16		(13)			
Aluminum Bars, Rods and Shapes	2SF (2SF)	QQ-A-411 Cond. F	As Fabricated	Rounds, Squares, Hexagons,						
				.375 - .750	(17)		(14)			
				.751 - 3.00	(15)		(13)			
				.301 - 8.00	(14)		(9)			
				Rectangles: Up to 1.50	(15)		(13)			
				1.51 - 3.00	(14)		(9)			

Recommended Uses and Notes - Use only for parts requiring welding or for very light loads. Strength reduced locally at weld and cannot be hardened by heat treatment.

Size Limits:

Rounds - min. .018, max. 2.5 in. dia.

Squares - min. .018, max. 1.5 in. across flats

Hexagons - min. .156, max. 1.5 in. across flats

Rectangles -

Thickness - the minimum and maximum thicknesses are as shown in the following table.

Width - the minimum and maximum widths depend on the thickness, as shown in the following table.

THICKNESS, IN.	WIDTH, IN.		THICKNESS, IN.	WIDTH, IN.	
	MIN.	MAX.		MIN.	MAX.
.188 - .390	.250	2.500	.751 - 1.000	.752	2.000
.391 - .500	.391	2.500	1.001 - 1.125	1.002	1.875
.501 - .625	.501	2.250	1.126 - 1.250	1.127	1.750
.626 - .750	.626	2.250	1.251 - 1.375	1.252	1.500



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.1 Bars, Rods, Shapes and Wire (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong.
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Alum. Alloy Bar, Rod & Shapes <u>Extruded</u>	24ST4 (24ST)	QQ-A-267 Temper T4	Solution Heat treated by Mill	.050 - .249	57	-	42	30	100.5	12
				.250 - .749	60	-	44	32	103.5	12
				.750 - 1.499	65	-	46	34	106.5	10
				1.5 - Up Area up thru 25 sq. in.	70	-	52	37	108.0	10
				1.5 - Up Area over 25 to 32 sq. in.	68	-	48			8
				Up to .249	57	-	38			12
	24ST42 (24ST)	QQ-A-267 Temper T42	Solution Heat treated by User	.250 - Up	57	-	38	30	91.5	12
				Area not over 32 sq. in.						

Recommended Uses and Notes - For primary structural applications except where 14S or 75S are required; not weldable; grain direction should be indicated on part drawing.

Use extruded bar wherever possible in preference to drawn bar.

Size Limits: (for other sizes see QQ-A-268)

Rounds - min. .532, max. 5.75 dia.

Hexagons - min. .532, max. 5.00 across flats

Shapes - Max. area 25 sq. in.

Size Limits - Squares and Rectangles

Width	Thickness		Width	Thickness	
	Min.	Max.		Min.	Max.
.626	.625	.626	4.0 - 4.999	.187	5
.75	.520	.75	5.0 - 5.999	.187	5
.875	.446	.875	6.0 - 6.999	.187	4.125
.937	.415	.937	7.0 - 7.999	.250	3.5
1.000	.391	1.000	8.0 - 8.999	.250	3.125
1.001 - 1.999	.125	2	9.0 - 9.999	.375	2.75
2.0 - 2.999	.125	3	10.0 - 10.999	.375	2.5
3.0 - 3.999	.125	4	11.0 - 12.0	.437	2.25

For larger sizes than listed above the drawing should state the minimum mechanical properties required, and these should be included on the purchase order. Availability should be checked with Purchasing Dept. before calling up on drawing.



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.1 Bars, Rods, Shapes and Wire (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong.																																																										
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.																																																											
Alum. Alloy Bar, Rod & Wire <u>Rolled or Drawn</u>	24ST4 (24ST)	QQ-A-268 Temper T4	Solution Heat treated by Mill or User	Up to .124 .125 - 2.5	62 62	60	40	37	96	14																																																										
	<p><i>Recommended Uses and Notes</i> - For primary structural applications except where 14S or 75S are required; not weldable; grain direction should be indicated on drawing.</p> <p>Size Limits: (for other sizes see QQ-A-267)</p> <p>Wire - up to .250</p> <p>Rounds - min. .250, max. 2.50 dia.</p> <p>Squares - min. .250, max. 1.500 across flats</p> <p>Hexagons - min. .250, max. 1.500 across flats</p> <p>Rectangles - the minimum and maximum thicknesses and widths are related to each other as shown in the table under 2SF.</p> <p style="text-align: right;">Drawn bar should be used only for sizes below the extruded limits, or where the closer tolerances of drawn bar are required.</p>																																																																			
Aluminum Alloy bars, Rods and Shapes, <u>Extruded</u>	14ST6 (26ST)	QQ-A-261 Temper T6	Solution Heat treated & artificially aged by Mill	.125 - .499	60	-	53	35	114	7																																																										
				.500 - .749	64	-	58	37	122	7																																																										
				.750 - Up Area up to 25 sq. in.	68	-	60	39	114	7																																																										
	14ST62	QQ-A-261 Temper T62	Solution Heat treated & aged by User	Area 25.1 to 32 sq. in. (See Notes)	68	-	58	-	-	6																																																										
				Sizes up to 32 sq. in. (See Notes)	60	-	53	35	(114)	7																																																										
<p><i>Recommended Uses and Notes</i> - For general mechanical and structural parts requiring higher strength than 24S. Not weldable. Material called up on drawing as T6 must not be re-heat treated.</p> <p>Use extruded bar wherever possible in preference to drawn bar.</p> <p>Size Limits:</p> <p>Rounds - min. .532, max. 5.75 dia.</p> <p>Hexagons - min. .532, max. 5.00 across flats</p> <p>Shapes - max. area 25 sq. in.</p> <p style="text-align: center;">Size Limits - Squares and Rectangles</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th rowspan="2">Width</th> <th colspan="2">Thickness</th> <th rowspan="2">Width</th> <th colspan="2">Thickness</th> </tr> <tr> <th>Min.</th> <th>Max.</th> <th>Min.</th> <th>Max.</th> </tr> </thead> <tbody> <tr> <td>.626</td> <td>.625</td> <td>.626</td> <td>4.0 - 4.999</td> <td>.125</td> <td>5</td> </tr> <tr> <td>.75</td> <td>.520</td> <td>.75</td> <td>5.0 - 5.999</td> <td>.125</td> <td>5</td> </tr> <tr> <td>.875</td> <td>.446</td> <td>.875</td> <td>6.0 - 6.999</td> <td>.187</td> <td>4.125</td> </tr> <tr> <td>.937</td> <td>.415</td> <td>.937</td> <td>7.0 - 7.999</td> <td>.187</td> <td>3.5</td> </tr> <tr> <td>1.000</td> <td>.391</td> <td>1.000</td> <td>8.0 - 8.999</td> <td>.250</td> <td>3.125</td> </tr> <tr> <td>1.001 - 1.999</td> <td>.125</td> <td>2</td> <td>9.0 - 9.999</td> <td>.250</td> <td>2.75</td> </tr> <tr> <td>2.0 - 2.999</td> <td>.125</td> <td>3</td> <td>10.0 - 10.999</td> <td>.250</td> <td>2.5</td> </tr> <tr> <td>3.0 - 3.999</td> <td>.125</td> <td>4</td> <td>11.0 - 12.0</td> <td>.250</td> <td>2.25</td> </tr> </tbody> </table>											Width	Thickness		Width	Thickness		Min.	Max.	Min.	Max.	.626	.625	.626	4.0 - 4.999	.125	5	.75	.520	.75	5.0 - 5.999	.125	5	.875	.446	.875	6.0 - 6.999	.187	4.125	.937	.415	.937	7.0 - 7.999	.187	3.5	1.000	.391	1.000	8.0 - 8.999	.250	3.125	1.001 - 1.999	.125	2	9.0 - 9.999	.250	2.75	2.0 - 2.999	.125	3	10.0 - 10.999	.250	2.5	3.0 - 3.999	.125	4	11.0 - 12.0	.250	2.25
Width	Thickness		Width	Thickness																																																																
	Min.	Max.		Min.	Max.																																																															
.626	.625	.626	4.0 - 4.999	.125	5																																																															
.75	.520	.75	5.0 - 5.999	.125	5																																																															
.875	.446	.875	6.0 - 6.999	.187	4.125																																																															
.937	.415	.937	7.0 - 7.999	.187	3.5																																																															
1.000	.391	1.000	8.0 - 8.999	.250	3.125																																																															
1.001 - 1.999	.125	2	9.0 - 9.999	.250	2.75																																																															
2.0 - 2.999	.125	3	10.0 - 10.999	.250	2.5																																																															
3.0 - 3.999	.125	4	11.0 - 12.0	.250	2.25																																																															

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3.2.2 Aluminum Alloys (Cont'd)

3.2.2.1 Bars, Rods, Shapes and Wire (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000 PSI T.S.	1000 PSI 1.5 Y.S.	1000 PSI Y.S.	1000 PSI S.S.	1000 PSI B.S.	% in 2 in. Elong.
Aluminum Alloy Bar, Rod and Wire Rolled or Drawn	61S-O (65S-O)	QQ-A-325 Temper O	Annealed	Wire up to .124	(15)	(9)	(6)			
				Wire .125 - .374	(15)	(9)	(6)			18
				Bars & Rods (See Notes)	(15)	(9)	(6)			18
Aluminum Alloy Bar, Rod and Wire Rolled or Drawn	61S-T4 (65S-W)	QQ-A-325 Temper T4	Heat treated by Mill or User	Wire up to .124	30					
				Wire .125 - .374	30	24	16			18
				Bars & Rods (See Notes)	30	24	16			18
Aluminum Alloy Bar, Rod and Wire Rolled or Drawn	61S-T6 (65S-T)	QQ-A-325 Temper T6	Heat treated & artificially aged by Mill or user	Wire up to .124	42					
				Wire .125 - .374	42	-	35			10
				Bars & Rods (See Notes)	42	-	35	25		10

Recommended Uses and Notes - When gas or arc welding or brazing is involved, the drawing must specify annealed temper "O". If higher than annealed strength is required, the drawing must specify "Heat treat to temper T4 after welding (or brazing)" or "Heat treat and age to temper T6 after welding (or brazing)". The full strength of the base metal is not obtained at the weld seam after welding. May be furnace brazed in temper T4 or T6, but reverts to annealed condition and must be reheat treated to obtain these tempers.

Size Limits: (for other sizes see QQ-A-270)
Wire - up to .250
Rounds - min. .250, max. 2.50 dia.
Squares - min. .250, max. 1.500 across flats
Hexagons - min. .250, max. 1.500 across flats

Rectangles - the minimum and maximum thicknesses and widths are related to each other as shown in the table under 2SF.

Drawn bar should be used only for sizes below the extruded limits, or where the closer tolerances of drawn bar are required.

Note - Temper O was previously cond. A.
Temper T4 was previously cond. W.
Temper T6 was previously cond. T.
Material to the previous condition designations may be used in place of new designations until old stock is used up.



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.1 Bars, Rods, Shapes and Wire (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy, Bars Rods and Shapes <u>Extruded</u>	61S-O (65S-O)	QQ-A-270 Temper O	Annealed	(See Notes)	(15)	(9)	(6)			16
	61S-T4 (65S-W)	QQ-A-270 Temper T4	Solution Heat-treated by Mill or user	(See Notes)	26	24	16			16
	61S-T6 (65S-T)	QQ-A-270 Temper T6	Solution Heat treated & artificially aged by Mill or user	(See Notes)	38	-	35	24	80	10

Recommended Uses and Notes - When gas or arc welding or brazing is involved, the drawing must specify annealed temper "O". If higher than annealed strength is required, the drawing must specify "Heat treat to temper T4 after welding (or brazing)" or "Heat treat and age to temper T6 after welding (or brazing)". The full strength of the base metal is not obtained at the weld seam after welding. May be furnace brazed in temper T4 or T6, but reverts to annealed condition and must be reheat treated to obtain these tempers.

Use extruded bar wherever possible in preference to drawn bar.

Size Limits: (for other sizes see QQ-A-325)

Rounds - min. .532, max. 5.75 dia.

Hexagons - min. .532, max. 5.00 across flats

Shapes - max. area 25 sq. in.

Size Limits for Squares and Rectangles

Width	Thickness		Width	Thickness	
	Min.	Max.		Min.	Max.
.626	.625	.626	4.0 - 4.999	.125	5
.75	.520	.75	5.0 - 5.999	.125	5
.875	.446	.875	6.0 - 6.999	.125	4.125
.937	.415	.937	7.0 - 7.999	.125	3.5
1.000	.391	1.000	8.0 - 8.999	.187	3.125
1.001 - 1.999	.125	2	9.0 - 9.999	.187	2.75
2.0 - 2.999	.125	3	10.0 - 10.999	.187	2.5
3.0 - 3.999	.125	4	11.0 - 12.0	.187	2.25

Aluminum Alloy Bars, Rods and Shapes <u>Extruded</u>	75S-T6 (75S-T)	QQ-A-277 Temper T6	Solution Heat treated & artificially aged by Mill or user	Up to .249	78	-	70	43	125	7
				.250 - 2.999 incl.	80*	-	72*	44*	128*	7
				3.00 - 4.499 incl.	80*	-	70*	-	-	7
				3.00 - 4.499 incl.	78*	-	70*	-	-	6
				4.50 - 5.00 incl.	78*	-	68*	-	-	6
				(See Notes)						

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3.2.2 Aluminum Alloys (Cont'd)

3.2.2.1 Bars, Rods, Shapes and Wire (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness	1000	1000	1000	1000	1000	% in
					PSI	PSI	PSI	PSI	PSI	2 in.
					T.S.	1.5 Y.S.	Y.S.	S.S.	B.S.	Elong
<p><i>Recommended Uses and Notes (Cont'd)</i> - For primary structural applications only, where high strength is required. Not weldable. Care must be taken always to avoid stress raisers, e.g. notches or sharp changes of section. Properties decrease from outer surface to core. Properties given are minimum core strengths for sizes up to 1-1/2 in. thick and average (midway between surface and core) for larger sizes. *See notes in ANC-5 for core strengths over 1-1/2 in. thick Use extruded bar wherever possible in preference to drawn bar.</p> <p>Size Limits: (for other sizes see QQ-A-282) Rounds - min. .532, max. 5.75 dia. Hexagons - min. .532, max. 5.00 across flats Shapes - max. area 25 sq. in. Square and Rectangular sections - see QQ-A-267.</p> <p>For larger sizes than listed above the drawing should state the minimum mechanical properties required, and these should be included on the purchase order. Availability should be checked with Purchasing Dept. before calling up on drawing.</p>										
Aluminum Alloy Bars, Rods, and Wire Rolled or Drawn	75S-T6 (75S-T)	QQ-A-282 Temper T6	Solution Heat treated & artificially aged by Mill or user	Wire .040 - .124 incl. .125 - .250 incl. Bars & Rods Round, Rectangular, Hexagonal etc. (See Notes)	77 77 77*	- -	66 66*	46 46*	123 123*	7 7
<p><i>Recommended Uses and Notes</i> - For primary structural applications only, where high strength is required. Not weldable. Care must be taken always to avoid stress raisers, e.g. notches or sharp changes of section. Properties decrease from outer surface to core. Properties given are minimum core strengths for sizes up to 1-1/2 in. thick and average (midway between surface and core) for larger sizes. *See ANC-5 for core strength of extrusions over 1-1/2 in. thick.</p> <p>Size Limits: (for other sizes see QQ-A-277) Wire - up to .250 Rounds - min. .250, max. 2.50 dia. Squares - min. .250, max. 1.500 across flats Hexagons - min. .250, max. 1.500 across flats</p> <p>Rectangles - the minimum and maximum thicknesses and widths are related to each other as shown in the table under 2SF. Drawn bar should be used only for sizes below the extruded limits, or where the closer tolerances of drawn bar are required.</p>										

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Sect II
Sub-Sect 3.2 (p.7)
Materials - Aluminum

3.2.2.2 Castings

(a) Permanent Mould

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy (5.0 Si)	43F (123)	QQ-A-596 Comp. 7	As cast		21*					5*
	43F	Temper F								
<i>Recommended Uses and Notes</i> - For small castings in large quantities. General use, good corrosion resistance, excellent casting properties, fair machinability. Cannot be hardened by heat treatment.										
Aluminum Alloy (7.0 Si)	356-T6 (135-T22)	QQ-A-596 Comp. 8	Solution Heat treated & artificially aged		33*	-	22	25		3*
		Temper T6								
<i>Recommended Uses and Notes</i> - For small castings in large quantities. For general use where high strength and corrosion resistance are required. Excellent casting properties, good machinability.										

(b) Sand

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Base Alloy (5.0 Si)	43F (123)	QQ-A-601 Comp. 2	As cast		17*	(12)*	(8)			3.0*
		Temper F								
<i>Recommended Uses and Notes</i> - For non-structural parts, weldable, very good casting qualities, can be used for intricate shapes. Not hardened by heat treatment, leakproof.										
Aluminum Base Alloy (7.0 Si)	356-T6 (135-T22)	QQ-A-601 Comp. 3	Solution Heat treated & artificially aged		30*	-	20	25	60	3.0*
		Temper T6								
<i>Recommended Uses and Notes</i> - For structural parts and parts requiring pressure tightness. Can be used for intricate shapes, good corrosion resistance. Not recommended for welding.										
Aluminum Base Alloy (10.0 Mg)	220-T4 (350-W)	QQ-A-601 Comp. 16	Solution Heat treated		42*	33*	22	30	66	12.0*
		Temper T4								
<i>Recommended Uses and Notes</i> - For structural parts requiring high strength and resistance to shock. Do not weld. Do not use for pressure tight castings. Fair castability. Corrosion resistance excellent.										

*Average tensile strength of specimen cut from castings will not be less than 75%, and elongation not less than 25%, of values given.



3.2.2.3 Forgings

(a) Die

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000 PSI T.S.	1000 PSI 1.5 Y.S.	1000 PSI Y.S.	1000 PSI S.S.	1000 PSI B.S.	% in. 2 in. Elong
Aluminum Alloy	14S-T6 (26S-T)	*QQ-A-367 Comp. 5 Temper T6 (Die Forging)	Heat treated and aged	up to 4.0	L 65		55	39	124	10
					T _L 62	-	52			3
<i>Recommended Uses and Notes</i> - For highly stressed parts, fittings etc. not weldable. Difficult to forge into intricate shapes. Mechanical properties are to be stated on drawing for parts over 4 inches diameter or over 4 inches section in least dimension.										
Aluminum Alloy	A51S-T6 (61S-T)	*QQ-A-367 Comp. 3 Temper T6 (Die Forging)	Heat treated and aged	up to 4.0	44	-	37	28	88.5	14
<i>Recommended Uses and Notes</i> - For lower stressed parts, fittings etc. Not weldable. More readily forged into intricate shapes. Mechanical properties are to be stated on drawing for parts over 4 inches dia. or over 4 inches section in least dimension.										
Aluminum Alloy	75S-T6 (75S-T)	*QQ-A-367 Comp. 10 Temper T6 (Die Forging)	Heat treated and aged	up to 3.0	L 75		65	45		10
					T _L 71	-	62			3
<i>Recommended Uses and Notes</i> - For very highly stressed parts, fittings etc. not weldable. Mechanical properties are to be stated on drawing for parts over 3 inches diameter or over 3 inches section in least dimension. Difficult to forge into intricate shapes.										

*Since this specification allows the manufacturer to use any method of forging most convenient unless specified, it is important to state die forging on drawing.

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Materials - Aluminum

3.2.2 Aluminum Alloys (Cont'd)

3.2.2.3 Forgings (Cont'd)

(b) Hand

Material	Comm Alloy	Speci- fication	Condition	Cross Sectional Area sq. inches	Length Width Ratio	1000	1000	1000	1000	% in 2 in. Elong
						PSI T.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy	14ST6 (26ST)	QQ-A-367 Hand Forging Comp. 5 Size Class I	Heat treated and aged	Up to 16	Up to 3	L 65	55	40	124	10
						T _L 62	55	40	124	6
						T _s 59	55	40	124	3
		Size Class II	Heat treated and aged	Up to 16	Over 3	L 65	55	40	124	10
						T _L 62	53	40	124	4
						T _s 59	53	40	124	2
		Size Class III	Heat treated and aged	1.61 - 36	Up to 3	L 65	53	40	124	9
						T _L 62	53	40	124	5
	T _s 59					53	40	124	2	
	Size Class IV	Heat treated and aged	16.1 - 36	Over 3	L 65	53	40	124	9	
					T _L 62	51	40	124	3	
					T _s 59	51	40	124	3	
	Size Class V	Heat treated and aged	36.1 - 144	Up to 3	L 62	50	38	118	7	
					T _L 59	50	38	118	3	
					T _s 56	50	38	118	1	
	Size Class VI	Heat treated and aged	36.1 - 144	Over 3	L 62	50		118	7	
T _L 59					50		118	2.5		
T _s 56					50		118	1		
Size Class VII	Heat treated and aged	144.1 - 256	Up to 3	L 60	48	37	114	5		
				T _L 57	48	37	114	2		
				T _s 54	48	37	114	1		
Size Class VIII	Heat treated and aged	144.1 - 256	Over 3	L 60	48		114	5		
				T _L 57	48		114	1.5		
				T _s 54	48		114	1		
75ST6 (75ST)	QQ-A-367 Hand Forging Comp. 10 Size Class I	Heat treated and aged	Up to 16	Up to 3	L 75	64			9	
					T _L 75	63			4	
					T _s (72)				(1)	
					L 73	62			8	
					T _L 73	61			3	
					T _s (70)	(61)			(1)	
	Size Class II	Heat treated and aged	Up to 16	Over 3	L 73	62			8	
					T _L 73	61			3	
					T _s (70)	(61)			(1)	
	Size Class III	Heat treated and aged	16.1 - 36	Up to 3	L 73	61			7	
					T _L 71	60			3	
					T _s (68)				(1)	
Size Class IV	Heat treated and aged	16.1 - 36	Over 3	L 71	59			6		
				T _L 69	58			2		
				T _s (66)	(58)			(1)		
Size Class V	Heat treated and aged	36.1 - 144	Up to 3	L 71	60			4		
				T _L 69	58			2		
				T _s (66)	(58)			(1)		
Size Class VI	Heat treated and aged	36.1 - 144	Over 3	L 69	58			3		
				T _L 67	56			1		
				T _s 64	(56)			(1)		
<i>Recommended Uses and Notes - (see following page)</i>										



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.3 Forgings (Cont'd)

(b) Hand (Cont'd)

Aluminum Alloy	<p><i>Recommended Uses and Notes</i> - Use for general mechanical and structural applications where quantities do not justify the cost of die forgings. Not weldable. <u>Class of forgings</u> covering ranges of cross sectional area and <u>length ratio</u>, should be specified on the drawing.</p> <p style="text-align: center;">width_L - refers to longitudinal direction (parallel to length) T_L - refers to long transverse direction (parallel to width) T_S - refers to short transverse direction (parallel to thickness)</p> <p>Note: Grain direction should be indicated on drawings.</p>
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3.2.2.4 Sheet, Plate and Strip

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong	
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.		
Aluminum Manganese Alloy	3S-0 (3SO)	QQ-A-359 Temper O	Annealed	.006 - .007	(14)	(7.5)	(5)			14	
				.008 - .012	(14)	(7.5)	(5)			18	
				.013 - .031	(14)	(7.5)	(5)			20	
				.032 - .050	(14)	(7.5)	(5)			23	
				.051 - .249	(14)	(7.5)	(5)			25	
				.250 - .500	(14)	(7.5)	(5)			23	
	<i>Recommended Uses and Notes</i> - For deep drawing and spinning; weldable; cannot be hardened by heat treatment.										
	3S-H14 3S 1/2H	QQ-A-359 Temper H 14	Strain Hardened Half-Hard	.008 - .012	20	-	(16)				1
				.013 - .019	20	-	(16)				2
				.020 - .031	20	-	(16)				3
				.032 - .050	20	-	(16)				4
				.051 - .113	20	-	(16)				5
.114 - .161				20	-	(16)				6	
.162 - .249	20	-	(16)				7				
.250 - .500	20	-	(16)				8				
<i>Recommended Uses and Notes</i> - Good forming qualities; weldable, but strength is reduced near weld. Cannot be hardened by heat treatment.											
Aluminum Alloy Sheet, Bare	24ST3 (24ST)	QQ-A-355 Temper T3	Solution Heat treated by Mill	.010 - .020	64	63	42	40	118.5	12	
				.021 - .051	64	63	42	40	118.5	15	
				.052 - .128	64	63	42	40	118.5	17	
				.129 - .249	64	63	42	40	118.5	15	
24ST4 (24ST)	QQ-A-355 Temper T4	Solution Heat treated by user	.012 - .020	62	60	40	37	96	12		
			.021 - .051	62	60	40	37	96	15		
			.052 - .249	62	60	40	37	96	17		
Aluminum Alloy Plate, Bare	24ST4 (24ST)	QQ-A-355 Temper T4	Solution Heat treated by Mill	.250 - .500	64	60	40	40	111	12	
				.501 - 1.000	62	60	40	38	105	8	
				1.001 - 1.500	60	-	40	36	102	7	
				1.501 - 2.000	60	-	40	36	102	6	
2.001 - 3.000	56	-	40	-	-	4					



AIRCRAFT ENGINEERING MANUAL

VOL. 1 (DESIGN)

Sect II
Sub-Sect 3.2
Materials - Aluminum

3.2.2 Aluminum Alloys (Cont'd)

3.2.2.4 Sheet, Plate and Strip (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy Plate, Bare	24ST42 (24ST)	QQ-A-355 Temper T42	Solution Heat treated by user	.250 - .500	64	57	38	38	91.5	12
				.501 - 1.000	62	57	38	37	91.5	8
				1.001 - 1.500	60	57	38	36	91.5	7
				1.501 - 2.000	60	57	38	36	91.5	6
				2.001 - 3.000	56	-	38	34	91.5	4
<p><i>Recommended Uses and Notes</i> - For general mechanical and primary structural applications where service requirements do not require the use of clad 24S material and for heavy plate where the surfaces are machined. Sheet called on drawing as temper T3 and plate as T4 must not be reheat treated. Not to be used for external surfaces.</p> <p>For parts made from sheet, the T3 temper is preferred unless manufacturing considerations, e.g. bend radii, require the use of T4. In this case the parts may be formed from material in either the O temper or AQ condition and then heat treated or aged to T4.</p> <p>For parts made from plate, the same procedure applies with respect to T4 and T42 tempers.</p> <p>Can be spot welded, but see ANC-5 for limitations. Cannot be fusion welded.</p>										
Aluminum Alloy Sheet, Clad	24ST3 Alclad (24ST Alclad)	QQ-A-362 Temper T3	Solution Heat treated by Mill	.010 - .020	59	58.5	39	38	109.5	12
				.021 - .063	59	58.5	39	38	109.5	15
				.064 - .128	62	60	40	40	111	15
				.129 - .249	62	60	40	40	111	13
	24ST4 Alclad (24ST Alclad)	QQ-A-362 Temper T4	Solution Heat treated by user	.010 - .020	58	55.5	37	35	88.5	12
				.021 - .040	58	55.5	37	35	88.5	15
Aluminum Alloy Plate, Clad	24ST Alclad (24ST Alclad)	QQ-A-362 Temper T4	Solution Heat treated by Mill	.250 - .499	62	60	40	40	111	11
				.500 - 1.000	62(a)	60(a)	40(a)	40	111	8(a)
				1.001 - 1.500	60(b)	57(b)	38(b)	37(b)	100.5(b)	-
					60(a)	60(a)	40(a)	40	111	7(a)
				1.501 - 2.000	58(b)	57(b)	38(b)	35(b)	96 (b)	-
					60(a)	60(a)	40(a)	40	111	6(a)
	2.001 - 3.000	58(b)	57(b)	38(b)	35(b)	96 (b)	-			
		56(a)	60(a)	40(a)	40	111	4(a)			
24ST42 Alclad (24ST Alclad)	QQ-A-362 Temper T42	Solution Heat treated by user	.250 - .499	62	57	38	37	91.5	11	
			.500 - 1.000	62(a)	57(a)	38(a)	37	91.5	8(a)	
			1.000 - 1.500	60(b)	54(b)	36(b)	36(b)	87 (b)	-	
				60(a)	57(a)	38(a)	38(a)	91.5	7(a)	
			1.501 - 2.000	58(b)	54(b)	36(b)	35(b)	87 (b)	-	
				60(a)	57(a)	38(a)	38(a)	91.5	6(a)	
			2.001 - 3.000	58(b)	54(b)	36(b)	35(b)	87 (b)	-	
				56(a)	57(a)	38(a)	38(a)	91.5	4(a)	
			54(b)	54(b)	36(b)	32(b)	87 (b)	-		
<p><i>Recommended Uses and Notes</i> - (see following page)</p>										

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3.2.2 Aluminum Alloys (Cont'd)

3.2.2.4 Sheet, Plate and Strip (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy Sheet and Plate, Clad	<p><i>Recommended Uses and Notes</i> - For general mechanical and primary structural applications. May be spot welded only. See ANC-5 for limitations. Cannot be fusion welded.</p> <p>Sheet called on drawing as temper T3 and plate as T4 must not be reheat treated.</p> <p>For parts made from sheet, the T3 temper is preferred unless manufacturing considerations, e.g. bend radii, require the use of T4. In this case the parts may be formed from material in either the O temper or AQ condition and then heat treated or aged to T4.</p> <p>For parts made from plate, the same procedure applies with respect to T4 and T42 tempers.</p>									
	(a) Core Properties					(b) Average Properties				
Aluminum Magnesium Silicon Alloy	61SO (65SO)	QQ-A-327 Temper O	Annealed	.010 - .020	(15)	(9)	(6)			14
				.021 - .128	(15)	(9)	(6)			16
				.129 - 1.000	(15)	(9)	(6)			18
				1.001 - 3.000	(15)	(9)	(6)			16
	61ST4 (65SW)	QQ-A-327 Temper T4	Solution Heat treated by Mill or user	.010 - .020	30	24	16	20	39	14
				.021 - .249	30	24	16	20	39	16
				.250 - 1.000	30	24	16			18
				1.001 - 3.000	30	24	16			16
	61ST6 (65ST)	QQ-A-327 Temper T6	Solution Heat treated & artificially aged by Mill or user	.010 - .020	42	-	35	27	84	8
				.021 - .249	42	-	35	27	84	10
				.250 - .500	42	-	35			10
				.501 - 1.000	42	-	35			9
			1.001 - 2.000	42	-	35			8	
			2.001 - 3.000	42	-	35			6	
<p><i>Recommended Uses and Notes</i> - Temper O is suitable for deep drawing and spinning. Temper T4 is suitable for light draws and moderate forming. Temper T6 is suitable for parts that are flat or have simple bends.</p> <p>When gas or arc welding or brazing is involved, the drawing must specify <u>annealed temper O</u>. If higher than annealed strength is required, the drawing <u>must</u> specify "Heat treat to temper T4 after welding (or brazing)" or "Heat treat and age to temper T6 after welding (or brazing)".</p> <p>The full strength of the base metal is not obtained at the weld seam after welding.</p> <p>May be <u>furnace</u> brazed in temper T4 or T6 but reverts to the annealed condition and must be reheat treated to obtain these tempers.</p> <p>Can be spot welded in any condition and does not require reheat treatment to the same condition.</p> <p>Note: Temper O was previously cond. A. Temper T4 was previously cond. W. Temper T6 was previously cond. T. Material to the previous condition designations may be used in place of new designations until old stock is used up.</p>										

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3.2.2 Aluminum Alloys (Cont'd)

3.2.2.4 Sheet, Plate and Strip (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy, Bare	75ST6 (75ST)	QQ-A-283 Temper T6	Solution Heat-treated & artificially aged by Mill or user	.016 - .039	76	-	65	46	144	7
				.040 - .500	77	-	66	46	146	8
				.501 - 1.000	77	-	66	47	150	6
				1.001 - 2.000	77	-	66	47	150	4
				2.001 - 2.500	73	-	62			3
				2.501 - 3.000	70	-	60			3
<p><i>Recommended Uses and Notes</i> - For primary structural applications where high strength is required and where service conditions do not require the use of clad 75S material and for heavy plate where surfaces are machined. More difficult to form than 24S material. For permissible spot welding combinations see ANC-5. Not to be used on external surfaces.</p>										
Aluminum Alloy, Clad	75ST6 Alclad (75ST Alclad)	QQ-A-287 Temper T6	Solution Heat treated & artificially aged by Mill or user	.016 - .039	70	-	60	42	133	7
				.040 - .499	72	-	62	43	137	8
				.500 - 1.000	77(a)	-	66(a)			6
					72(b)	-	62(b)	44(b)	141(b)	-
				1.001 - 2.000	77(a)	-	66(a)			4
					72(b)	-	62(b)	44(b)	141(b)	-
				2.001 - 2.500	73(a)	-	62(a)			3
					68(b)	-	58(b)			-
2.501 - 3.000	70(a)	-	60(a)			3				
	65(b)	-	56(b)			-				
<p><i>Recommended Uses and Notes</i> - For primary structural applications only, where high strength is required. More difficult to form than 24S material. May be spot welded only.</p>										

- (a) Core properties
- (b) Average properties across core and cladding.

3.2.2.5 Tubing

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy	24ST3 (24ST)	WW-T-785 Temper T3	Solution Heat treated by Mill	Dia. 1/4 in. to 2 in.						
				.018 - .024	64	63	42	39	100.5	10
				.025 - .049	64	63	42	39	100.5	12
				.050 - .259	64	63	42	39	100.5	14
				.260 - .500	64	63	42	39	100.5	16
				Dia. greater than 2 inches to 8 inches & tubing other than rounds						
				.025 - .259	64	63	42	39	100.5	10
				.260 - .500	64	63	42	39	100.5	12



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.5 Tubing (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000 PSI T.S.	1000 PSI 1.5 Y.S.	1000 PSI Y.S.	1000 PSI S.S.	1000 PSI B.S.	% in 2 in. Elong
Aluminum Alloy	24ST4 (24ST)	WW-T-785 Temper T4	Solution Heat treated by user	.018 - .500	62	60	40	39	96	
<p><i>Recommended Uses and Notes</i> - For general mechanical and structural application. For permissible spot welding combinations see ANC-5.</p> <p>Tubing called up on drawing as Temper T3 must not be reheat treated.</p>										
R→ Aluminum Alloy	52SO (57SO)	WW-T-787 Temper O	Annealed		(26)	(15)	(10)			
<p><i>Recommended Uses and Notes</i> - For general mechanical and non-structural applications, such as fuel and oil lines, etc. Weldable, cannot be hardened by heat treatment.</p>										
Aluminum Alloy	61SO (65SO)	WW-T-789 Temper O	Annealed	All sizes	(15)	(9)	(6)			15
	61ST4 (65SW)	WW-T-789 Temper T4	Solution Heat treated by Mill or user	Dia. 1/4 in. to 2 in. .025 - .049 .050 - .259 .260 - .500	30 30 30	24 24 24	16 16 16	(20) (20) (20)	(30) (39) (39)	16 18 20
				Dia. greater than 2 in. and tubing other than round:						
				.025 - .049 .050 - .259 .260 - .500	30 30 30	24 24 24	16 16 16			
	61ST6 (65ST)	WW-T-789 Temper T6	Solution Heat treated & artificially aged by Mill or user	Dia. 1/4 in. to 2 in. .025 - .049 .050 - .259 .260 - .500	42 42 42	- - -	35 35 35	27 27 27	84 84 84	10 12 14
				Dia. greater than 2 to 8 in. and tubing other than round:						
				.025 - .049 .050 - .259 .260 - .500	42 42 42	- - -	35 35 35			8 10 12
<p><i>Recommended Uses and Notes</i> - When gas or arc welding or brazing is involved, the drawing must specify <u>annealed temper O</u>. If higher than annealed strength is required, the drawing <u>must</u> specify "Heat treat to temper T4 after welding (or brazing)" or "Heat treat and age to temper T6 after welding (or brazing)."</p> <p>The full strength of the base metal is not obtained at the weld seam after welding.</p> <p>May be <u>furnace</u> brazed in temper T4 or T6 but reverts to the annealed condition and must be reheat treated to obtain these tempers.</p>										



3.2.2 Aluminum Alloys (Cont'd)

3.2.2.5 Tubing (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Aluminum Alloy	<p><i>Recommended Uses and Notes – (Cont'd)</i></p> <p>Note: Temper O was previously cond. A. Temper T4 was previously cond. W. Temper T6 was previously cond. T. Material to the previous condition designations may be used in place of new designations until old stock is used up.</p>									
Aluminum Alloy, Flexible	3S Flexible (3S Flexible)	WW-T-816 Type II		I/Dia. 1/4 to 8						
<p><i>Recommended Uses and Notes – For air ducts, speaking tubing and other aircraft applications requiring flexible tubing, but not involving the conveying of liquids.</i></p>										

A. V. ROE CANADA LIMITED
INTER-DEPARTMENTAL MEMORANDUM

Reference 7096/31A/Gen.
DATE October 22, 1953
TO Mr. F. Mitchell - Stress Office, Aircraft Division
FROM R. Smallman-Tew - Chief Metallurgist, Aircraft Division
SUBJECT MAGNESIUM CASTING ALLOY TO MIL-M-4204, SOME MECHANICAL PROPERTIES

As requested by Mr. H. Shoji, I give below mechanical properties covering ultimate shear strength, bearing yield and bearing ultimate.

No figures are actually available to cover material to this specification and I have, therefore, taken figures, as a guide, from a Magnesium casting alloy of different chemical composition, but having very similar strength properties in terms of 0.2% proof and ultimate. The figures below must, therefore, be regarded as not being accurate for this alloy, but should work out as a pretty close approximation.

Condition	Shear Ultimate	Bearing	
		Yield	Ultimate
F	16,000 p.s.i.	45,000 p.s.i.	50,000 p.s.i.
T4	16,000 "	50,000 "	55,000 "
T6	18,000 "	55,000 "	60,000 "

J11

R. Smallman-Tew

cc Mr. S.E. Harper



3.2.3 Magnesium Alloy

3.2.3.1 Extruded Bar, Rod and Shapes

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Magnesium Aluminum Zinc Alloy	AMC 52S DOW FS-1 AZ 31B	QQ-M-31 Comp. AZ 31B	As extruded	Bars, Rods or Shapes: up to .249	33	30	20			7
				.250 - 2.499	32	30	20			7
				Hollow Shapes: .050 - .250	32	24	16			8
<p><i>Recommended Uses and Notes</i> - For lightly stressed parts. Weldable using argon-arc process, but mechanical properties reduced at heated area. These are specification values; for strength calculations see ANC-5.</p>										
Magnesium Aluminum Zinc Alloy	AMC 58S DOW 0-1 AZ80A	QQ-M-31 Comp. AZ80A Cond. as extruded	As extruded	Up to .249	43	42	28			9
				.250 - 1.499	43	42	28			8
				1.500 - 2.499	43	42	28			6
	QQ-M-31 Comp. AZ80A Cond. T51	Heat Treated and aged	Up to .249	47	45	30			4	
			.250 - 1.499	47	-	32			3	
			1.500 - 2.499	47	-	32			2	
<p><i>Recommended Uses and Notes</i> - For moderately stressed parts. Compression yield is substantially the same as tension yield in condition T51. See ANC-5 for shear, bearing and other values. Weldable using argon-arc process, but mechanical properties are reduced at heated area. These are specification values; for strength calculations see ANC-5.</p>										

3.2.3.2 Sand Castings

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Magnesium Alloy	AZ 91	MIL-M-4204 Temper F	As cast		18*		10			
	AZ 91	MIL-M-4204 Temper T4	Heat Treated		32*		10			7*
	AZ 91	MIL-M-4204 Temper T6	Heat Treated and aged		34*		16			3*
<p><i>Recommended Uses and Notes</i> - Good casting qualities. Weldable for repairs only.</p>										

*Average tensile strength of specimen cut from castings will not be less than 75%, and elongation not less than 25%, of values given.



3.2.3 Magnesium Alloy (Cont'd)

3.2.3.2 Sand Castings (Cont'd)

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in
					PSI	PSI	PSI	PSI	PSI	2 in.
					T.S.	1.5 Y.S.	Y.S.	S.S.	B.S.	Elong
Magnesium Alloy	ZK 61	C28-97. Cond. AC	As cast		35		18			8
	ZK 61	C28-97 Cond. HTA	Fully Heat Treated		42		26			5
<i>Recommended Uses and Notes</i> - For structural parts. Maximum strength with minimum weight but expensive, and should be used only where really necessary. Castability fair. Weldable for repairs only.										

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3.2.3.3 Forgings

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in
					PSI	PSI	PSI	PSI	PSI	2 in.
					T.S.	1.5 Y.S.	Y.S.	S.S.	B.S.	Elong
Magnesium Aluminum Zinc Alloy	AMC 58 DOW 0-1 AZ80A	QQ-M-40 Comp. AZ80A Cond. F	As forged		42	39	26	20		5
			Forged and aged		42	-	28	20	70	2
<i>Recommended Uses and Notes</i> - High strength. Press forgings only.										

3.2.3.4 Sheet and Plate

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in
					PSI	PSI	PSI	PSI	PSI	2 in.
					T.S.	1.5 Y.S.	Y.S.	S.S.	B.S.	Elong
Magnesium Aluminum Zinc Alloy	AMC52SO FS-1a AZ31X	QQ-M-44 Cond. A	Annealed	.020 - .249	32	22.5	15	17	43.5	12
			Hard rolled	.020 - .128 .129 - .249	39	-	29	18	64.5	4
					39	-	29			4
<i>Recommended Uses and Notes</i> - May be spot welded or argon-arc welded, but mechanical properties are reduced by welding in the heated area. Hot forming reduces strength of Cond. H material.										



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3.2.3 Magnesium Alloy (Cont'd)

3.2.3.5 Tubing

Material	Comm Alloy	Speci- fication	Condition	Thickness Inches	1000	1000	1000	1000	1000	% in 2 in. Elong
					PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.	
Magnesium Alloy	AMC52S DOW FS-1 AZ31X	WW-T-825 Comp. AZ31X	As extruded	Tubing wall .050 - .250	32	24	16			8
<p><i>Recommended Uses and Notes</i> — May be spot welded or argon-arc welded. Mechanical properties are reduced by arc welding in the heated area.</p>										



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Sect II
Sub-Sect 3.2
Materials - Steel

3.2.4 Steels - Carbon and Alloy

3.2.4.1 Bar and Rod

MATERIAL	COMM. ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Mild Steel	1018 1020 1025	MIL-S-7097 Cond. B	As Rolled	Case Harden		55	54	36	35	90	22	B65	1.5
						70	69	46	(45)	(105)	18		
<i>Recommended Uses and Notes</i> - For low strength parts. May be welded. May be used for case hardened parts requiring resistance to abrasion only with low core strength. 1020 preferred over 1025 for case hardening.													
Nickel Steel	2330	AN-QQ-S-689 Cond. F (125,000)	Hardened & Tempered	Harden & Temper	up to 1.5	125	-	100	75	175	17	C27	0.50
						up to 1.5*	150	-	135	90	190	14	
		AN-QQ-S-689 Cond. C	Annealed	Harden & Temper	over 1.5	55	54	36	35	90	-	B65	4.00
						100	-	80	65	140	21	C16	4.00
						125	-	100	75	175	17	C27	1.50
						150	-	135	90	190	14	C34	0.50
<i>Recommended Uses and Notes</i> - For general structural parts, bolts, etc. requiring high impact strength and elasticity at low temperatures. Not weldable. For conditions see 4130 below except for thickness.													
Chrome Molybdenum Steel	4130	MIL-S-6758 Cond. F (125,000)	Hardened & Tempered	Harden & Temper	up to 1.5	125	-	100	75	175	17	C27	0.50
						up to 1.5*	150	-	135	90	190	14	
		MIL-S-6758 Cond. C	Annealed	Harden & Temper	over 1.5	55	54	36	35	90	-	B65	4.00
						100	-	80	65	140	21	C16	4.00
						125	-	100	75	175	17	C27	1.50
						150	-	135	90	190	14	C34	0.50
<i>Recommended Uses and Notes</i> - For general structural parts, bolts, etc. Bars up to 1.5" in least dimension are purchased in Condition F and machined in this condition. Bars over 1.5" in least dimension are purchased annealed and parts subsequently heat treated. Weldable. All welded part drawings should specify "Stress Relieve" or "Harden and Temper after welding". Bar hardened and tempered before welding will drop, when stress relieved, to lower properties. For steel at 125,000 over 1.5" thick use 4140 up to 2.0" thick.													
NOTE: Material is usually ordered in black rolled surface condition, making it necessary to use bar slightly thicker than the finished part.													
Chrome Molybdenum Steel	4140	MIL-S-5626 Cond. F (125,000)	Hardened & Tempered	Harden & Temper	over 1.0 to 2.0	125	-	100	75	175	17	C27	1.5
						over 1.0 to 2.0**	150	-	135	90	190	14	
								160	-	145	95	200	12
						180	-	165	105	200	12	C40	1.00
		MIL-S-5626 Cond. C	Annealed	Harden & Temper	over 2.0	55	54	36	35	90	-	B65	3.5
						125	-	100	75	175	17	C27	3.5
						150	-	135	90	190	14	C34	1.5
						160	-	145	95	200	13	C36	1.25
						180	-	165	105	200	12	C40	1.00
<i>Recommended Uses and Notes</i> - For general structural parts, bolts, etc. Not weldable. Heat treatment is to be specified on drawing when higher than 125,000 is required.													

*Provided it is heat treated with ruling section not more than 0.50".

**Provided it is heat treated with ruling section not more than stated in critical size column.



3.2.4 Steels - Carbon and Alloy (Cont'd)

3.2.4.1 Bar and Rod (Cont'd)

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Chrome Nickel Molybdenum Steel	4340	MIL-S-5000 Cond. C	Annealed	Harden & Temper	up to 1.5 over 1.5	65	-	45	40	110			
						55	54	36	35	90		B65	
						125	-	100	75	175	17	C27	6.00
						150	-	135	90	190	14	C34	4.00
						160	-	145	95	190	13	C36	3.25
					180	-	165	105	200	12	C40	2.50	
<i>Recommended Uses and Notes</i> - For highly stressed parts requiring good hardenability; heat treatment is to be specified on drawing. Not weldable.													
Carburizing Steel	4620	AMS 6294	Machinable (Brinell 229 max.)	Case Harden		100	-	75	65	140	20		2.00
						125	-	100	75	175	15		0.75
<i>Recommended Uses and Notes</i> - For case hardened parts only. The strengths refer to core strength.													

3.2.4.2 Castings

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	BRINELL HARDNESS	IZOD In. ft - lbs.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Steel		Avrocan M-7-2	Hardened & Tempered			90		65			15		35
Medium High Tensile Steel		D.T.D. 666**	Hardened & Tempered			134		100*			12	269	30
High Tensile Steel		D.T.D. 705**	Hardened & Tempered			168		134*			7	340	10
High Manganese Steel		MIL-C-5698											
<i>Recommended Uses and Notes</i> - For non-structural parts with high wear resistance.													

*0.1% Proof Stress

**NOTE: Magnaflux Inspection required. See Sect II, 12.7.2.

3.2.4.3 Forgings

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Chrome Molybdenum Steel	4130	MIL-S-6758 in accordance with Avrocan M-7-1*	Hardened & Tempered			100	-	80	65	140	21	C16	4.00
						125	-	100	75	175	17	C27	1.50
						150	-	135	90	190	14	C34	0.50
<i>Recommended Uses and Notes</i> - For general structural parts, heat treatment and properties are to be specified on forging drawing. Weldable. All welded part drawings should specify "Stress Relieve" or "Harden and Temper after Welding". Forgings hardened and tempered before welding will drop, when stress relieved, to the properties of normalized bar.													

*Avrocan M-7-1 forging spec. is to be included on forging drawings, also heat treatment block specifying the heat treatment as noted in "Condition Received" column and properties



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Materials - Steel

3.2.4 Steels - Carbon and Alloy (Cont'd)

3.2.4.3 Forgings (Cont'd)

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Chrome Molybdenum Steel	4140	MIL-S-5626 in accordance with Avrocan M-7-1*	Hardened & Tempered			100	-	80	65	140	21	C16	5.50
						125	-	100	75	175	17	C27	3.50
						150	-	135	90	190	14	C34	1.50
						180	-	165	105	200	12	C40	1.00
						<i>Recommended Uses and Notes</i> - For general structural parts in which thickness exceeds one inch or have a wide variation in thickness, Not weldable. Heat treatment and properties to be specified on forging drawing.							
Chrome Nickel Molybdenum Steel	4340	MIL-S-5000 in accordance with Avrocan M-7-1*	Hardened & Tempered			125	-	100	75	175	17	C27	6.00
						150	-	135	90	190	14	C34	4.00
						180	-	165	105	200	12	C40	2.50
						<i>Recommended Uses and Notes</i> - For general structural parts, heat treatment and properties to be specified on forging drawing. Not weldable.							
Carburizing Steel	4620	AMS 6294 in accordance with Avrocan M-7-1*	Normalized	Case Harden		100	-	75	65	140	20		2.00
						125	-	100	75	175	15		0.75
						<i>Recommended Uses and Notes</i> - For case hardened parts only. The strengths refer to core strength only. Forging drawings should specify normalized condition and no properties. The machining drawing should specify case harden quoting required properties.							

*Avrocan M-7-1 forging spec. is to be included on forging drawings, also heat treatment block specifying the heat treatment as noted in "Condition Received" column and properties.

3.2.4.4 Sheet

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Low Carbon Steel	1010	QQ-S-636 Temper 4	Soft Skin rolled temper									B65 Max	
						<i>Recommended Uses and Notes</i> - Soft ductile cold rolled product suitable for fairly deep drawing. It is capable of being bent flat upon itself. Weldable.							
Mild Steel	1020 1025	AN-S-11	Cold rolled			55	54	36	35	90	22	B65	
						<i>Recommended Uses and Notes</i> - For a/c fittings and misc. parts; not suitable for deep forming or cupping operations. Weldable but requires stress relief after spot welding where used on primary structure.							



3.2.4 Steels - Carbon and Alloy (Cont'd)

3.2.4.4 Sheet (Cont'd)

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
High Carbon	1095	AN-QQ-S-666	Annealed	Harden & Temper		180					7*	C40 to C45	
<i>Recommended Uses and Notes</i> - For springs, clips, etc. Heat treatment after forming is to be specified on drawing. Not weldable.													
Chrome Molybdenum Steel	4130	AN-QQ-S-685 Cond. N	Normalized	Harden & Temper	Up to .061	95	-	75	55	140	10	B94	
					.062 - .125	95	-	75	55	140	12	B94	
					.126 - .188	95	-	75	55	140	15	B94	
					.189 - .749	90	-	70	55	140	20	B92	
					.750 - 1.50	90	-	70	55	140	18	B92	
					Up to .031	125	-	100	75	175	5	C27	
					.032 - .060	125	-	100	75	175	7	C27	
					.061 - .249	125	-	100	75	175	10	C27	
					.250 - up	125	-	100	75	175	17*	C27	
					Up to .031	150	-	135	90	190	4	C34	
.032 - .060	150	-	135	90	190	6	C34						
.061 - .249	150	-	135	90	190	9	C34						
.250 - up	150	-	135	90	190	14*	C34						
<i>Recommended Uses and Notes</i> - For a/c fittings and misc. parts requiring higher strength than AN-S-11; heat treatment is to be specified if required. May be arc or torch welded but not spot welded. Drawings for all welded parts should specify "Stress Relieve" or "Harden and Temper after Welding". Stress relief treatment does not reduce normalized properties. Yield strength is reduced if normalized by user. See ANC-5.													

*Round test specimens used of QQ-M-151, Type 1 or 4.

3.2.4.5 Tubing

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Carbon Steel	1025 Seamless	MIL-T-5066 Type II	Cold Drawn & Stress Relieved or Normalized			55	54	36	35	90	22	B65	
<i>Recommended Uses and Notes</i> - For low strength parts; may be welded but requires stress relief when spot welded and used in primary structure.													
Chrome Molybdenum Steel	4130 Seamless	MIL-T-6736 Cond. N	Normalized		Up to .035	95	-	75	55	140	10*		
					.036 - .188	95	-	75	55	140	12*	B94	
					.189 - up	90	-	70	55	140	15*	B92	
<i>Recommended Uses and Notes</i> - See following page.													

*Values for tubing tested in full section



3.2.4 Steels - Carbon and Alloy (Cont'd)

3.2.4.5 Tubing (Cont'd)

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL SIZE In.	
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.				ELONG
		MIL-T-6736 Cond. N (Cont'd)		Harden & Temper		125	-	100	75	175	12*	C27		
						150	-	135	90	190	10*	C34		
						180	-	165	105	200	8*	C40		
		MIL-T-6736 Cond. HT125 Cond. HT150 Cond. HT180	Hardened & Tempered				125	-	100	75	175	12*	C27	
							150	-	135	90	190	10*	C34	
							180	-	165	105	200	8*	C40	

Recommended Uses and Notes - For general mechanical and structural parts. When tubing is to be used in conditions HT125, HT150 or HT180 it should be called up in that condition rather than heat treated by the user, unless bending or other working must be done in Cond. N. Where straightness is a factor, tubing should not be called up for heat treatment by user. May be arc or torch welded but not spot welded. It is difficult to obtain the higher heat treated properties on thicknesses over 3/16". Drawings for all welded parts should specify "Stress Relieve" or "Harden and Temper after welding". Stress relief treatment does not reduce normalized properties. Yield strength is reduced if normalized by user. See ANC-5.

*Values for tubing tested in full section

3.2.4.6 Wire

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Mild Steel Zinc Coated	Mild Steel	QQ-W-461 Grade FS 1005 Annealed Finish 4 Type 1	Annealed										
						<i>Recommended Uses and Notes</i> - For locking wire on nuts, turnbuckles etc. and for serving non-flexible cable splices.							
High Carbon Spring Steel (Music Wire)	Music Wire	MIL-W-6101	Cold Drawn			.048							
						.051	306						
						.055	303						
						.059	300						
						.063	296						
						etc. Up to .25"	293						

Recommended Uses and Notes - For small springs. No strength-improving heat treatment is required. See spec. for strength of smaller and larger sizes.



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3.2.5 Steels - Stainless

3.2.5.1 Bar and Rod

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	IZOD ft. - lbs.	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion Resisting Steel	303	QQ-S-763 Class 7 Type A*	Annealed			75	(45)	(30)	(40)	(75)	35		
	303	MIL-S-7720 Comp. FM Cond. B	Cold Worked			Up to 0.75	125	-	100			12	
						.76-1.00	115	-	80			15	
						1.01-1.25	105	97.5	65			20	
						1.26-1.50	100	75.0	50			28	
						1.51-3.00	95	67.5	45			28	
3.01-Up	80	52.5	35			28							
<p><i>Recommended Uses and Notes</i> - For parts requiring good corrosion resistance. Should not be used where working temperature exceeds 800°F. Slightly magnetic in condition B. Not recommended for gas or arc welding.</p>													
16-2 Corrosion Resisting Steel	431	AN-QQ-S-770 Cond. QT Class II	Quenched & Tempered		Up to 1.5"	115	-	90			15	50	
		AN-QQ-S-770 Cond. A	Annealed		All Sizes	115	-	90			15	50	1.5
				Quench & Temper		175	-	135			13	35	1.5
<p><i>Recommended Uses and Notes</i> - Use for stainless steel machined parts. Not weldable. Not to be used when working temperature exceeds 700°F. Magnetic. Material seizes when threaded to material of similar composition but not when threaded to material of 18-8 type SS. If parts require heat treatment in larger than 1.5" ruling section the Metallurgical Dept. should be consulted for reduced properties. These should have note added: "These properties refer to positions midway between centre and surface of ruling section". Izod value to be specified in heat treatment block when heat treated by user.</p>													
18-8 Corrosion and Heat Resisting Steel	321 or 347	QQ-S-763 Class 8 Type A	Annealed			75	(45)	(30)	(40)	(75)	35		
<p><i>Recommended Uses and Notes</i> - For parts requiring corrosion and heat resistance; weldable. Cannot be hardened by heat treatment; non-magnetic in annealed condition.</p>													

*Type A is used for swaged cable terminals, in which case a note must be added to spec. call-up requiring tensile strength not to exceed 115,000 psi.

3.2.5.2 Casting

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	BRINELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
Corrosion & Heat Resisting Steel		AMS 5363	Solution Heat treated									180 max	
<p><i>Recommended Uses and Notes</i> - For non-structural parts.</p>													



3.2.5 Steels - Stainless (Cont'd)

3.2.5.3 Forging

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	IZOD ft.-	CRITICAL IN. SIZE
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion Resisting Steel	303	MIL-S-7720 Comp. FM In accordance with Avrocan M-7-1*	Annealed			(75) to 100 max	(45)	(30)	(40)	(75)	35		
<i>Recommended Uses and Notes</i> - For parts requiring good corrosion resistance; should not be used when working temperature exceeds 800°F. It is non-magnetic. Not recommended for gas or arc welding.													

*Avrocan M-7-1 forging spec. is to be included on forging drawings. Also, anneal is to be specified in heat treatment block on forging drawings.

3.2.5.4 Sheet

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL IN. SIZE
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion Resisting Steel	301	MIL-S-5059 Comp. D Cond. 1/2 Hard	1/2 Hard		Up to .015 .016 and over	150 150	- -	110 110	80 80	180 180	15 18	C32	
<i>Recommended Uses and Notes</i> - For parts requiring good corrosion resistance; not recommended for gas or arc welding; may be spot welded only; should not be used when working temperature exceeds 800°F; slightly magnetic. The maximum thickness that can be supplied at the present time is .125".													
18-8 Corrosion and Heat Resisting Steel	321 or 347 or Cb-Ta	MIL-S-6721	Annealed			(75) to 100 max	(45)	(30)	(40)	(75)	40	(B80)	
<i>Recommended Uses and Notes</i> - For parts requiring corrosion and heat resisting properties; non-magnetic; weldable; cannot be hardened by heat treatment.													

3.2.5.5 Tubing

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in.	ROCKWELL HARDNESS	CRITICAL IN. SIZE
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion Resisting Steel	302	AN-WW-T-855 Cond. B	1/4 Hard			120	112.5	75			15*		
<i>Recommended Uses and Notes</i> - For general mechanical and structural parts requiring good corrosion resistance, but where gas or arc welding and elevated temperatures are not involved; slightly magnetic.													

*Tubing tested in full section



3.2.5 Steels - Stainless (Cont'd)

3.2.5.5 Tubing (Cont'd)

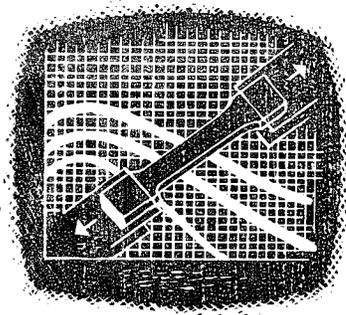
MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion and Heat Resisting Steel	321 or 347	AN-WW-T-858	Annealed			(75)	(45)	(30)	(40)	(75)	35*		
						<i>Recommended Uses and Notes</i> - For parts requiring corrosion and heat-resistance; weldable; cannot be hardened by heat treatment; non-magnetic.							
18-8 Corrosion Resisting Steel (For Hydraulic Systems)	304	MIL-T-6846 Seamless	Annealed			75 to 100 max	45	30			40*		
						<i>Recommended Uses and Notes</i> - For high pressure hydraulic systems. Non-magnetic. Not weldable, must not be brazed or exposed to temperatures higher than 800°F. Seamless to be specified on drawing.							

*Tubing tested in full section

3.2.5.6 Wire

MATERIAL	COMM ALLOY	SPECIFICATION	CONDITION RECEIVED	HEAT TREATED BY USER	THICKNESS INCHES	1000	1000	1000	1000	1000	% in 2 in. ELONG	ROCKWELL HARDNESS	CRITICAL SIZE In.
						PSI T.S.	PSI 1.5 Y.S.	PSI Y.S.	PSI S.S.	PSI B.S.			
18-8 Corrosion Resisting Steel Wire (Grade G)	302	MIL-W-6713 Cond. B	Spring Temper		.048 - .051 .052 - .055 .056 - .063 etc.	255							
						245							
<i>Recommended Uses and Notes</i> - For parts where corrosion resisting steel is required such as hinge pins, etc. Not to be welded or subjected to elevated temperatures. Not to be used where pitting type of corrosion is of primary importance such as springs subjected to salt spray and salt water. Slightly magnetic. See spec. for strength of smaller and larger sizes.													

Stress





SECTION V

STRESS

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2.2.2

PLANE TENSION FIELD BEAMS

2.2.2.1 General

Beam Analysis

The following sections present a method of analysis for partial tension field beams with parallel flanges and perpendicular uprights. Information, largely in the form of graphs, is provided for the analysis of beams constructed of 24S-T and 75S-T alloys.

Beam Design

For convenience, a table (Table 2.2.2-2) giving properties of standard extruded angles used as single uprights is included. A table of design information (Table 2.2.2-1) is presented which should enable the designer to select approximate web gauge, upright size, rivet sizes, etc. For a given shear flow, the necessary web gauge is listed, along with a large and small upright spacing, thus allowing the designer to determine the effect of this spacing on the remaining design variables. The information in this table applies only when the beam flanges are very stiff; nevertheless it should be useful as a first approximation for tension field beam design.

Effective Beam Depth

It is considered sufficiently accurate to take the distance between flange centroids, h , as a common dimension for the effective depth of the beam, depth of the web, and length of the upright.

Limitations

The curves given herein are believed to give reasonable assurance of conservative strength predictions provided that normal design practices and proportions are used. The most important points are:

- (i) The upright should not be too thin, keep $\frac{t_u}{t} > 0.6$
- (ii) The upright spacing should be in the range $0.2 < \frac{d}{h} < 1.0$
- (iii) The method of analysis presented here is applicable only to beams with webs in the range $115 < \frac{h}{t} < 1500$.

References

1. "Strength Analysis of Stiffened Beam Webs", by Paul Kuhn and James P. Peterson, NACA, T.N. 1364, July 1947.

2. "Ultimate Stresses Developed by 24S-T and Alclad 75S-T Aluminum-Alloy Sheet in Incomplete Diagonal Tension", by L. Ross Levin, NACA, T.N. 1756, November 1948.
3. "Strength Analysis of Stiffened Thick Beam Webs", by L. Ross Levin and Charles W. Sandlin, Jr. NACA, T.N. 1820, March 1949.
4. A summary of Diagonal Tension Part I by Kuhn, Peterson, and Levin, NACA T.N. 2661.
5. Strength of Aircraft Elements ANC-5, June 1951.

Symbols

A_f	Average area of tension and compression flanges, in. ²
A_u	Actual area of upright, in. ²
A_{u_e}	Effective area of upright, in. ²
C_1, C_2, C_3	Stress concentration factors
C_r	Rivet factor = $\frac{p-D}{p}$
D	Rivet diameter, in.
E	Modulus of elasticity, p.s.i.
F_{co}	Column yield stress, p.s.i.
F_{col}	Allowable column stress, p.s.i.
F_{max}	Ultimate allowable compressive stress for natural crippling, p.s.i.
F_o	Ultimate allowable compressive stress for forced crippling, p.s.i.
F_s	Ultimate allowable web shear stress, p.s.i.
$F_{s_{cr}}$	Critical shear stress for initial buckling of web, p.s.i.
I_f	Average moment of inertia of beam flanges, in. ⁴
M_{sb}	Secondary bending moment in the flange, in. lb.
P	Applied shear load, lb.



P_u	Upright end load, lb.
d	Spacing of uprights, in.
e	Distance from upright centroid to web, in.
f_{cent}	Compressive stress at centroid of upright, p.s.i.
f_f	Compressive stress in flange due to horizontal component of diagonal tension, p.s.i.
f_s	Shear stress in web, p.s.i.
f_u	Average lengthwise compressive stress in upright, p.s.i.
$f_{u_{max}}$	Maximum compressive stress in upright, p.s.i.
h	Effective depth of beam, centroid of compression flange to centroid of tension flange, in.
k	Diagonal tension factor
p	Rivet pitch, in.
q	Beam shear flow, lb/in.
t	Thickness of web, in.
t_u	Thickness of upright, in.
α	Angle of diagonal tension, degrees
ρ	Radius of gyration of upright with respect to its centroidal axis parallel to web, in. (No portion of web to be included.)

2.2.2.2 Analysis of Web

Web Shear Stress

The web shear flow can be closely approximated by, $q = \frac{P}{h}$

From this it follows that the web shear stress is,

$$f_s = \frac{q}{t}$$

Diagonal Tension Factor

The diagonal tension field factor, k , can be found from Figure 2.2.2-4 as a function of the web shear

stress, f_s , and the ratios d/t and d/h . This factor specifies the portion of the total shear that is carried by the diagonal tension action of the web. The curves contain a practical approximation for values in the plastic range and the simplifying assumption is made that the shear panel has simply supported edges. The more exact method which attempts to account for varying degrees of edge restraint, as employed for shear resistant webs, is considered an unnecessary refinement in the case of tension field webs because the effect of a more exact value of k is small. Moreover, the effect of edge restraint is considered in the allowable web shear stresses where the difference is appreciable.

It is recommended that the diagonal tension factor at ultimate load be limited to a maximum value,

$$k_{max} = .75 - 2.6t$$

in order to avoid excessive wrinkling and permanent set at limit load and thereby inviting fatigue failure.

Allowable Web Shear Stresses

The allowable web shear stress, F_s , can be obtained from Figure 2.2.2-11 for 75S-T Alclad sheet or Figure 2.2.2-12 for 24S-T Alclad sheet. These values are based on tests of long webs subjected to loads approximating pure shear, and contain an allowance for the rivet factor, C_r . This factor is included because tests have shown that the ultimate allowable shear stress based on the gross section is almost constant in the normal range of the rivet factor. ($C_r > 0.6$). The values for F_s are given as a function of the stress concentration factor, C_2 , which can be found from Figure 2.2.2-6 as a function of ωd . Unfortunately the higher values of C_2 are largely theoretical since very few tests, if any, were made with beams having extremely flexible flanges. On the basis of a few scattered tests, the indications are that the values of C_2 become increasingly conservative for the higher values of the flange flexibility parameter. However, until more test data is available, the present values remain the closest conservative approximation.

The NACA also gives an additional stress concentration factor, C_1 , due to the diagonal tension angle differing from the 45° angle associated with the state of pure diagonal tension. However, this correction is always small and is felt to be unwarranted in view of the conservatism employed throughout the analysis.



2.2.2.3 Analysis of Uprights

Effective Area of Upright

The total cross sectional area of the upright for double or single uprights is given as A_u , and no part of the web is to be included in any case. However, if the upright is attached to a rib or bulkhead an effective width of the bulkhead web may be included. In order to make the design charts apply to both single and double uprights the following effective upright areas, A_{ue} , are to be used in the analysis.

- (i) For double upright symmetrical with respect to web,

$$A_{ue} = A_u$$

- (ii) For single uprights,

$$A_{ue} = \frac{A_u}{1 + (e/\rho)^2}$$

See Table 2.2.2-2 for properties of standard extruded angles commonly used as uprights.

Design Criteria

The uprights or web stiffeners must have sufficient moment of inertia to prevent buckling of the web panel as a whole before formation of the tension field, and to prevent column failure under the loads imposed on the upright by the tension field. Furthermore, the upright must be thick enough to prevent forced crippling failure caused by the waves of the buckled web. The latter mode of failure is almost always the most critical. From the values given in Table 2.2.2-1 a good first approximation for A_u seems to be in the range A_u equals 0.6 dt to 0.8 dt.

Moment of Inertia

The required moment of inertia of the upright about its own neutral axis parallel to the web is obtained from Figure 2.2.2-7 and is a function of ht^3 and the ratio d/h . Essentially, the curve is derived by equating the critical buckling stress of the sheet between stiffeners to the general instability stress of the stiffened web as a whole. As the stiffener spacing decreases, the required moment of inertia increases as a consequence of the higher critical buckling stress of the sheet.

Stresses

The lengthwise average stress in the upright at the faying surface of the leg attached to the web, f_u , is obtained from Figure 2.2.2-5 as a function of k , $\frac{A_{ue}}{dt}$, and f_s as

$$f_u = f_s (f_u / f_s)$$

The upright stress varies from a maximum at the neutral axis of the beam to a minimum at the ends of the upright due to the gusset effect. The maximum stress, $f_{u\max}$, is obtained from Figure 2.2.2-5 as a function of k , d/h , and f_u as,

$$f_{u\max} = f_u (f_{u\max} / f_u)$$

Allowables

The maximum upright stress, $f_{u\max}$, should be checked against an allowable forced crippling stress, F_o , which is obtained from Figure 2.2.2-10. In the case of double uprights only, $f_{u\max}$ should also be checked against an allowable natural crippling stress, F_{\max} , as calculated from the information given in Section 3.1.1 of this Manual.

The stress f_u should be no greater than the column yield stress, F_{co} .

The centroidal upright stress, $f_{cent} = f_u \frac{A_{ue}}{A_u}$ should be checked against an allowable column stress, F_{col} , which is obtained from Figure 2.2.2-10 for the slenderness ratio, h'/ρ .

For simplicity, h' may be taken as h , since this effect is rarely critical. However if necessary, an effective column length h' , may be used where,

$$h' = \frac{h}{\sqrt{1 + k^2 (3 - \frac{2d}{h})}} \dots\dots \text{for double uprights}$$

$$h' = \frac{h}{2} \dots\dots \text{for single uprights}$$



The theoretical "column" types of failure are greatly at variance with test results but strength checks given are the best available to date.

2.2.2.4 Analysis of Rivets

Rivets: Web-to-Flange

The shear load per inch acting on the web-to-flange rivets is given in Figure 2.2.2-8. It should be pointed out that the rivet factor, C_r , should be not less than 0.6 in order to justify the allowable web stresses used and to avoid undue stress concentration.

Rivets: Web-to-Upright

The tensile load per inch acting on the web-to-upright is given in Figure 2.2.2-9. The tensile loads on the rivets are exerted by the prying action of the buckled web. The criteria used is a variation of that suggested by the NACA because the degree of diagonal tension is accounted for which is considered to be a more reasonable and realistic approximation. However, these loads should be considered only tentative because of the meager test data available at this time, but nevertheless, they do reflect time tested practice.

The tensile load criteria is believed to insure a satisfactory design as far as shear strength is concerned. The NACA has suggested a lesser tensile load criteria for double uprights but added a formula to check the total rivet strength in shear. This latter formula does not account for the restraining action of the web among other things and consequently is not recommended.

Rivets: Upright-to-Flange

The shear load on the upright to flange rivets is given by the formula,

$$P_u = (f_u) (A_{u_e})$$

(Note: $A_{u_e} = A_u$ for double uprights)

This formula ignores the gusset effect in order to be conservative and also neglects certain other forces, the net effect of which makes the above formula a close approximation.

2.2.2.5 Analysis of Flange

The total stress in the flanges is the result of the superposition of three individual stresses; the primary beam stress, the compressive stress caused by the horizontal component of the diagonal tension in the web, and the secondary bending stress, caused by the distributed vertical component of the diagonal tension.

Compressive stresses due to the horizontal component of diagonal tension

$$f_f = \frac{kqh}{2A_f + .5(1-k)th}$$

Secondary bending moment due to the distributed vertical component of diagonal tension:

$$M_{sb} = \frac{1}{12} k f_s t d^2 C_3$$

where C_3 is given in Figure 2.2.2-6 (Compression is on the innermost fiber of the flange). M_{sb} is the maximum moment in the bay and exists at the ends of the bay over the uprights. If C_3 and k are near unity, the moment in the middle of the bay is half as large as that given and of opposite sign. Shear beam flanges are often attached to upper and lower surfaces etc. The effects of these webs must be computed in the same manner and superimposed upon those from the main shear web.

2.2.2.6 Combined Loading

Interaction Curves

This section may be used to solve plane rectangular panels and cylinders subjected to shear and normal stress or shear and transverse stress. Reference N.A.C.A. T.N. 1223.

The combination of shear and axial or transverse stress that causes plane panels to buckle may be obtained from Figures 2.2.2-13 and 2.2.2-14. When axial or transverse stress are known to exist in a plane shear web, it is necessary to alter the beam analysis in a manner similar to the solution of combined loadings shown in Section 2.2.5-1.



TABLE 2.2.2.-1

TENSION FIELD BEAM DESIGN DATA

75S-T6 Alclad Webs - 75S-T6 Extruded Angle Uprights

The data in this table has been worked out to give positive margins of safety for ultimate allowable shear flows indicated and applies only to beams with very stiff flanges. (See Sect. 2.2.2.1). It is intended that this table serve only as a general guide in proportioning tension field beams. For each shear flow the necessary web gauge is given and two widely different upright spacings are listed. This should enable the designer to evaluate this effect and interpolate accordingly. From the values given in the table, a good first approximation for A_u seems to be in the range A_u equals 0.6 dt to 0.8 dt.

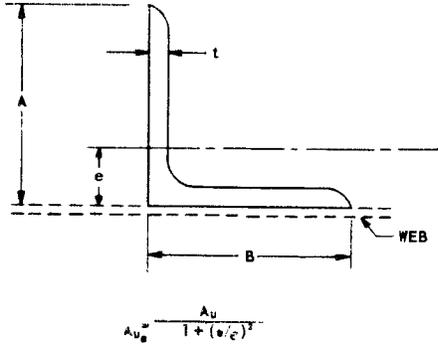
ULT. ALLOW SHEAR FLOW	WEB THICKNESS	UPRIGHT SPACING	BEAM DEPTH	UPRIGHT (SMALL LEG AT WEB)		RIVET SIZE AND EFFECTIVE SPACING			
				DIMENSIONS	A_u/dt	Web to Flange	Web to Upright	Upright to Flange	
q	t	d	h						
500	.020	4	6	5/8 x 1/2 x .050	.67	AD3 at 0.30	AD3 at 0.60	2 - AD4	
500	.020	7	11	1 x 3/4 x .063	.77	AD3 at 0.30	AD3 at 0.60	2 - DD6	
625	.025	4	6	5/8 x 5/8 x .063	.75	AD4 at 0.45	AD4 at 0.80	2 - AD4	
625	.025	8	12	3/4 x 3/4 x .094	.65	AD4 at 0.40	AD4 at 0.80	2 - DD6	
800	.032	4	6	3/4 x 3/4 x .063	.71	AD5 at 0.55	AD5 at 1.05	2 - AD5	
800	.032	9	14	1-1/4 x 1 x .094	.70	AD5 at 0.50	AD5 at 0.95	2 - DD8	
1050	.040	4	6	1-1/4 x 3/4 x .063	.775	AD5 at 0.45	AD5 at 1.10	2 - AD5	
1000	.040	10	15	1-1/4 x 1-1/4 x .125	.745	AD5 at 0.45	AD5 at 0.95	2 - DD8	
1350	.051	5	8	1 x 7/8 x .094	.655	DD6 at 0.70	DD6 at 1.30	1 - DD6	
1300	.051	11	17	1-3/4 x 1-1/2 x .156	.85	DD6 at 0.65	DD6 at 1.10	2 - ALPPH-T6	
1800	.064	5	8	1-1/4 x 1 x .094	.63	DD6 at 0.60	DD6 at 1.30	1 - DD6	
1650	.064	12	18	2 x 1-1/2 x .156	.675	DD6 at 0.55	DD6 at 1.00	2 - ALPPH-T6	
2300	.081	6	9	1-1/4 x 1-1/4 x .125	.615	DD6 at 0.45	DD6 at 1.15	1 - DD6	
2100	.081	13	20	2-1/2 x 2 x .188	.77	DD6 at 0.45	DD6 at 0.90	3 - ALPPH-T6	
2900	.102	7	11	1-1/2 x 1-1/4 x .156	.56	DD8 at 0.65	DD8 at 1.50	1 - DD8	
2600	.102	14	21	2 x 2 x .250	.645	DD8 at 0.65	DD8 at 1.30	3 - ALPPH-T6	
3650	.125	8	12	2 x 1-1/2 x .156	.52	DD8 at 0.55	DD8 at 1.40	1 - DD8	
3200	.125	15	23	2-1/2 x 2-1/2 x .250	.625	DD8 at 0.55	DD8 at 1.20	2 - ALPPH-T8	
4700	.156	9	14	2 x 1-3/4 x .188	.47	DD8 at 0.40	DD8 at 1.25	1 - DD8	
4300	.156	16	24	3 x 2-1/2 x .313	.64	DD8 at 0.40	DD8 at 1.05	2 - ALPPH-T8	



TABLE 2.2.2.-2

PROPERTIES OF STANDARD EXTRUDED UPRIGHTS

Reference: AN Standards, Volume 2



AND 10133						
ANGLE - EQUAL LEG, EXTRUDED						
Dash No.	A	t	A _u	ρ _{yy}	I _{yy}	A _{ue}
0401	.500	.063	.0582	.147	.0013	.0297
0501	.625	.063	.0749	.186	.0026	.0400
0502		.078	.0908	.185	.0031	.0467
0503	.750	.094	.1068	.181	.0035	.0525
0601		.063	.0908	.227	.0047	.0498
0602	.875	.094	.130	.220	.0063	.0665
0603		.125	.167	.216	.0078	.0802
0701	1.000	.063	.107	.265	.0075	.0597
0702		.094	.154	.262	.0106	.0819
0703	1.250	.125	.198	.256	.0129	.0990
1001		.063	.128	.302	.0118	.0735
1002	1.500	.094	.183	.296	.0160	.0989
1003		.125	.235	.293	.0202	.121
1201	1.750	.063	.159	.383	.0234	.0932
1202		.094	.230	.379	.0330	.129
1203	2.000	.125	.298	.373	.0414	.160
1204		.188	.427	.362	.0561	.210
1401	2.500	.063	.191	.464	.0411	.113
1402		.094	.277	.460	.0585	.158
1403	3.000	.125	.360	.454	.0741	.199
1404		.188	.521	.443	.1021	.268
1601	3.500	.094	.330	.536	.0947	.192
1602		.125	.429	.532	.1214	.242
1603	4.000	.168	.621	.521	.1683	.329
2001		.094	.377	.617	.1436	.222
2002	4.500	.125	.491	.612	.1838	.281
2003		.188	.715	.601	.2582	.388
2004	5.000	.250	.924	.591	.3222	.475
2401		.125	.616	.774	.3689	.359
2402	5.500	.188	.903	.762	.5248	.504
2403		.250	1.17	.751	.6628	.628
3001	6.000	.188	1.09	.924	.9306	.621
3002		.250	1.42	.912	1.1834	.783
3003	6.500	.313	1.75	.900	1.4200	.932

AND 10134							
ANGLE - UNEQUAL LEG, EXTRUDED							
Dash No.	A	B	t	A _u	ρ _{yy}	I _{yy}	A _{ue}
0501	.625	.500	.050	.0533	.193	.0020	.0276
0601	.750	.500	.063	.0739	.235	.0041	.0357
0602	.875	.625	.063	.0818	.232	.0044	.0427
0701		.625	.063	.0922	.274	.0069	.0470
0702	1.000	.750	.063	.100	.270	.0073	.0539
0703		.125	.184	.258	.0123	.0872	
1001	1.250	.625	.063	.100	.316	.0100	.0494
1002		.125	.184	.304	.0170	.0807	
1003	1.500	.750	.063	.108	.313	.0106	.0565
1004		.125	.200	.302	.0182	.0937	
1005	1.750	.875	.063	.116	.316	.0111	.0637
1006		.125	.267	.305	.0155	.0872	
1201	2.000	.750	.063	.124	.400	.0197	.0616
1202		.094	.179	.393	.0276	.0845	
1203	2.500	.125	.231	.387	.0346	.104	
1204		.063	.139	.394	.0217	.0756	
1205	3.000	.094	.202	.389	.0306	.105	
1206		.125	.262	.382	.0383	.130	
1401	3.500	.750	.094	.204	.476	.0463	.0927
1402		.125	.264	.465	.0584	.114	
1403	4.000	.094	.228	.474	.0512	.114	
1404		.125	.295	.468	.0647	.142	
1405	4.500	.156	.361	.463	.0772	.169	
1406		.094	.251	.468	.0550	.135	
1407	5.000	.125	.327	.462	.0698	.170	
1408		.156	.400	.457	.0834	.201	
1601	6.000	1.000	.125	.327	.553	.0998	.153
1602		1.250	.125	.358	.549	.1078	.180
1603	7.000	1.500	.125	.389	.542	.1144	.208
1604		.156	.478	.537	.1377	.248	
2001	8.000	1.000	.125	.358	.637	.1453	.162
2002		.156	.439	.631	.1745	.193	
2003	9.000	.125	.392	.643	.1576	.195	
2004		.156	.480	.629	.1893	.229	
2005	10.000	.188	.568	.622	.2197	.263	
2006		.125	.423	.629	.1675	.221	
2007	11.000	.156	.519	.623	.2016	.263	
2008		.188	.615	.618	.2346	.304	
2009	12.000	.125	.454	.623	.1760	.249	
2010		.156	.558	.617	.2125	.298	
2011	13.000	.188	.662	.611	.2472	.341	
2401		.156	.558	.796	.3540	.252	
2402	14.000	.188	.662	.790	.4137	.292	
2403		.156	.597	.795	.3773	.288	
2404	15.000	.188	.709	.789	.4416	.334	
2405		.156	.636	.791	.3976	.294	
2406	16.000	.188	.756	.785	.4660	.376	
2407		.156	.681	.783	.4174	.363	
2408	17.000	.188	.809	.777	.4889	.421	
2409		.156	.720	.776	.4333	.398	
2410	18.000	.188	.856	.770	.5077	.462	
3001		.188	.809	.956	.7387	.367	
3002	19.000	.250	1.050	.943	.9323	.458	
3003		.188	.903	.950	.8159	.451	
3004	20.000	.250	1.17	.938	1.034	.564	
3005		.250	1.30	.927	1.115	.676	
3006	21.000	.313	1.60	.915	1.334	.803	



AIRCRAFT ENGINEERING MANUAL

VOL. 1 (DESIGN)

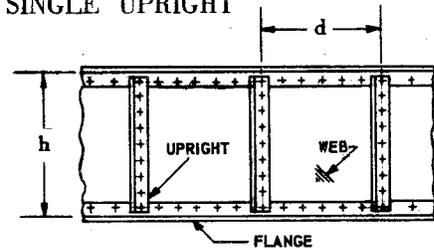
Sect V
Sub-Sect 2.2.2
Sample Calculations

TABLE 2.2.2.-3
SAMPLE CALCULATIONS FOR BEAM WITH SINGLE UPRIGHT

Given: $q = 1300 \text{ lb./in.}$, $h = 10''$, $d = 8''$

Web - .051 75S-T6 Alclad
Beam flanges - $1.25 \times 1.25 \times .188$
Upright - $1.00 \times 1.00 \times .125$
Rivets - web to flange,
 - web to upright,
 - flange to upright,

75S-T6 Extruded Angle
75S-T6 Extruded Angle
DD6 at .7 O.C.
AD5 at .8 O.C.
2 - DD6



	EQUATION	REFERENCE	VALUE	ROW
Shear Flow	q		1300	1
Web Thickness	t		.051	2
Beam Depth	h		10	3
Upright Spacing	d		8	4
Upright Thickness	t_u		.125	5
Upright Area	A_u		.235	6
Upright Radius of Gyration	ρ		.293	7
Upright Moment of Inertia	I_u		.0202	8
Upright Effective Area	A_{ue}		.121	9
Flange Moment of Inertia	I_f		.0561	10
Web Shear Stress	f_s	(1) / (2)	25,500	11
P A R A M E T E R S	d/t	(4) / (2)	157	12
	d/h	(4) / (3)	.80	13
	ωd	Figure 2.2.2.-3	.46	14
	A_{ue}/dt	(9) / (4) (2)	.296	15
	$ht^3 \times 10^3$	(3) (2) ³ $\times 10^3$	1.33	16
	t_u/t	(5) / (2)	2.45	17
	h/ρ	(3) / (7)	34.2	18
Diagonal Tension Factor	k	Figure 2.2.2.-4	.42	19
Ratio - Upright to Web Stresses	f_u/f_s	Figure 2.2.2.-5	.61	20
Ratio - Upright Stresses	$f_{u_{max}}/f_u$	Figure 2.2.2.-5	1.15	21
Stress Concentration Factor - Flange Flexibility	C_2	Figure 2.2.2.-6	.22	22
Stress Concentration Factor - Secondary Bend. Mom.	C_3	Figure 2.2.2.-6	.90	23
Minimum Mom. of Inertia Required for Upright	I_s	Figure 2.2.2.-7	.0013	24
Rivet Load - Web to Flange	q_r	Figure 2.2.2.-8	1520	25
Rivet Load - Web to Upright	q_t	Figure 2.2.2.-9	760	26
Upright Allowable - Column Yield Stress	F_{co}	Figure 2.2.2.-10	81,500	27
Upright Allowable - Column	F_{col}	Figure 2.2.2.-10	67,000	28
Upright Allowable - Forced Crippling	F_o	Figure 2.2.2.-10	19,700	29
Allowable Web Stress	F_s	Figure 2.2.2.-11	25,500	30
Upright Stress, Average	f_u	(11) (20)	15,550	31
Upright Stress, Centroidal	$f_{cent.}$	(31) (9) / (8)	8010	32
Upright Stress, Maximum	$f_{u_{max}}$	(31) (21)	17,900	33
Rivet Load, Upright to Flange	P_u	(31) (9)	1880	34
Secondary Bending Moment	M_{sb}	$1/12$ (9) (1) (2) (4) ² (23)	2620	35
Allowable Rivet Load - Web to Flange		See Fig	1640	36
Allowable Rivet Load - Web to Upright		See Fig	885	(37)
Allowable Rivet Load - Upright to Flange		See Fig	2360	38
Margin of Safety - Web		(30) / (11) - 1	.00	39
Margin of Safety - Uprights		*	.10	40
Margin of Safety - Rivets, Web to Flange		(33) / (25) - 1	.08	41
Margin of Safety - Rivets, Web to Upright		(37) / (26) - 1	.16	42
Margin of Safety - Rivets, Upright to Flange		(38) / (34) - 1	.26	43

* Either (8) -1, (27) -1, (28) -1, or (29) -1, whichever is least.
(24) (31) (32) (33)

Beam flanges are usually affected by webs and forces other than the vertical shear web and hence require separate analysis.

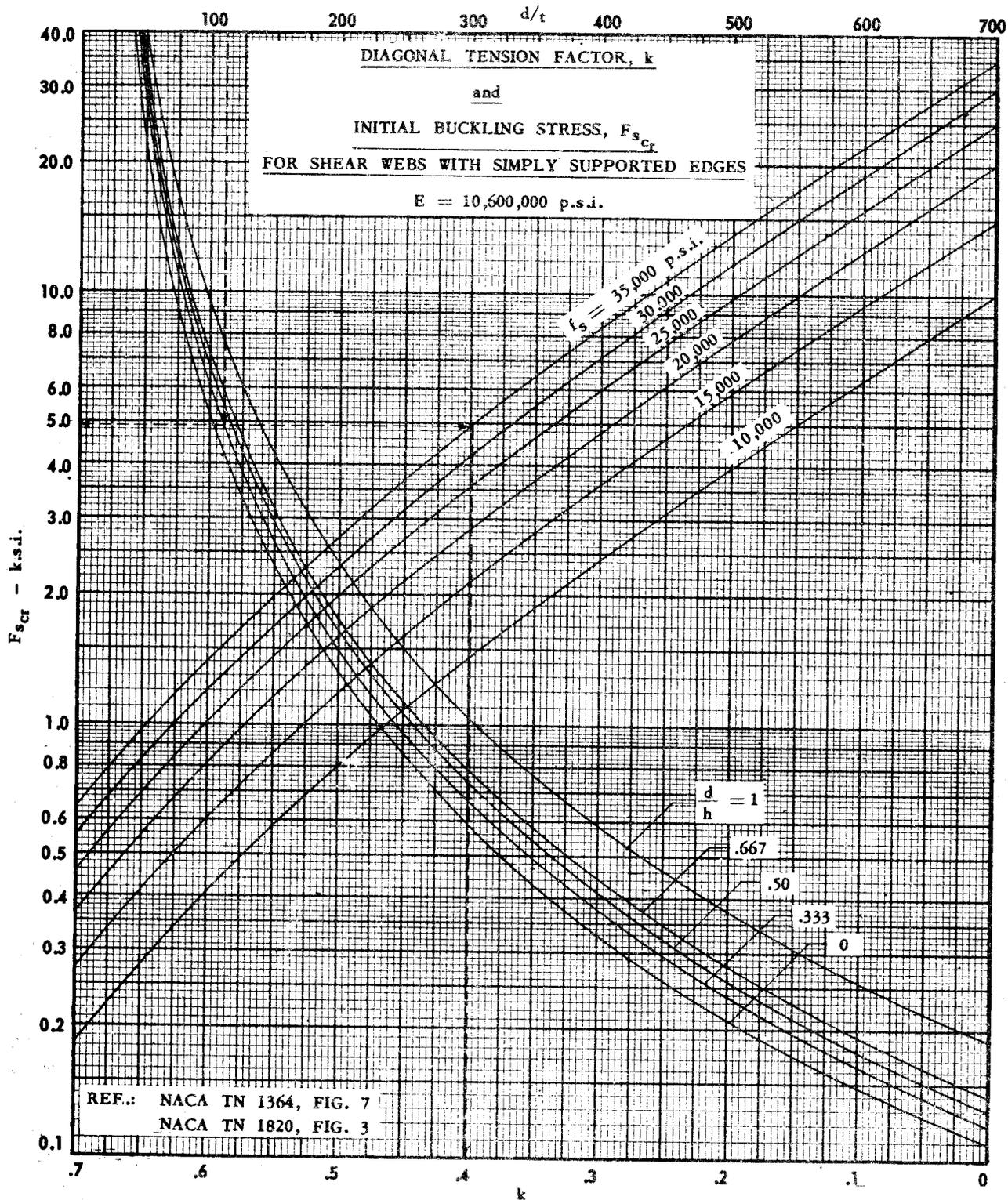


FIGURE 2.2.2-4

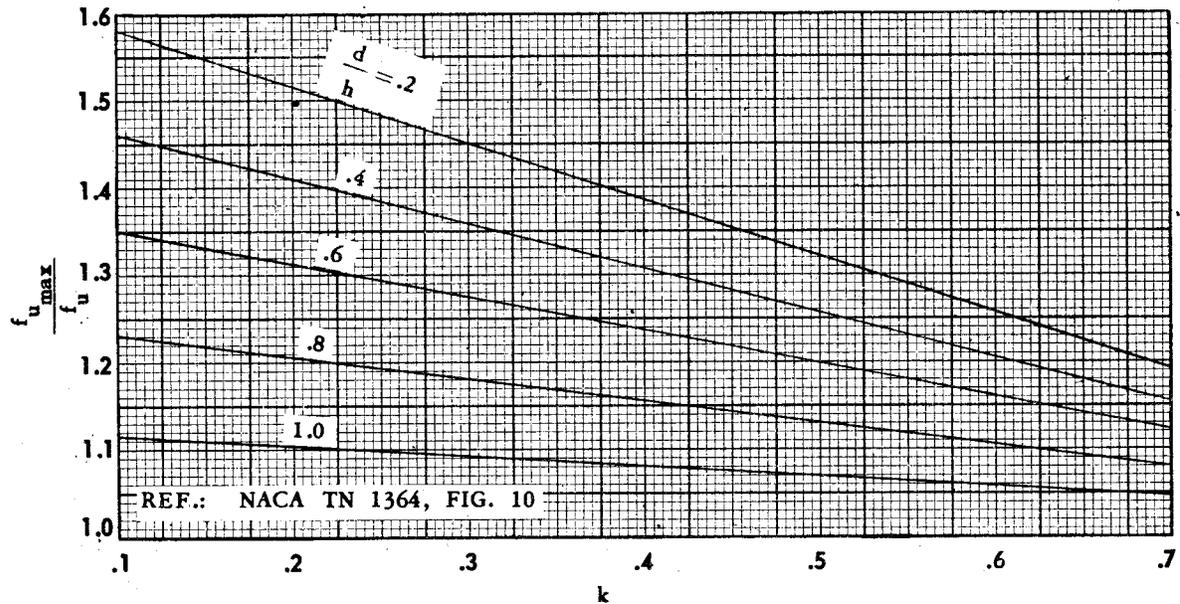
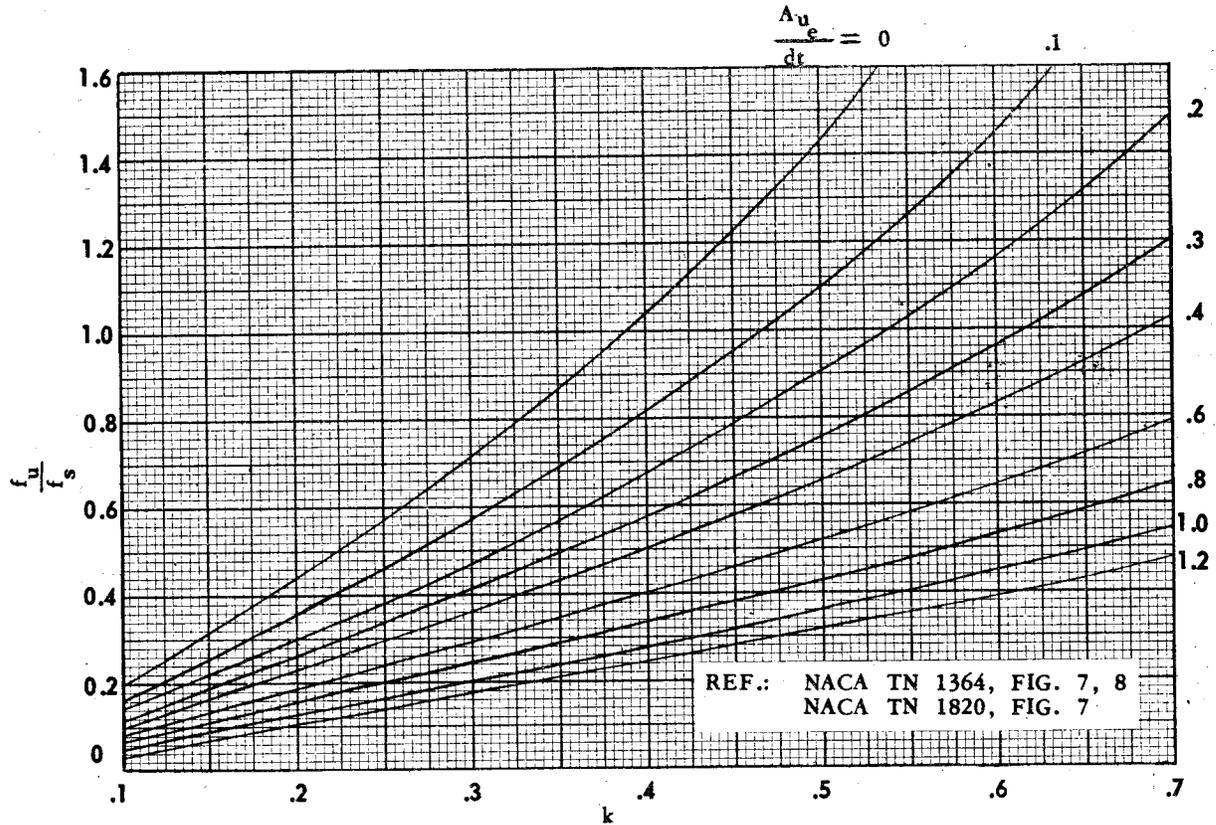


FIGURE 2.2.2-5
UPRIGHT STRESS RATIOS

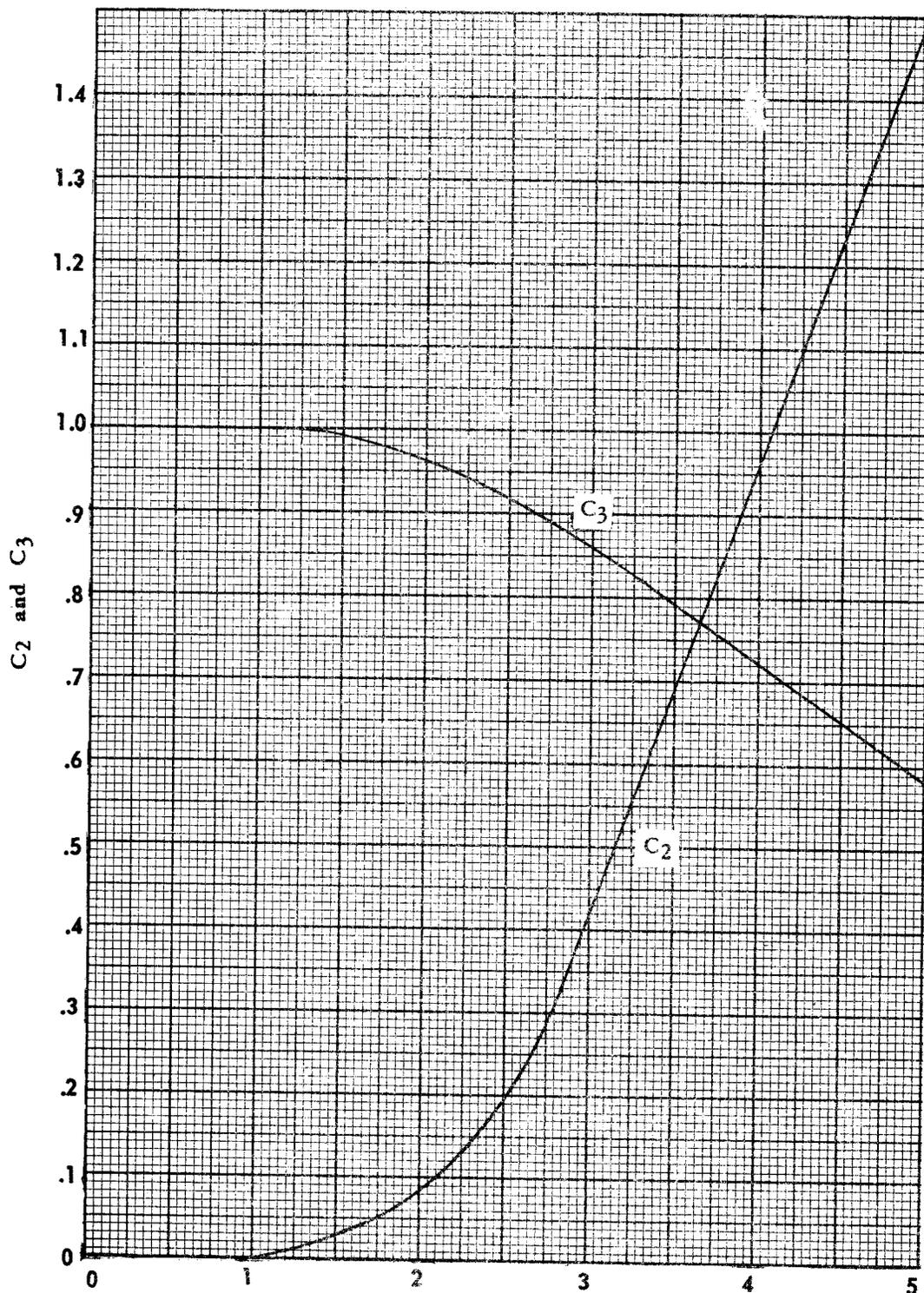


FIGURE 2.2.2-6

$$\omega d = 0.7d \sqrt{\frac{4\tau}{2lgh}}$$

STRESS CONCENTRATION FACTORS C_2 AND C_3

REF. N.A.C.A. T.N. 1364 FIG. 13

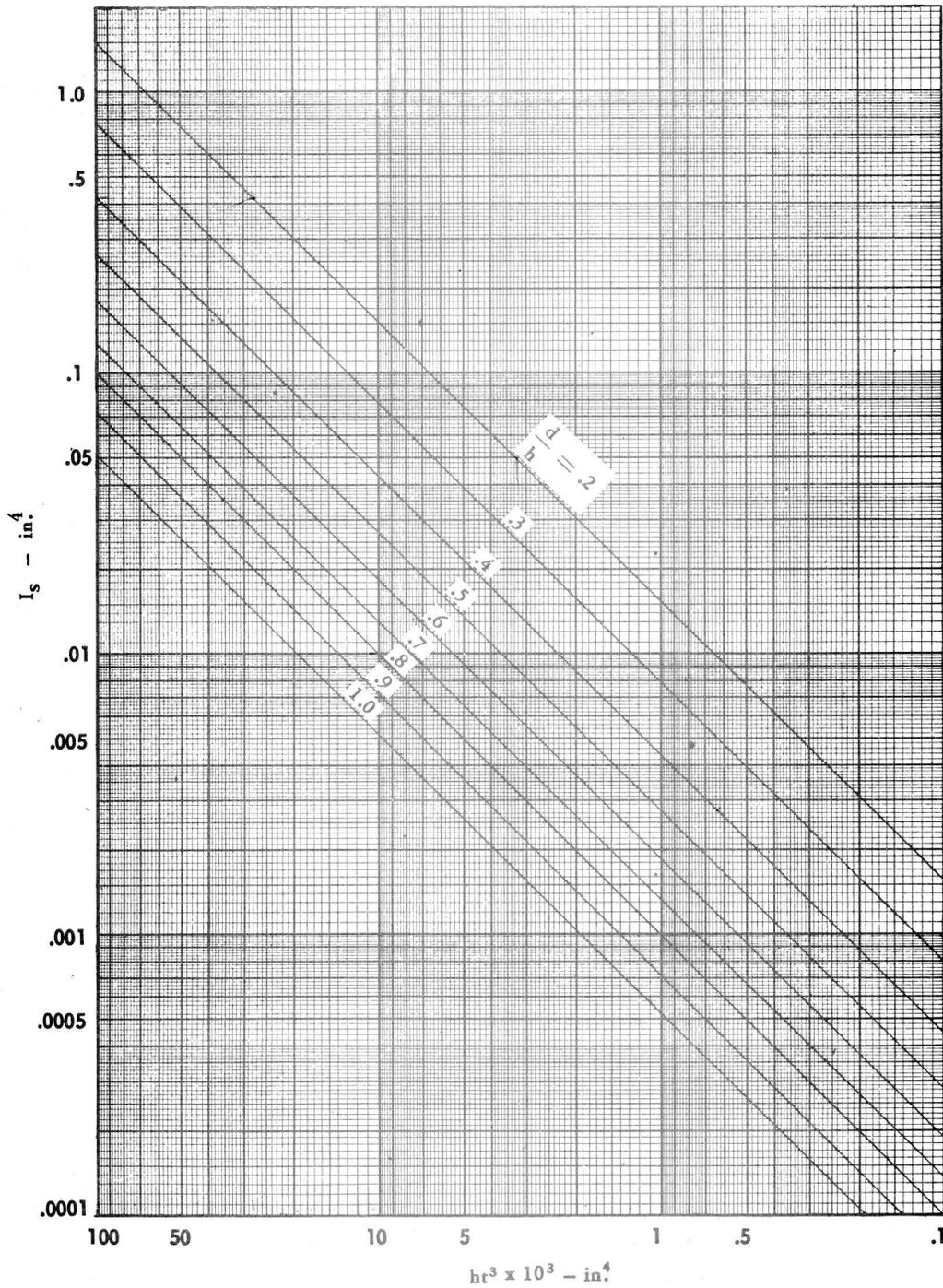


FIGURE 2.2.2-7
MINIMUM MOMENT OF INERTIA REQUIRED
FOR TENSION FIELD BEAM UPRIGHTS

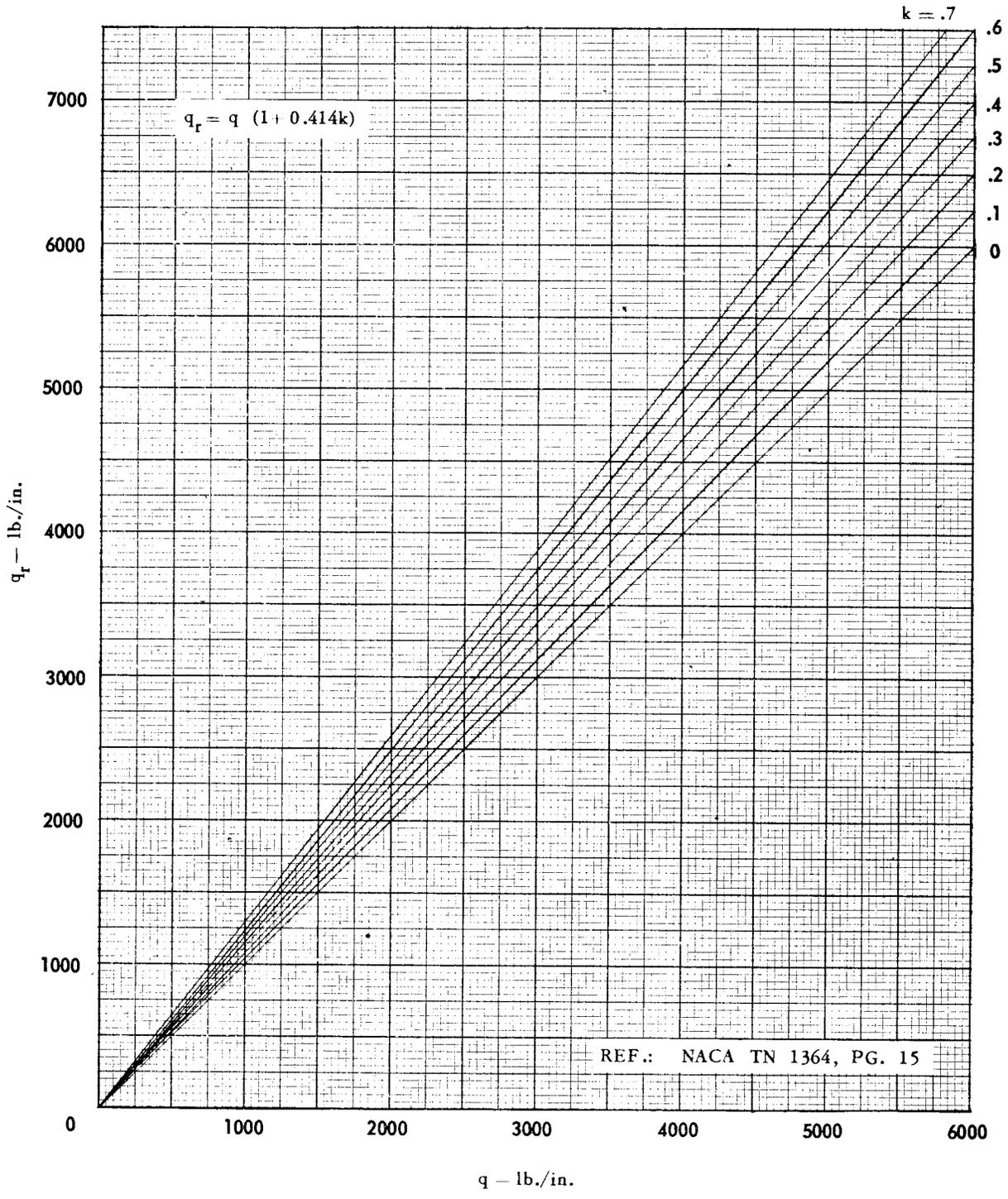


FIGURE 2.2.2-8
RIVET LOADS
WEB-TO-FLANGE AND WEB SPLICES

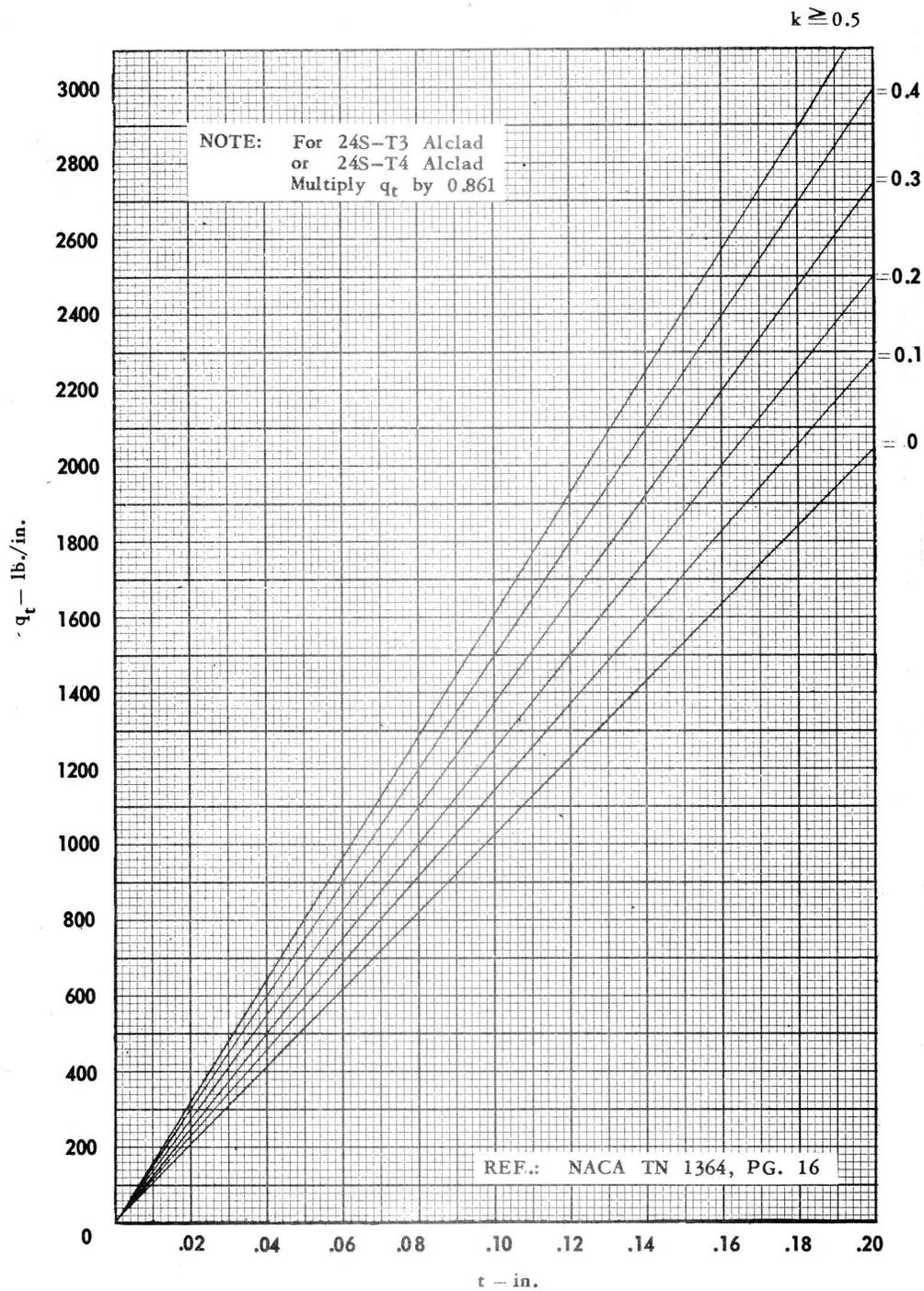


FIGURE 2.2.2-9
RIVET TENSION LOADS
WEB TO UPRIGHT
75S-T6 ALCLAD

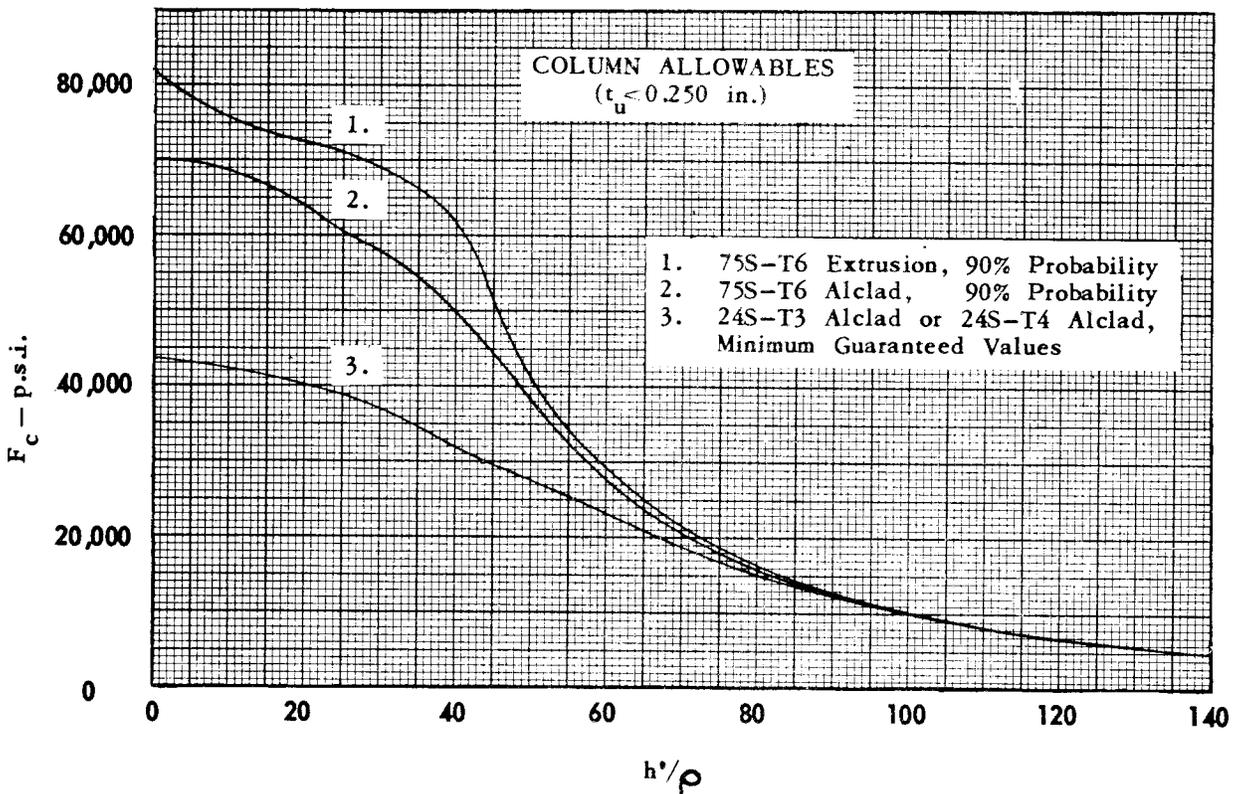
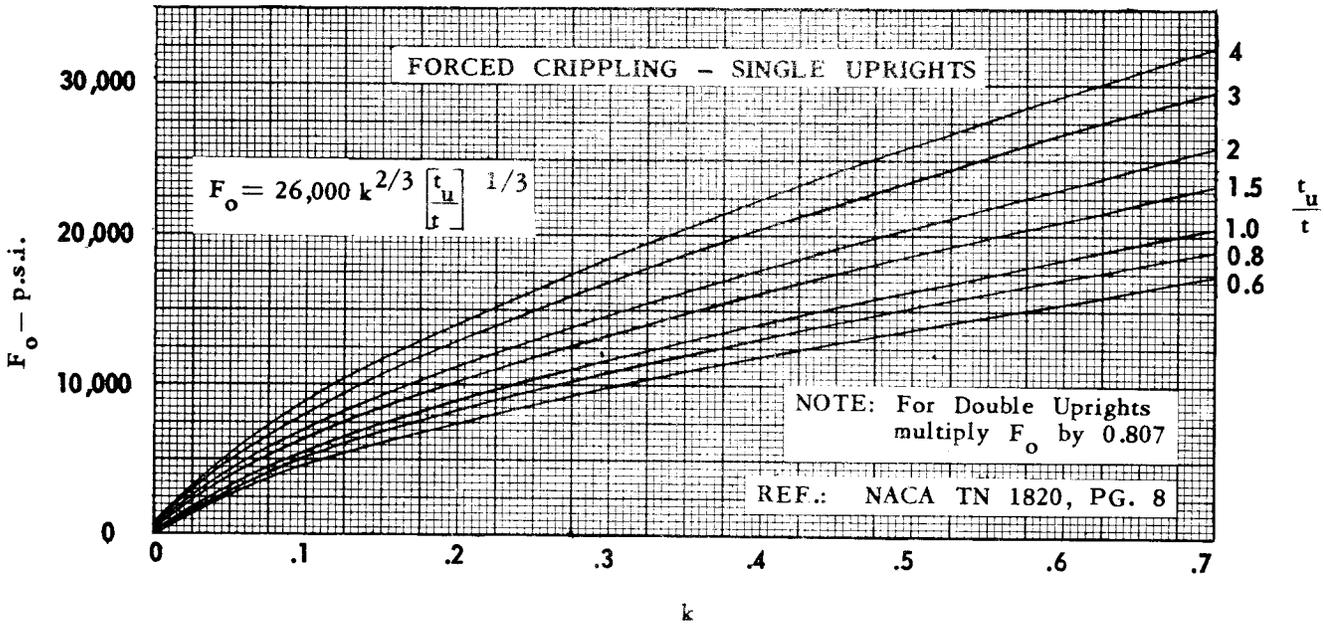


FIGURE 2.2.2-10
UPRIGHT ALLOWABLES

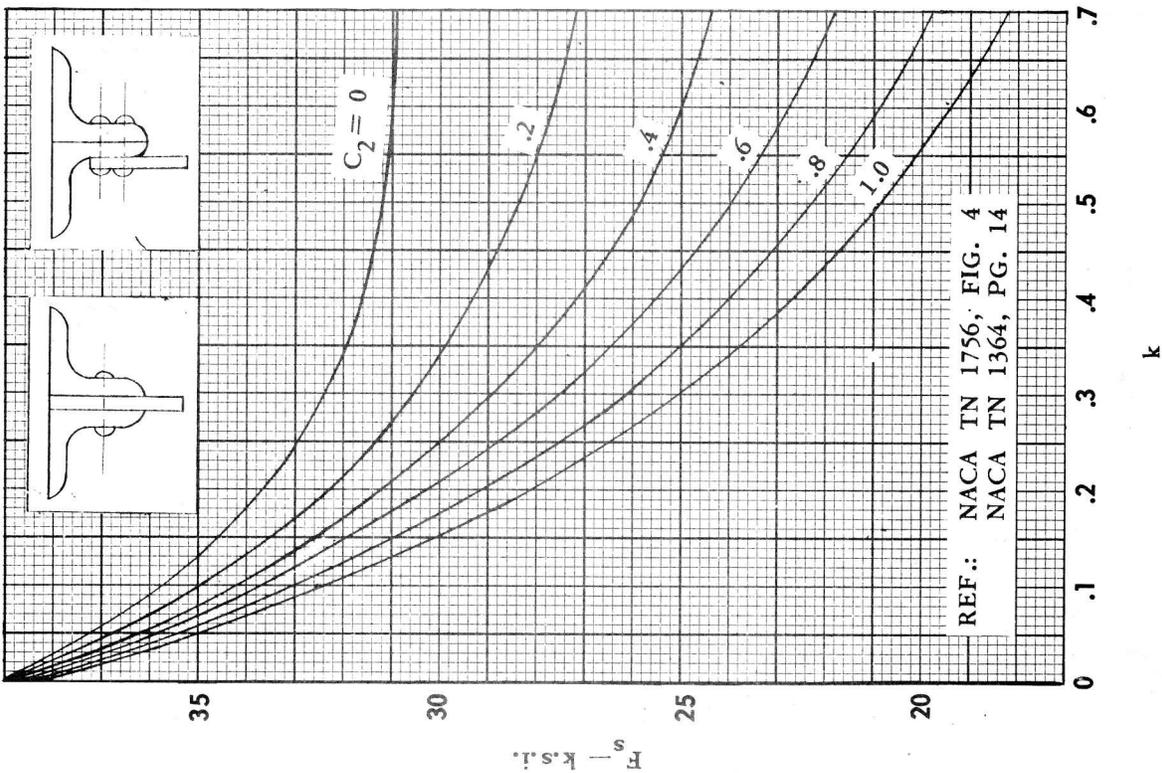
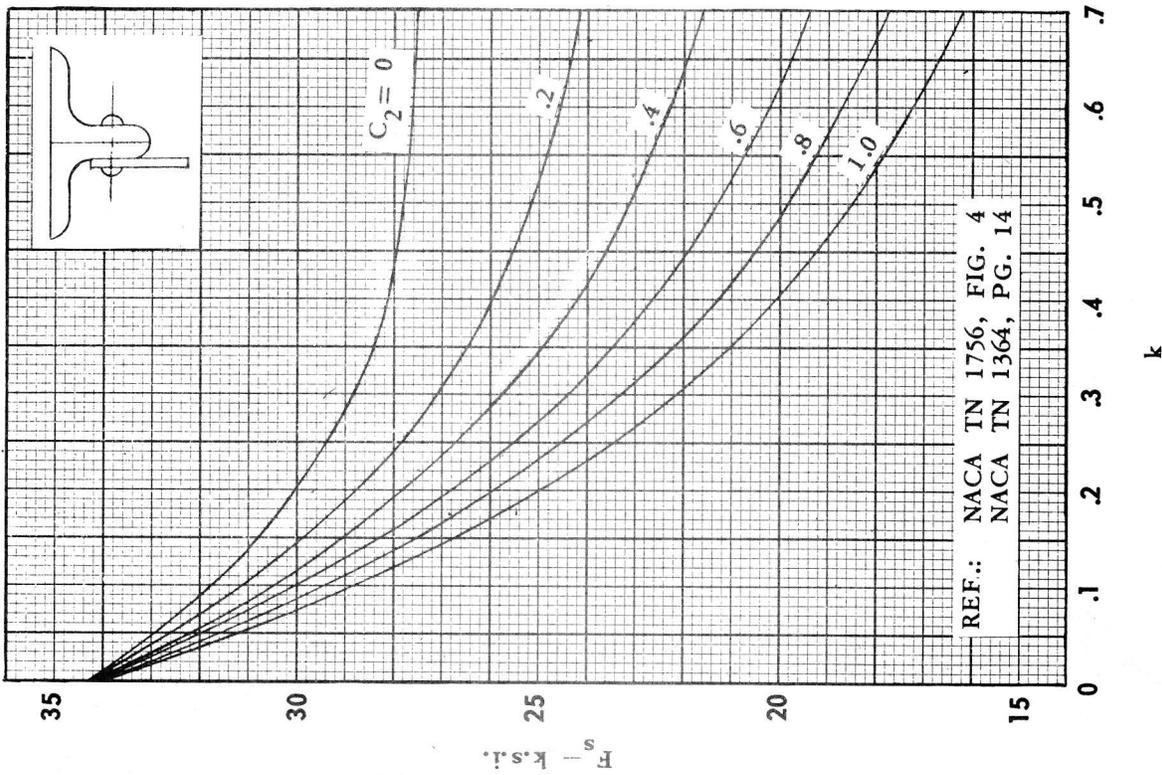


FIGURE 2.2.2-11
ALLOWABLE WEB SHEAR STRESS
75S-T6 ALCLAD

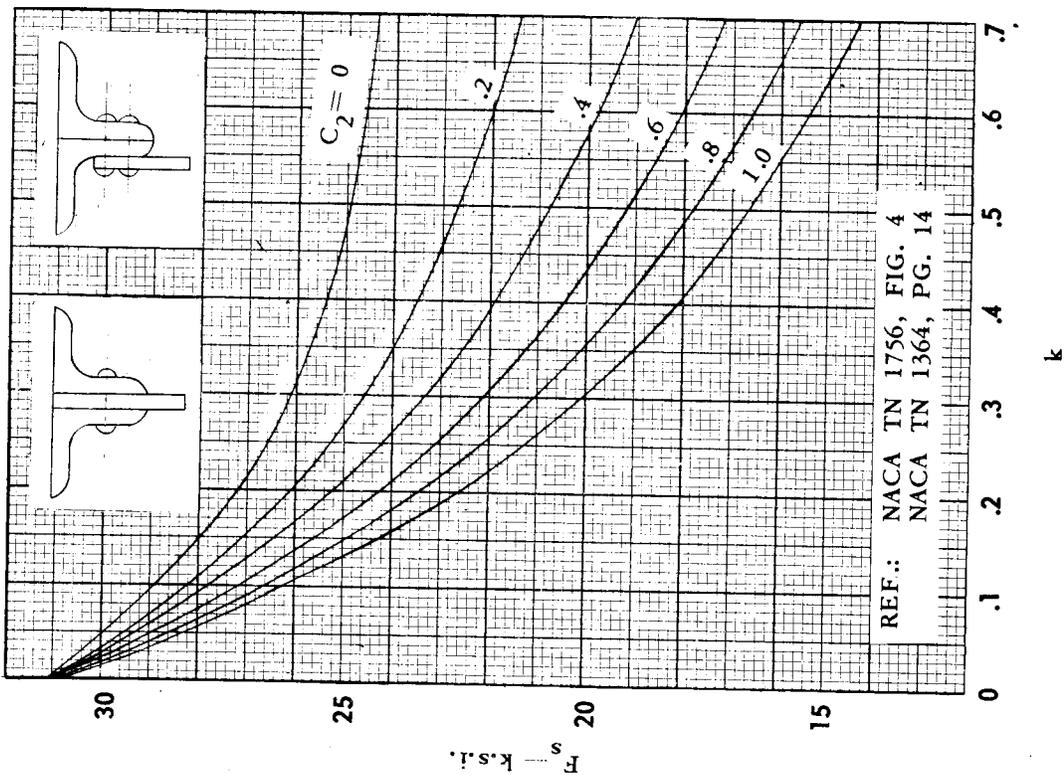
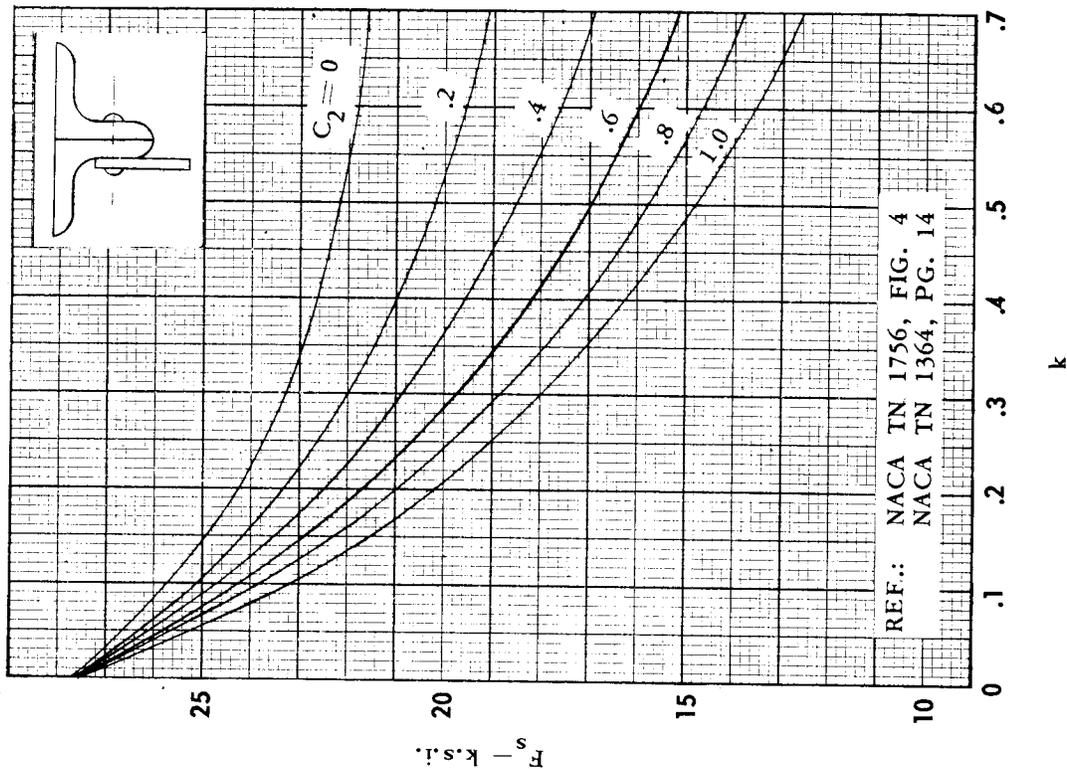
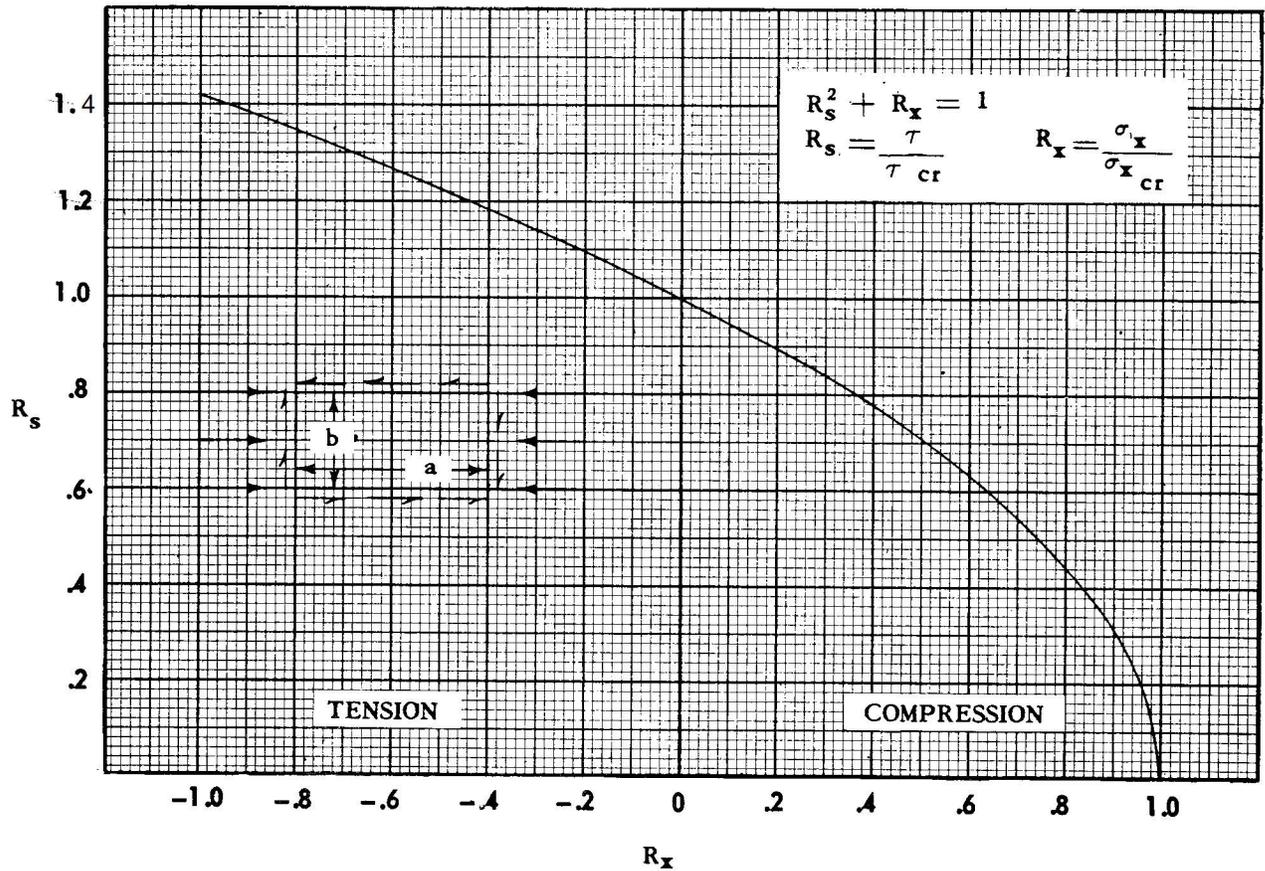
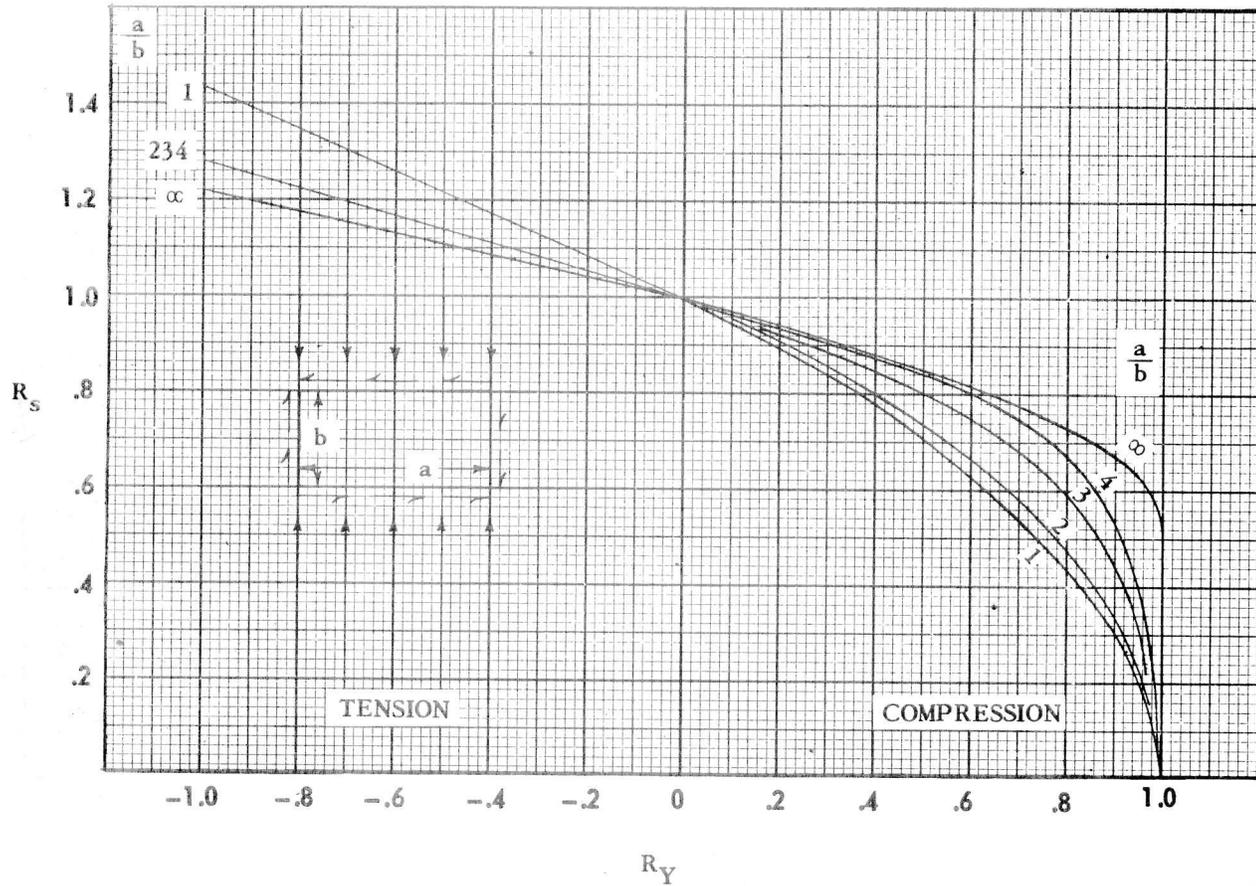


FIGURE 2.2.2-12
ALLOWABLE WEB SHEAR STRESS
24S-T3 ALCLAD



BUCKLING INTERACTION CURVE FOR
SHEAR AND NORMAL STRESS FOR SIMPLY
SUPPORTED FLAT PLATES WHEN $\frac{a}{b} > 1$.

FIGURE 2.2.2-13
NACA TN 1223



BUCKLING INTERACTION CURVE FOR
SHEAR AND TRANSVERSE STRESS FOR
SIMPLY SUPPORTED FLAT PLATES

$$R_s = \frac{\tau}{\tau_{cr}}$$

$$R_Y = \frac{\sigma_Y}{\sigma_{Y_{cr}}}$$

FIGURE 2.2.2-14
NACA TN 1223



2.2.3

PLANE TENSION FIELD BEAMS WITH ACCESS HOLES

2.2.3.1 General

Beam Design – This section presents a method of analysis for partial tension field beams with access holes. It consists of a series of empirical corrections which are made to the analysis in Section 2.2.2 to account for the effect of the access hole. The limitations and methods of Section 2.2.2 are used in conjunction with the curves and formulas presented in this section in making the analysis. A sample design procedure is given in Table 2.2.3-1 which combines the methods of Section 2.2.2 and this section, and gives a complete solution.

Effect of a Flange on the Access Hole – The problem of flanging the access hole is entirely different for a shear resistant beam and a partial tension field beam. In a partial tension field beam, the web buckles may cause the flange around the access hole to buckle. Care must therefore be taken to design the flange so that permanent set and cracking will not take place. An integral or lip type flange does tend to develop permanent set and is definitely not recommended for partial tension field beams. The most satisfactory flange is a flat doubler ring reinforcement of about the same gauge as the web which will buckle without permanent set. However, the flange may effect no significant improvement in the static strength of the panel, since yielding of the material around small access holes takes place before failure, and the effect of stress concentration is almost eliminated. The purpose of the flat doubler ring is primarily to protect the hole against notches or cracks which materially increase the stress concentration. This stress concentration can be effectively reduced by the reinforcing rings. Since notches or cracks could be caused by accidental damage or vibration, the desirability of putting ring flanges on access holes in partial tension field beams is a design problem and should be based on the expected service experience of the beam.

2.2.3.2 References

1. Consolidated Report No. SG-673, Plate Girder Webs with Access Holes, October 1, 1941
2. Consolidated Report No. GS-766, A method for the Design of Plate Girder Webs with Access Holes, January 2, 1941
3. Tests of 10-inch 24 S-T Aluminum-Alloy Shear Panels with 1½ Inch Holes. II-Panels Having Holes with Notched Edges. by Paul Kuhn and L.R. Levin. NACA RB No. L4D01, April 1944.

2.2.3.3 Symbols

Symbols used include those given in Section 2.2.2 and the following, which are used in accounting for the effect of the access hole:

- A_e Effective area of upright, and web, in.²
- A'_u Reduced area of upright, in.²
- A_w Effective area of web, in.²
- C_4, C_5 Design reduction factors
- D Diameter of access hole, in.
- F'_{o} Reduced ultimate allowable compressive stress for forced crippling p.s.i.
- F'_s Reduced allowable web shear stress, p.s.i.
- I'_s Increased minimum moment of inertia required for uprights, in.⁴
- q'_t Increased rivet tensile load – web to upright, lb./in.

2.2.3.4 Web Design

Use the method given in Section 2.2.2.2 with a reduced allowable web shear stress F'_s , calculated as follows:

$$F'_s = \frac{F_s (A_e/dt)}{C_5}$$

where F_s = allowable web shear stress found from Figure 2.2.2-11 as a function of k and C_2

C_5 = design reduction factor found from Figure 2.2.3-2 as a function of D/h

$$A_e = A_w + A'_u$$

$$A_u = t (d-D)$$

$$A'_u = (A_u) \left(\frac{D}{d}\right) (C_4)$$

C_4 = design reduction factor found from Figure



2.2.3-2 as a function of $\frac{t_u/t}{d/h}$

This method gives good correlation with 58 test beams in the following range:

$$.020'' < t < .132''$$

$$.040'' < t_u < .079''$$

$$7.4'' < h < 19.4''$$

$$7.0'' < d < 18.0''$$

$$2.375'' < D < 5.875''$$

This method is *not* recommended for beams outside this range but it can be used as a guide in view of the fact that there is no other information or test results available at the present time.

2.2.3.5 Upright Design

Use the method given in Section 2.2.2.3 with a reduced forced crippling allowable, F'_o , and an increased minimum required moment of inertia, I'_s , as follows:

$$F'_o = \frac{F_o}{1 + D/d}$$

$$I'_s = I_s (1 + D/d)$$

where $(1 + D/d)$ is a correction factor which takes into account the effect of the access hole. The remainder of the stiffener analysis is not appreciably affected by the access hole and the methods of Section 2.2.2 may be used without correction.

2.2.3.6 Rivet Design

Rivets: Web-to-Flange

Use the method given in Section 2.2.2.4

Rivets: Web-to-Upright

Use the method given in Section 2.2.2.4 with an increased rivet tensile load, q'_t , given by the following equation:

$$q'_t = q_t (1 + D/d)$$

Rivets: Upright-to-Flange

Use the method given in Section 2.2.2.4

2.2.3.7 Flange Design

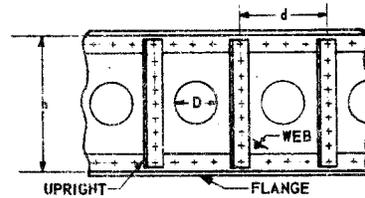
Use the method given in Section 2.2.2.5

TABLE 2.2.3-1

SAMPLE CALCULATIONS FOR BEAM WITH SINGLE UPRIGHT AND ACCESS HOLE

Given: $q = 525 \text{ lb./ins.}$, $h = 10''$, $d = 8.5''$, $D = 4.5''$

- Web - .051" 75S-T6 Alclad
- Beam Flanges 2-1 1/4" x 1 1/4" x .188" 75S-T6 Extruded Angle
- Upright - 3/8" x 3/4" x .063" 75S-T6 Extruded Angle
- Rivets - web to flange ADS at 1.00"
- web to upright DD6 at .85"
- upright to flange, 1-DD6



	EQUATION	REFERENCE	VALUE	ROW	
Shear Flow	q		525	1	
Web Thickness	t		.051	2	
Beam Depth	h		10	3	
Upright Spacing	d		8.5	4	
Access Hole Diameter	D		4.5	5 a	
Upright Thickness	t_u		.063	6	
Upright Area	A_u		.0908	7	
Upright Radius of Gyration	ρ		.227	8	
Upright Moment of Inertia	I_u		.0047	9	
Upright Effective Area	A_{ue}		.0498	10	
Flange Moment of Inertia	I_f		.1122	11	
Web Shear Stress	f_s	(1) / (2)	10,300	12	
P A R A M E T E R S	d_f	(4) (2)	.433	13	
	d/t	(4) / (2)	167	14	
	d/h	(4) / (3)	.85	15	
	ωd	Figure 2.2.2.-3	2.31	16	
	D/h	(5) / (3)	.45	17 a	
	D/d	(5) / (4)	.53	18 a	
	$1 + D/d$	1 + (18)	1.53	19 a	
	A_w	(2) ((4) - (5))	.204	20 a	
	A_{ue}/dt	(10) / (13)	.115	21	
	$ht^3 \times 10^3$	(3) (2)^3 x 10^3	1.325	22	
	t_u/t	(6) / (2)	1.235	23	
	h/ρ	(3) / (8)	44.0	24	
	t_u/t d/h	(23) / (15)	1.45	25 a	
	Diagonal Tension Factor	k	Figure 2.2.2.-4	.26	26
	Ratio - Upright to Web Stresses	f_u/f_s	Figure 2.2.2.-5	.47	27
Ratio - Upright Stresses	$f_{u\max}/f_u$	Figure 2.2.2.-5	1.17	28	
Stress Concentration Factor - Flange Flexibility	C_2	Figure 2.2.2.-6	.17	29	
Stress Concentration Factor - Secondary Bend. Mom.	C_3	Figure 2.2.2.-6	.93	30	
Minimum Mom. of Inertia Required for Upright	I_s	Figure 2.2.2.-7	.00115	31	
Rivet Load - Web to Flange	q_r	Figure 2.2.2.-8	580	32	
Rivet Load - Web to Upright	q_t	Figure 2.2.2.-9	670	33	
Upright Allowable - Column Yield Stress	F_{co}	Figure 2.2.2.-10	81,500	34	
Upright Allowable - Column	F_{col}	Figure 2.2.2.-10	54,000	35	
Upright Allowable - Forced Crippling	F_o	Figure 2.2.2.-10	11,200	36	
Allowable Web Stress	F_s	Figure 2.2.2.-11	28,000	37	
Design Reduction Factor Due to Access Hole	C_4	Figure 2.2.3.-2	.465	38 a	
Design Reduction Factor Due to Access Hole	C_5	Figure 2.2.3.-2	1.377	39 a	
Reduced Area of Upright	A_u	(7) (18) (39)	.0224	40 a	
Effective Area of Upright and Web	A_e	(20) + (40)	.226	41 a	
Parameter	A_e/dt	(41) / (13)	.522	42 a	
Increased Upright Min. Mom. of Inertia Required	I_s	(51) (19)	.00176	43 a	
Reduced Upright Allowable - Forced Crippling	F_o	(36) / (19)	7320	44 a	
Reduced Allowable Web Shear Stress	F_s	(37) (42) / (39)	10,600	45 a	
Upright Stress, Average	f_u	(12) (27)	4840	46	
Upright Stress, Centroidal	$f_{cent.}$	(48) (10) / (7)	2660	47	
Upright Stress, Maximum	$f_{u\max}$	(46) (28)	5660	48	
Rivet Load, Upright to Flange	P_u	(49) (10)	241	49	
Secondary Bending Moment	M_{sb}	1/12 (28) (12) (2) (4)^2 (50)	765	50	
Increased Rivet Load - Web to Upright	q_t	(33) (19)	1025	51 a	
Allowable Rivet Load - Web to Flange		6.1	594	52	
Allowable Rivet Load - Web to Upright		6.1	1080	53	
Allowable Rivet Load - Upright to Flange		6.1	1148	54	
Margin of Safety - Web		(45) / (12) - 1	.03	55	
Margin of Safety - Upright		*	.29	56	
Margin of Safety Rivets, Web to Flange		(52) / (32) - 1	.02	57	
Margin of Safety Rivets, Web to Upright		(53) / (51) - 1	.05	58	
Margin of Safety Rivets, Upright to Flange		(54) / (49) - 1	3.76	59	

* Either (9)/(45) -1, (34)/(46) -1, (35)/(47) -1, or (44)/(48) -1, whichever is least.

* The above steps are those added to account for the effect of the access hole. The rest of the table is the same as Table 2.2.2-3.

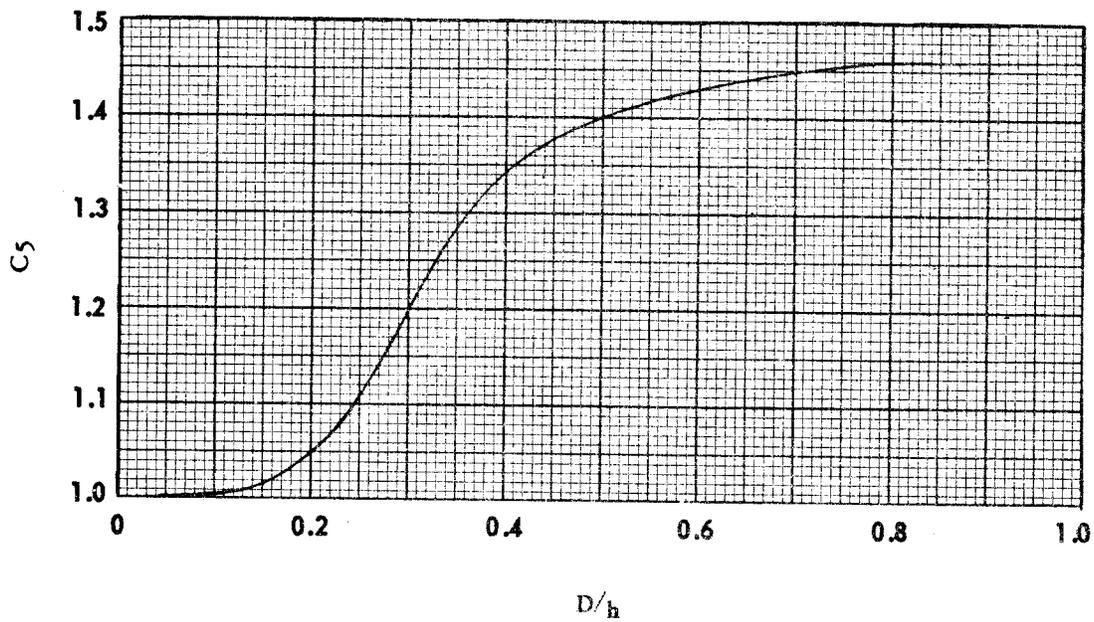
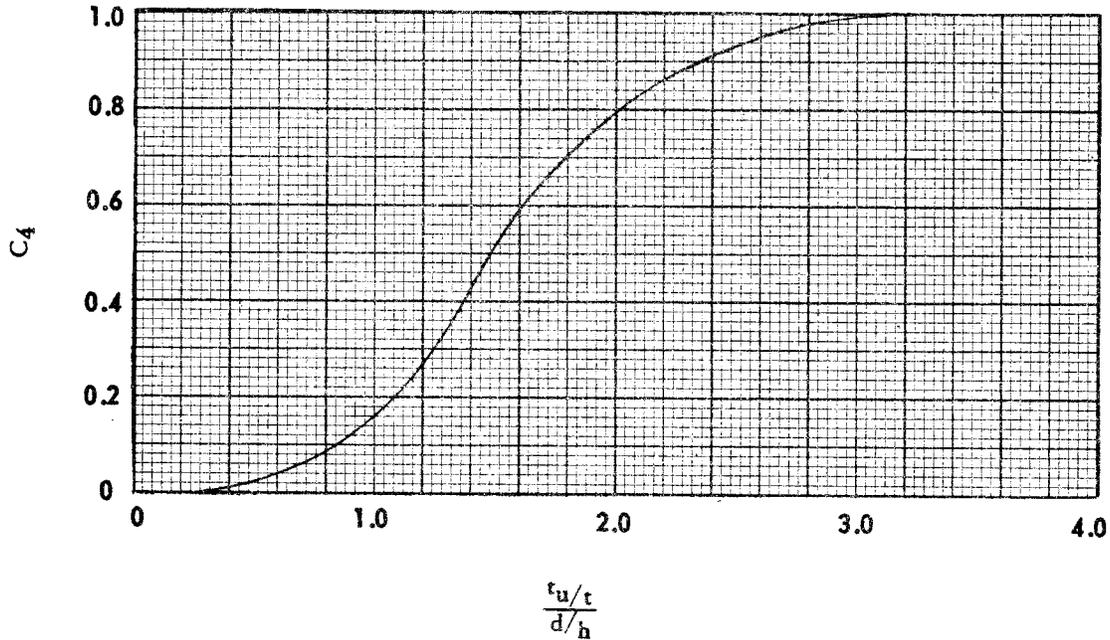


FIGURE 2.2.3-2
DESIGN REDUCTION FACTORS
DUE TO ACCESS HOLE



2.2.4

CURVED WEB TENSION FIELD BEAMS

2.2.4.1 General

The following section applies strictly to complete cylinders or to beams with single curvature subjected to uniform shear stresses. However, it may be used as an approximation for cylinders or beams having non-uniform shear stress distribution. The method is an adaptation of that presented in section 2.2.2 for tension field beams with plane webs. Information is included for both 24 S-T and 75 S-T aluminum alloys.

References:

1. "Diagonal Tension in Curved Webs", by Paul Kuhn and George E. Griffith, NACA TN 1481, 1947.
2. "A Summary of Diagonal Tension Part I" by Kuhn, Peterson and Levin, NACA, TN 2661, June 1951.
3. "Critical Combinations of Shear and Direct Axial Stress for Curved Rectangular Panels" by Scheldcrout and Stein, NACA TN 1928, August 1949.
4. Airplane Structural Analysis and Design, by Sechler and Dunn, 1942.
5. Experimental Investigation of Stiffened Cylinders Subjected to Torsion and Compression, NACA, T.N. 2188, 1950. by James P. Peterson.

Symbols:

- A Cross sectional area, in²
- E Young's modulus, psi.
- P Load, lbs.
- Q Beam shear, lbs.
- R Radius of curvature, in.
- T Applied torque, in.-lbs.
- Z Curvature parameter $\frac{.954h^2}{Rt}$ or $\frac{.954d^2}{Rt}$
(Use whichever is smaller of h or d)
- d Spacing of rings, in.
- ε Normal strain, in./in.
- h Spacing of stiffeners, in.
- k Diagonal - tension factor
- η Ratio of tangent modulus at the critical stress to the elastic modulus
- q Shear flow, lbs./in.
- t Thickness, in. (Without subscript signifies thickness of web)

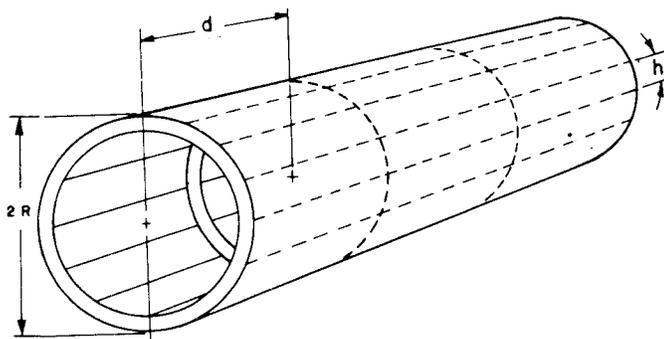
- α Angle of diagonal tension, deg.
- μ Poissons ratio
- σ Normal stress, psi.
- τ Shear stress, psi.
- ρ Radius of gyration of cross section, in.
- Δ Correction factor for allowable shear stress

Subscripts:

- all. Allowable
- cr Critical
- e Effective
- max. Maximum
- DT Diagonal Tension
- PDT Pure Diagonal Tension
- R Rivet
- RG Ring
- S Shear
- ST Stringer

Special Combinations:

- K_s Critical shear-stress coefficient, established by geometry of the type of edge support.
- σ_o "Basic" allowable compressive stress for forced crippling of stiffeners (valid for stresses below proportional limit of material), psi.
- τ_{all}* "Basic" allowable shear stress, psi.





2.2.4.2 Initial Buckling Stress of Web

The stress at which the web starts to buckle is given by the formula

$$\frac{\tau_{cr}}{\eta_s} = K_s E \left(\frac{t}{h}\right)^2 \quad \text{if } h < d \quad (1a)$$

$$= K_s E \left(\frac{t}{d}\right)^2 \quad \text{if } d < h \quad (1b)$$

where K_s is obtained from Fig. 2.2.4-1a or 2.2.4-1b

In no case should the stringer or ring gauge be less than one gauge lighter than the web gage.

2.2.4.3 Determination of Diagonal Tension Field Factor, k

The factor, k, specifies the degree to which the diagonal tension is developed; when $k = 0$, there is no diagonal tension action; when $k = 1.0$, the diagonal tension is fully developed.

When the parameter, $\frac{300 td}{Rh}$, has been calculated, the value of k may be read from Fig. 2.2.4.-2

2.2.4.4 Determination of Stresses in Stringers, Rings, and Web.

The stress in a stringer is given by:

$$\sigma_{ST} = \frac{k \tau \cot \alpha}{\frac{A_{ST}}{ht} + 0.5(1-k)} \quad (2)$$

and the stress in a ring by

$$\sigma_{RG} = \frac{k \tau \tan \alpha}{\frac{A_{RG}}{dt} + 0.5(1-k)} \quad (3)$$

For floating rings (not attached to skin but its stringers only) the factor .5(1-k) representing effective skin in formula (3) is omitted.

The above stresses are compressive stresses and must be added to other stresses not arising from diagonal-tension action where such stresses exist. The shear stress in the web is found as follows:

$$\tau = \frac{T}{2 \pi R^2 t} \quad (\text{complete cylinder})$$

$$\tau = q/t \quad (\text{curved web beam})$$

The angle α between a generatrix at the cylinder and the direction of the diagonal tension is given by the formula:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_{ST}}{\epsilon - \epsilon_{RG} + \frac{1}{24} \left(\frac{h}{R}\right)^2} \quad \text{if } d > h \quad (4a)$$

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_{ST}}{\epsilon - \epsilon_{RG} + \frac{1}{8} \frac{d}{R} \tan^2 \alpha} \quad \text{if } h > d \quad (4b)$$

where ϵ is the strain in the sheet along the direction of diagonal tension, $\epsilon_{ST} = \frac{\sigma_{ST}}{E}$ is the strain in the

stringer, and $\epsilon_{RG} = \frac{\sigma_{RG}}{E}$ is the strain in the ring.

The strain ϵ in the sheet is given by the formula:

$$\epsilon = \frac{\tau}{E} \left[\frac{2K}{\sin 2\alpha} + (1-k)(1+\mu) \sin 2\alpha \right] \quad (5)$$

which is evaluated in Fig. 2.2.4.-3

From the above it may be seen that formulas (2) through (5) are interdependent and must be solved by successive approximation. The process is rapidly convergent and normal accuracy is usually obtained within three cycles.

A first approximation to the angle α may be made by means of the formula

$$\alpha \approx \alpha_{PDT} \left(\frac{a}{\alpha_{PDT}} \right) \quad (6)$$

with α_{PDT} taken from Fig. 2.2.4.-4 and the ratio $\frac{a}{\alpha_{PDT}}$ from Fig. 2.2.4.-5.

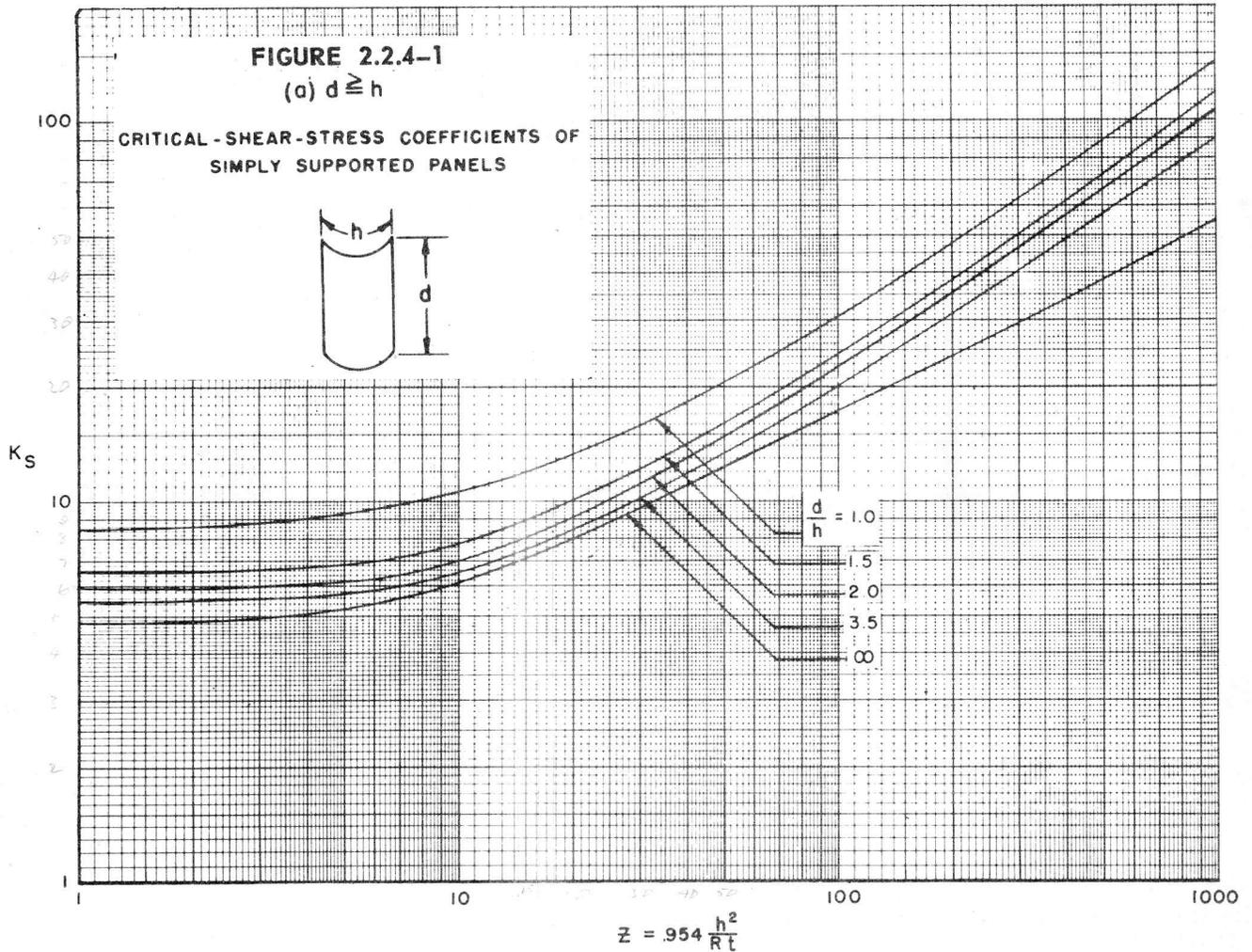
This first approximation will give results that are probably close enough for most cases of curved tension field webs if a M.S. $\geq .20$ is held for the stringers and rings and a M.S. $\geq .15$ is held for the web. If more accuracy is desirable or necessary the proper value of α may be determined by successive approximation as given above.

2.2.4.5 Analysis of Web

The ultimate strength of the web can be estimated by means of the empirical formula

$$\tau_{all} = \tau_{all}^* (0.65 + \Delta) \quad (7)$$

where τ_{all}^* is the "basic" allowable shear stress taken from Fig. 2.2.4.-6 or 2.2.4.-7 and Δ is obtained from the graph in Fig. 2.2.4.-8.



2.2.4.6 Analysis of Stringers

The stringer stress given by formula (2) is the average stress over the length of the stringer. The maximum stress at the midpoint of a panel is given by the formula

$$\sigma_{ST_{max.}} = \sigma_{ST} \left(\frac{\sigma_{ST_{max.}}}{\sigma_{ST}} \right) \quad (8)$$

where the ratio $\frac{\sigma_{ST_{max.}}}{\sigma_{ST}}$ is obtained from Fig. 2.2.4.-9 using h/d as the other parameter.

Because of the polygonal shape acquired by the cross section of the cylinder as diagonal tension develops each tension diagonal experiences a change in direction as it crosses the stringer. The magnitude

of this inward radial pressure per running inch is

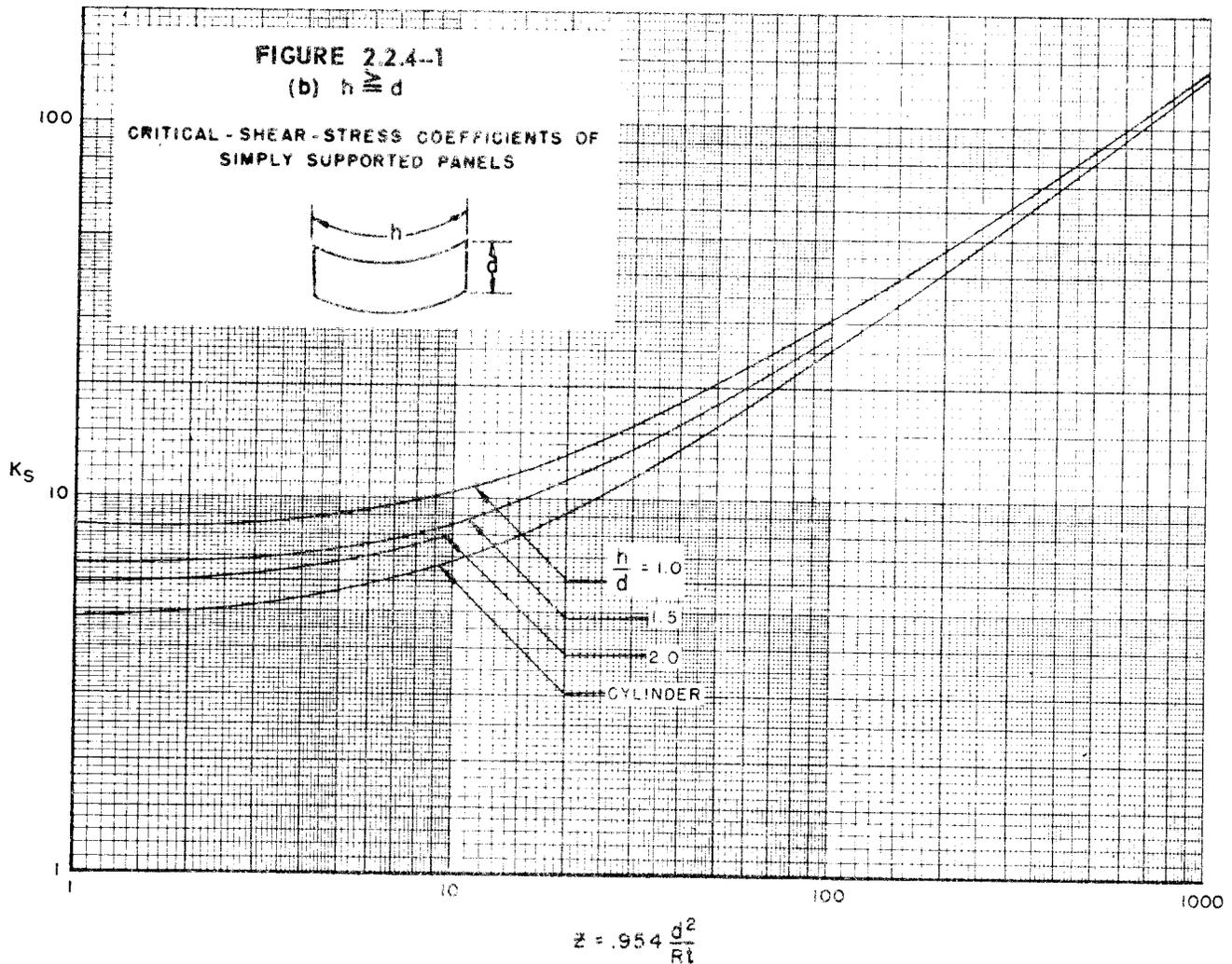
$$p = \frac{k r t h}{R} \tan \alpha \quad (9)$$

If this pressure were distributed uniformly along the length of the stringer, the primary peak bending moment (at the junction with the ring) would be given by the formula

$$M_{ST} = k r \frac{t h d^2}{12R} \tan \alpha \quad (10)$$

A secondary peak moment would exist half-way between rings and its magnitude would be one-half the primary peak.

Since the rings are actually circular (or curved) a portion of the tension diagonal near each end of a



panel will be forced to remain in the original cylindrical surface and experience little change in direction. Tests indicate that the secondary peak moment (half-way between rings) agrees roughly with the calculated value

$$M_{ST} = k \tau \frac{t h d^2}{24 R} \tan \alpha \quad (11)$$

Strength tests on cylinders have led to the conclusion that the maximum moment at the ring is no larger than the calculated secondary moment above.

- (1) A check should be made against column failure normal to the skin (using the local crippling stress for the stringer as allowable for $\frac{h}{d} = 0$) with some reduction to allow for effect of skin buckles unless t_{ST}/t is larger than 3. Some allowance should be made for beam column effect.

- (2) The stiffened cylinder should be checked against general instability by Fig. 2.2.4-10

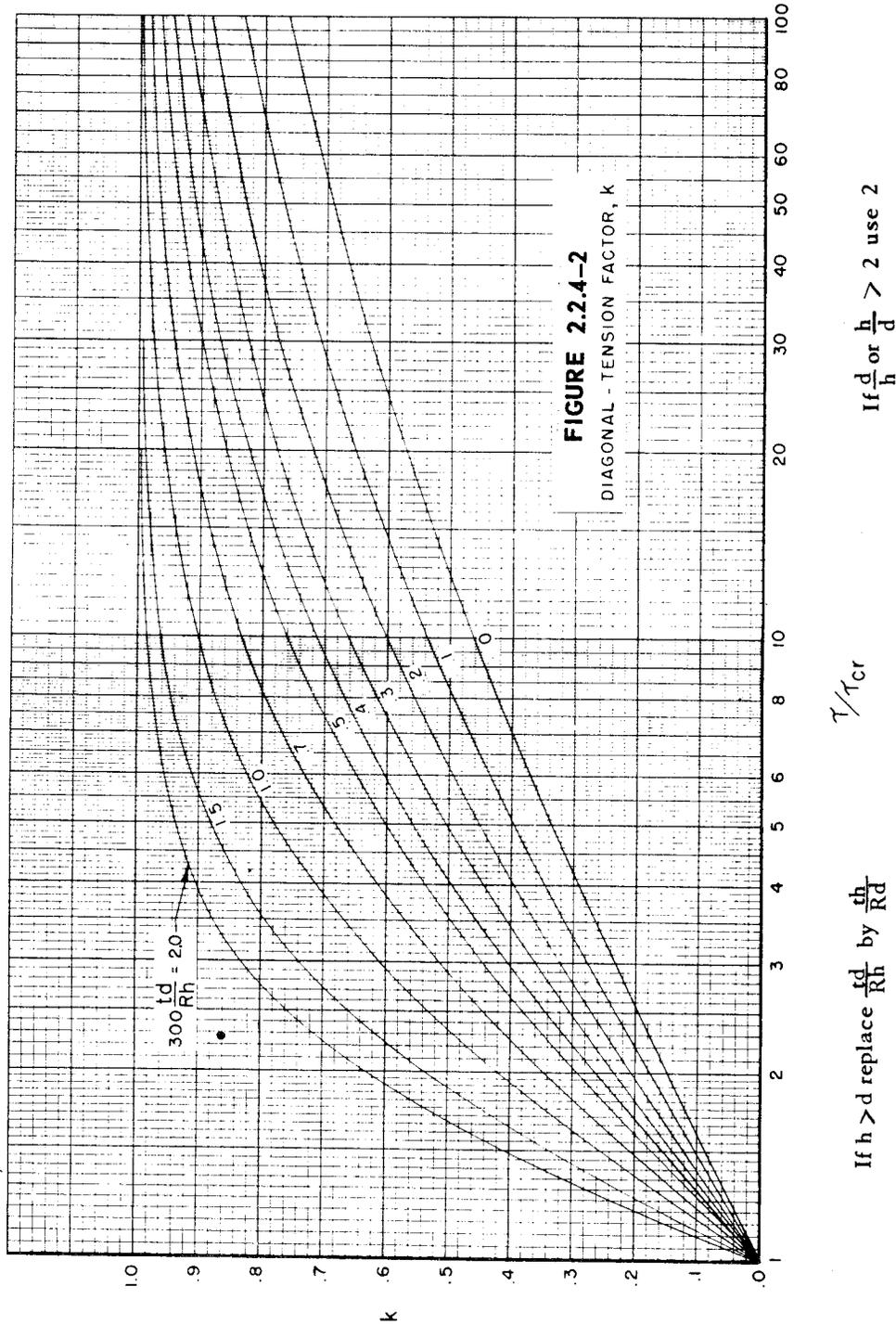
- (3) The stringers should be checked for forced crippling in the same manner as for plain webs. See Sec. 2.2.2.3.

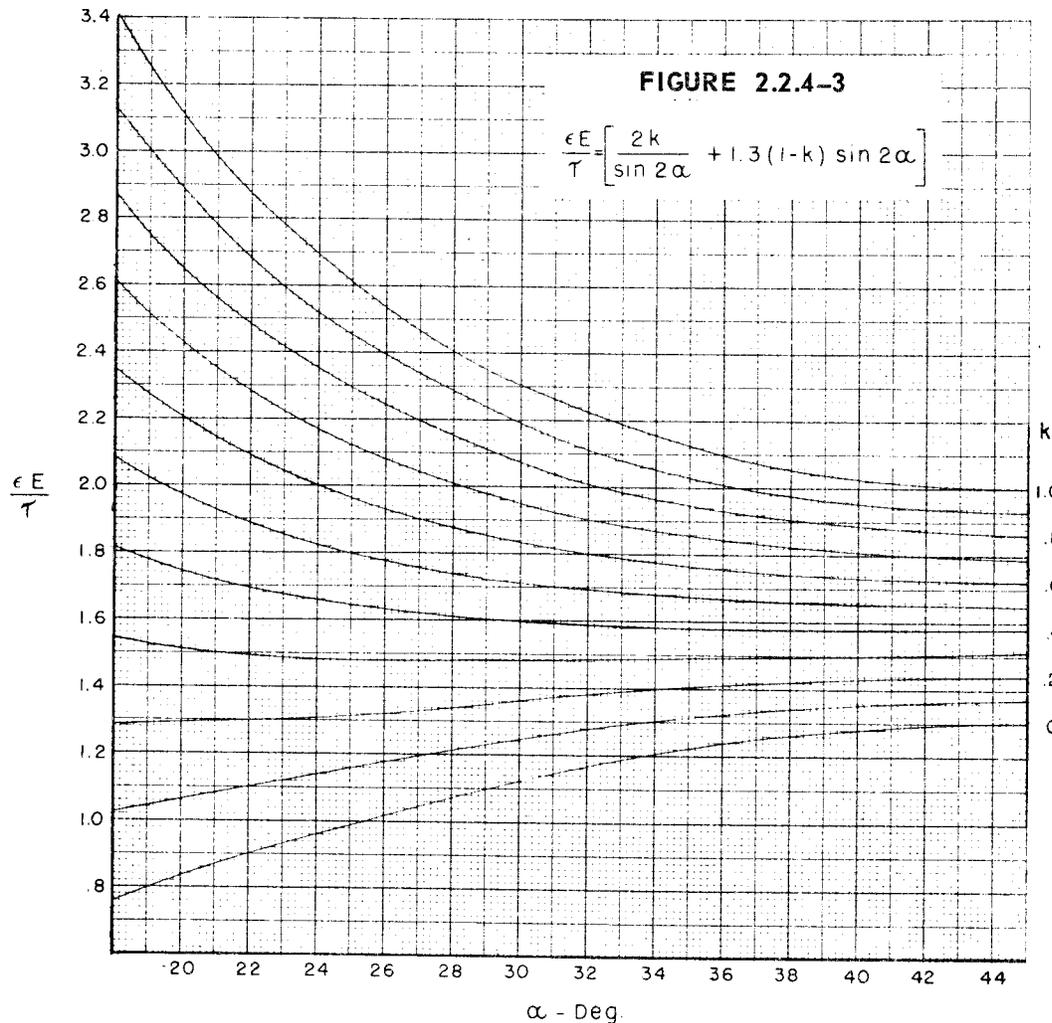
2.2.4.7 Analysis of Rings

The ring stress given by formula (3) is the average stress in the ring. The maximum stress exists at the mid-point of the panel and is given by the formula

$$\sigma_{RGmax.} = \sigma_{RG} \frac{\sigma_{RGmax.}}{\sigma_{RG}} \quad (12)$$

where the ratio $\frac{\sigma_{RGmax.}}{\sigma_{RG}}$ is obtained from Figure 2.2.4-9 using d/h as the other parameter.





Floating Rings (not attached to skin but to stringers only)

The primary bending moment in a floating ring (at the junction with a stringer) is

$$M_{RG} = k \tau \frac{t h^2 d}{12 R} \tan \alpha \quad (13)$$

the secondary maximum half-way between stringers is half as large. This type of ring should be designed to carry the loads shown below:

- (1) Hoop compression σ_{RG} from equation (3)
- (2) Bending moment due to M_{RG} shown above at the junction with stringer. A check at the station mid-way between stringers (where the moment is only half as large, but of opposite sign) may be necessary if the cross section of the ring is such that

allowable stresses in the outer and inner fibers differ greatly.

Rings Attached to Skin

This type of ring should be checked for the following stresses.

- (1) Hoop compression σ_{RG} from equation (3)
- (2) Forced crippling is checked in the same manner as for stringers
- (3) General instability from Figure 2.2.4.-10

Unless the stringers are made intercostal (which leads to loss of efficiency in bending strength of the cylinder and therefore is seldom done) the ring must be notched to permit the stringers to pass through. At

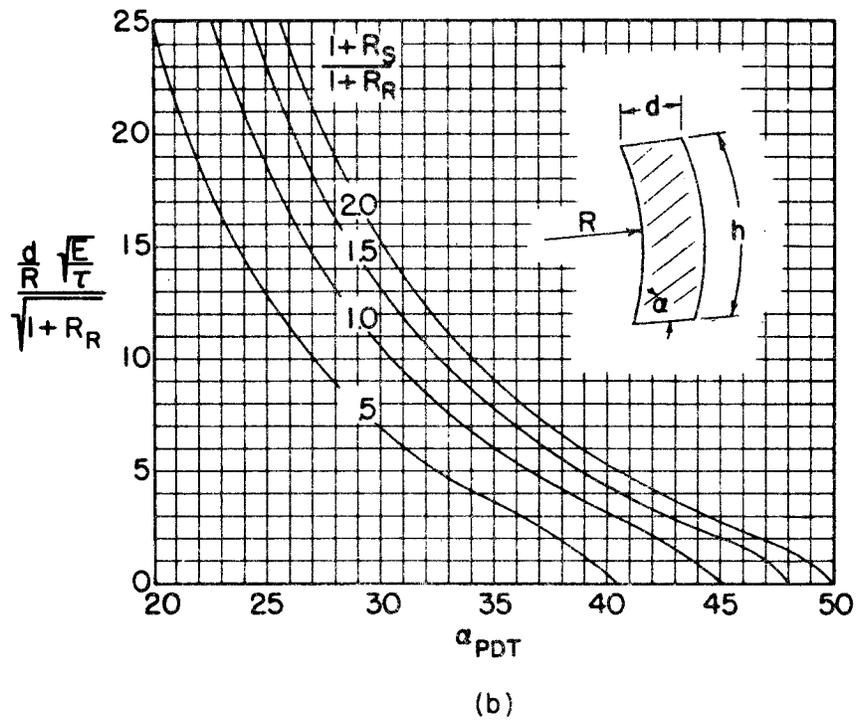
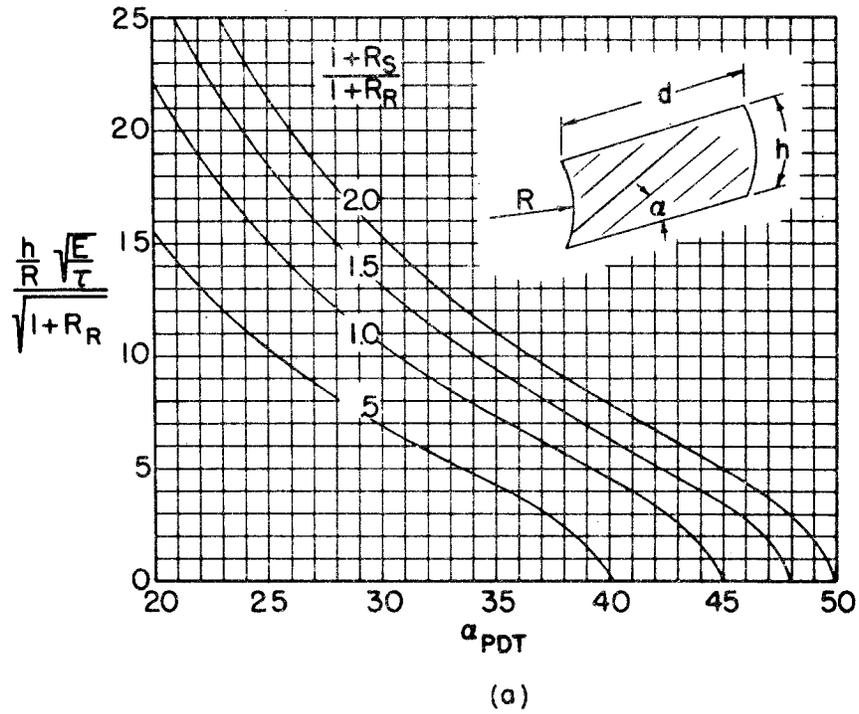


FIGURE 2.2.4-4 - Angle of pure diagonal tension

$$R_R = \frac{dt}{A_{RG}} ; R_S = \frac{ht}{A_{ST}}$$



the notch the ring stress is increased because the cross section is reduced; this effect is aggravated also by the suddenness of reduction. The free edge of the notch should be checked against local crippling failure. If the stringer is connected to the ring by a clip-angle of sufficient length riveted to the web of the ring, the section at the notch is increased, and the edge of the notch can be stiffened so much that there is no danger of this type of failure. The section at the notch should be checked for axial load on the net section plus the bending moment caused by the displacement of the centroid of the net section from that of the gross section. See Section 4.4.

2.2.4.8. Analysis of Rivets

For a complete cylinder only the rivets in a web splice need be checked. The load per inch for rivets in a splice may be taken as

$$P_R = q(1 + 0.414k) \quad (14a)$$

For a curved web beam the rivets need be checked at two points only.

Web-to-flange rivets

The load per inch run is, according to eq. 15 of NACA TN 1364:

$$P_R = q(1 + 0.414k) \quad (14b)$$

Upright-to-flange rivets

The upright-to-flange rivets must carry the load existing in the upright into the flange.

$$P_{RG} = A_{RG} \sigma_{RG} \quad (15)$$

2.2.4.9 Analysis of Flanges

Flanges on a curved web exist only where the member is not a complete cylinder. Where they exist the method given in Section 2.2.2-5 should be followed.

2.2.4.10 Sample Calculation

A cylinder having the following dimensions and subjected to a pure torque of 934,000 in. lbs. will be analysed as a sample problem to show the use of the method. The tabular form used in this problem is to be used as a guide only and no standard form will be adopted unless experience shows the desirability of doing so.

Material - 24 S-T Alc.

Skin thickness, $t = .040$ in.

Radius of curvature, $R = 15.0$ in.

Spacing of rings, $d = 15.0$ in.

Spacing of stringers, $h = 7.85$ in.

Thickness of stringers, $t_{ST} = .081$ in.

Thickness of rings, $t_{RG} = .102$ in.

Area of stringer (1.0 x 2.0 x .081 Channel, 24 S-T3 Alc.), $A_{ST} = .287$ in.²

Area of ring (1.0 x 2.0 x .102 Channel, 24 S-T4 Alc.), $A_{RG} = .365$ in.²

$$q = \frac{T}{2A} = \frac{934,000}{2 \times \pi (15.00)^2} = 660 \text{ lbs./in.}$$

Computations of stringer as a column

$C = 3.0$ (assumed)

$I = .165$ in.⁴

$$\rho_{ST} = \sqrt{\frac{I}{A_{ST}}} = \sqrt{\frac{.165}{.287}} = .758 \text{ in.}$$

$$L'/\rho = \frac{15.00}{.758 \sqrt{3.0}} = 11.42$$

$$\sigma_{col.} = 48,800 \text{ lb./in.}^2 \quad \text{See Sec. 3.1}$$

Computations of general instability

$$\rho_{RG} = 1.919 \sqrt{\frac{\frac{1.919}{.960} + 6}{12 \left(\frac{1.919}{.960} + 2 \right)}} = .784$$

See Table 10.5, p. 310. Niles & Newell, Vol. 1, Second Edition

$$\sqrt{\frac{\rho_{ST} \times \rho_{RG}}{hd}} \left(\frac{\sqrt{\rho_{ST} \times \rho_{RG}}}{R} \right)^{3/4} =$$

$$\sqrt{\frac{.758 \times .784}{15.0 \times 7.85}} \left(\frac{\sqrt{.758 \times .784}}{15.0} \right)^{3/4} = .00756$$

Fig. 65 of NACA TN 1197 shows that the general instability failure stress would be very high for this case. Therefore, no further investigation of this phenomena will be made.



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NO.	ITEM	EQUATION	VALUE
1	Shear Flow, q, lb./in.	$\frac{T}{2A}$ or $\frac{Q}{h}$	660
2	Skin Thickness, t, in.		.040
3	Shear Stress, τ , psi.	(1) / (2)	16,500
4	Radius of curvature, R, in.		15.00
5	Spacing of Rings or Upright, d, in.		15.00
6	Spacing of Stringers h, in.		7.85
7	Thickness of Stringers t_{ST} , in.		.081
8	Thickness of Ring, t_{RG}		.102
9	Area of Stringer Cross Section, A_{ST} , in. ²		.287
10	Area of Ring Cross Section, A_{RG} , in. ²		.365
11	Curvative Parameter, Z	$\frac{.954 \times (5)^2}{(4) \times (2)}$ or $\frac{.954 \times (6)^2}{(4) \times (2)}$ whichever is least	98.22
12	K_s	See Fig. 1, NACA TN 1481 or Fig. 2.2.4-1	22.7
13	Critical Shear Stress, τ_{cr} , psi.	See Eq. (1a) or (1b)	6000
14	τ / τ_{cr}	(3) / (13)	2.75
15	300 td/Rh	$\frac{300 \times (2) \times (5)}{(4) \times (6)}$	1.53
16	Diagonal Tension Factor, k	See Fig. 2, NACA TN 1481 or Fig. 2.2.4-2	.71
17	$\frac{h}{R} \sqrt{E/\tau}$	$\frac{(6)}{(4)} \sqrt{\frac{(8)}{(3)}}$	14.5
18	$\sqrt{1 + 0.5 \left(\frac{ht}{A_{ST}} + \frac{dt}{A_{RG}} \right)}$	$\sqrt{1 + 0.5 \frac{(6) \times (2)}{(9)} + \frac{(5) \times (2)}{(10)}}$	1.54
19		(17) / (18)	9.42
20	α PDT	See Fig. 5, NACA TN 1481 or Fig. 2.2.4-4	31.6°
21	α / α PDT	See Fig. 6, NACA TN 1481 or Fig. 2.2.4-5	.929
22	α	(20) x (21)	29° 20'
23	$\cot \alpha$		1.7796
24	$\tan \alpha$.56194
25	$\frac{A_{ST}}{ht} + 0.5(1-K)$	$\frac{(9)}{(6) \times (2)} + 0.5(1 - (16))$	1.059
26	$\frac{A_{RG}}{dt} + 0.5(1-K)$	$\frac{(10)}{(5) \times (2)} + 0.5(1 - (16))$.754
27	σ_{ST}	$-\frac{(16) \times (3) \times (23)}{(23)}$	-19,700
28	ϵ_{ST}	(27) / E	-.00193
29	σ_{RG}	$-\frac{(16) \times (3) \times (24)}{(26)}$	-8740
30	ϵ_{RG}	(29) / E	-.00086



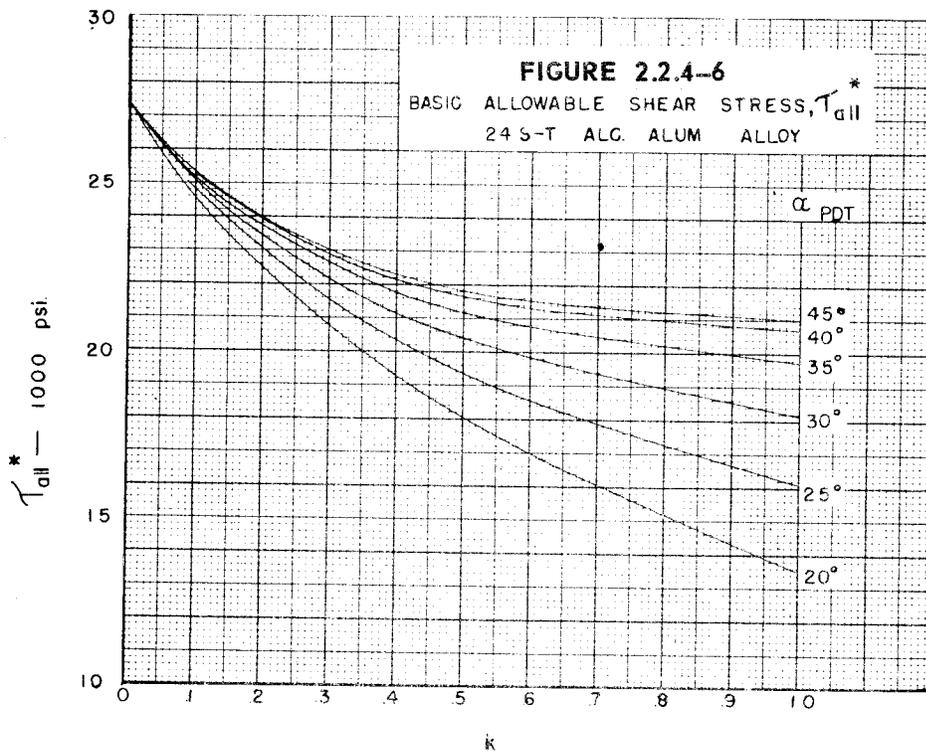
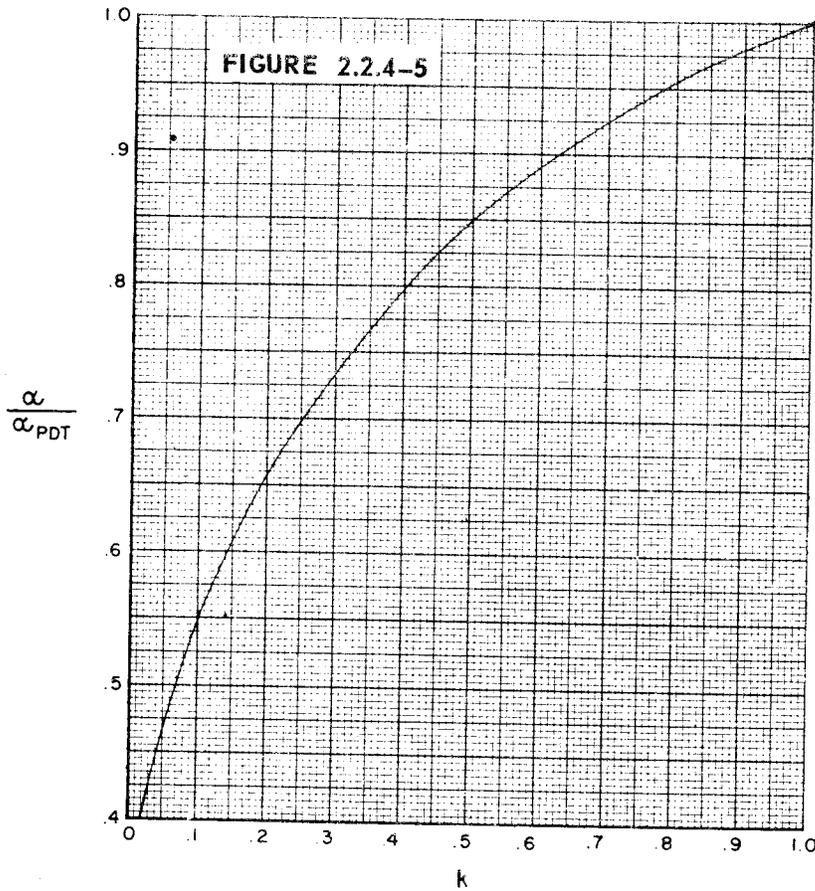
NO.	ITEM	EQUATION	VALUE
31	$\epsilon E / \tau$	See Fig. 4, NACA TN 1481 or Fig. 2.2.4-3	1.98
32	ϵ	$\frac{(31) \times (3)}{E}$.00320
33	$\tan \alpha_1$	$\sqrt{\frac{(32) - (28)}{(32) - (30) + \frac{1}{24} \left(\frac{6}{4}\right)^2}}$.576
34	α_1	arc tan (33)	29° 57'
35	$\cot \alpha_1$		1.7361
36	σ_{ST}	$-\frac{(16) \times (3) \times (35)}{(25)}$	- 20,200
37	ϵ_{ST}	(36) / E	- .00198
38	σ_{RG}	$-\frac{(16) \times (3) \times (33)}{(26)}$	- 8510
39	ϵ_{RG}	(38) / E	- .00084
40	$\epsilon E / \tau$	See Fig. 4, NACA TN 1481 or Fig. 2.2.4-3	1.96
41	ϵ	$\frac{(40) \times (3)}{E}$.00317
42	$\tan \alpha_2$	$\sqrt{\frac{(41) - (27)}{(41) - (39) + \frac{1}{24} \left(\frac{6}{4}\right)^2}}$.578
43	α_2		30° 2'
44	$\cot \alpha_2$		1.7301
45	σ_{ST}	$-\frac{(16) \times (3) \times (44)}{(25)}$	- 20,250
46	ϵ_{ST}	(45) / E	- .00199
47	σ_{RG}	$-\frac{(16) \times (3) \times (42)}{(26)}$	- 8490
48	ϵ_{RG}	(47) / E	- .00083
49	$\epsilon E / \tau$	See Fig. 4, NACA TN 1481 or Fig. 2.2.4-3	1.95
50	ϵ	$\frac{(49) \times (3)}{E}$.00315
51	$\tan \alpha_3$	$\sqrt{\frac{(50) - (46)}{(50) - (48) + \frac{1}{24} \left(\frac{6}{4}\right)^2}}$.57791
52	α_3		30° 1.4'
53	$\cot \alpha_3$		1.7302
54	σ_{ST}	$-\frac{(16) \times (3) \times (53)}{(25)}$	20,250

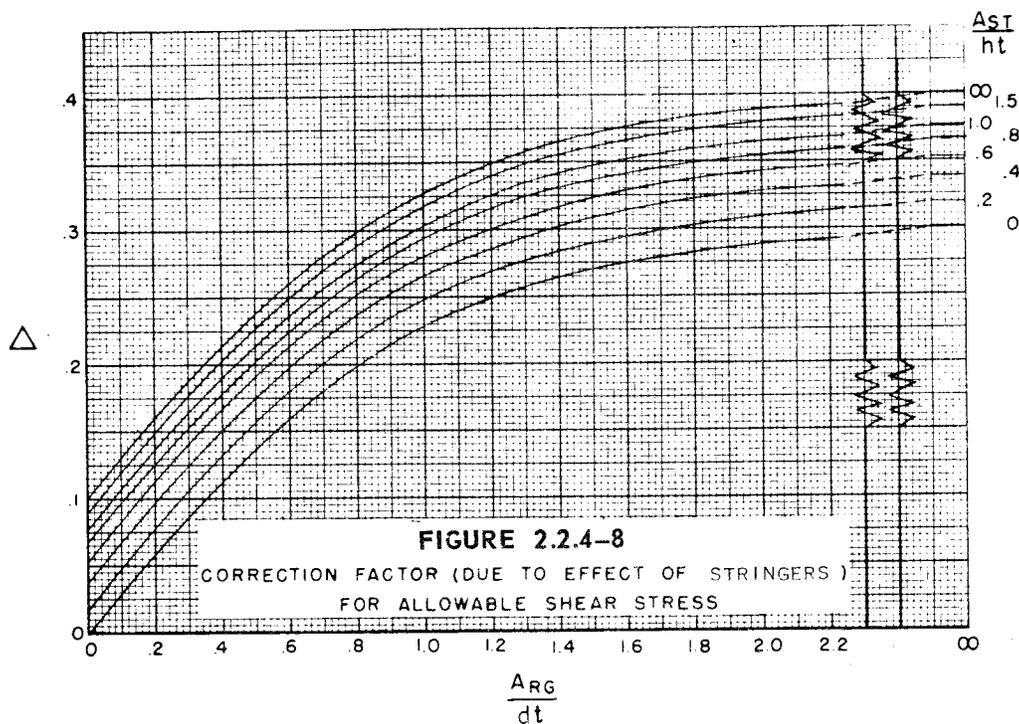
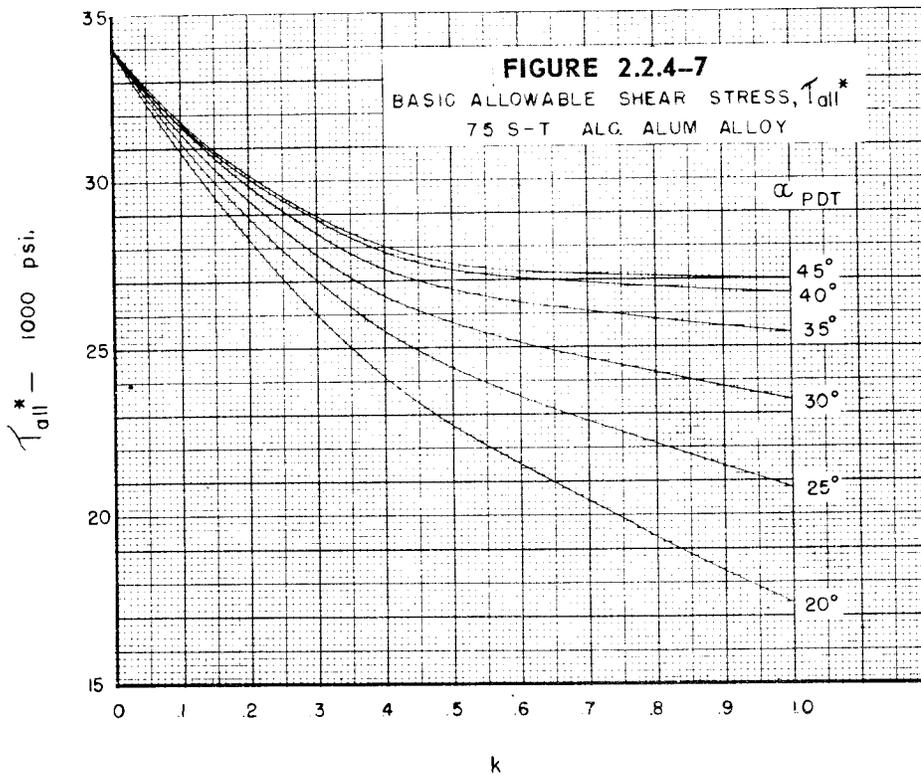


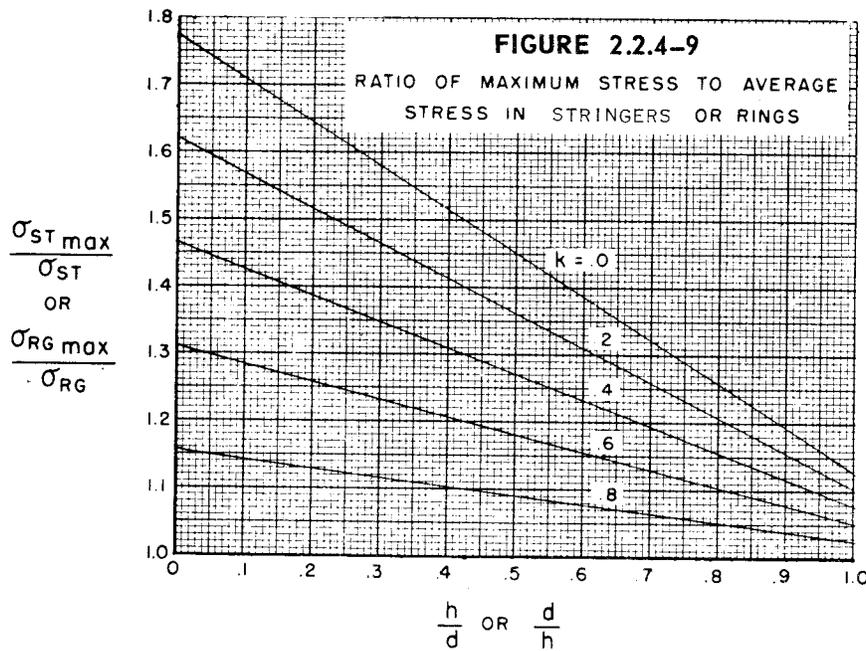
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NO.	ITEM	EQUATION	VALUE
55	σ_{RG}	$\frac{(16) \times (3) \times (51)}{(26)}$	8490
56	$\tau_{all} *$	See Fig. 7, NACA TN 1481 or Fig. 2.2.4-6 or 7	18,940
57	Δ	See Fig. 8, NACA TN 1481 or Fig. 2.2.4-8	.235
58	τ_{all}	$(56) (.65 + (57))$	16,780
59	<u>M.S. of Web</u>	$\frac{(58)}{(3)} - 1$.01
60	$\sigma_{ST_{max.}} / \sigma_{ST}$	See Fig. 3, NACA TN 1481 or Fig. 2.2.4-9	1.126
61	$\sigma_{ST_{max.}}$	$(54) \times (60)$	22,800
62	col.	See Sample Calculation	48,800
63	<u>M. S. (Column)</u>	$\frac{(62)}{(61)} - 1$	1.14
64	$\frac{r_u}{r}$	$\frac{(7)}{(2)}$	2
65	σ_O	See Figure 2.2.2 -7	27,500
66	<u>M. S. (Forced Crippling)</u>	$\frac{(65)}{(61)} - 1$.21
67	$\sigma_{max.}$	See Section 3.1.1	38,500
68	<u>M. S. (Natural Crippling)</u>	$\frac{(67)}{(61)} - 1$.75
69	$\sigma_{RG_{max.}} = \sigma_{RG}$ (Assumed)	(55)	8,490
70	$\frac{r_u}{t}$	$\frac{(8)}{(2)}$	2.6
71	O	See Figure 2.2.2-7	29,500
72	<u>M. S. (Forced Crippling)</u>	$\frac{(71)}{(69)} - 1$	2.48







For rings read abscissa $\frac{d}{h}$

For stringers read abscissa $\frac{h}{d}$

Empirical criterion for general instability failures of stiffened 24S-T3 aluminum alloy cylinders subjected to torsion. From N.A.C.A T.N. 1197.

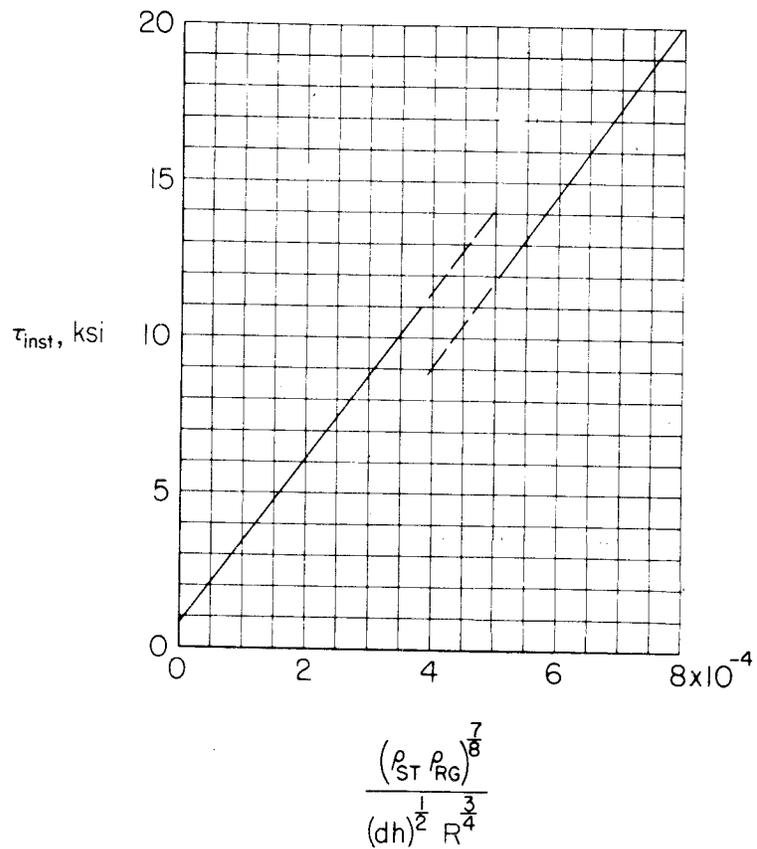
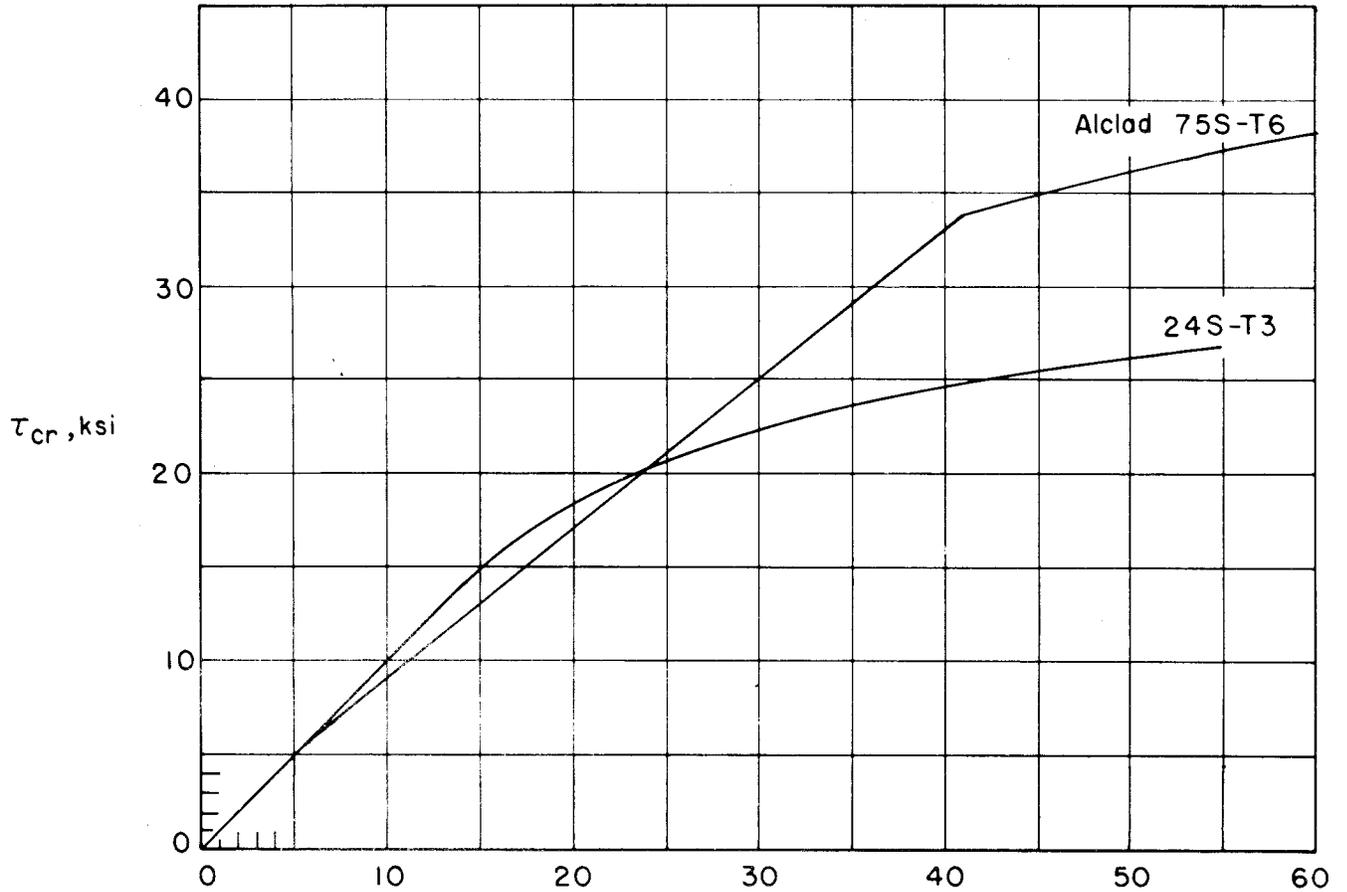


FIGURE 2.2.4-10



$$\tau_{cr,elastic} \text{ , ksi} = \frac{\tau_{cr}}{m_s}$$

(c) Plasticity correction.

FIGURE 2.2.4-11



2.2.4.11 Combined Loading

The following method of analysis has been found to yield a reasonably accurate solution for cylinders subjected to torsion and compression.

The critical shear stress is calculated with the aid of Figure 2.2.4-1. This stress is now denoted by $\tau_{cr,0}$ where the additional subscript zero indicates the condition of shear acting alone. Next the critical compression stress is calculated with the aid of Section 3.2, and denoted by $\sigma_{cr,0}$. Figures 2.2.4-14 to 2.2.4-17 are plotted from data given in Reference 3 Section 2.2.4.1. Each intercept of these interaction curves represents a value of $k_{s,0}$ or $k_{x,0}$ which can be computed from formulas 16 and 17 if $\tau_{cr,0}$ is substituted for τ and $\sigma_{cr,0}$ is substituted for σ

$$k_s = \frac{\tau t b^2}{D \pi^2} \quad (16)$$

$$k_x = \frac{t b^2}{D \pi^2} \quad (17)$$

b = axial or circumferential dimension of panel whichever is smaller.

D = flexural stiffness of panel per unit length

$$= \left(\frac{E t^3}{12 (1 - \mu^2)} \right)$$

Each point on the interaction curve characterizes a pair of critical stress coefficients $k_{s,cr}$ and $k_{x,cr}$ that, acting in conjunction, will produce buckling of the sheet.

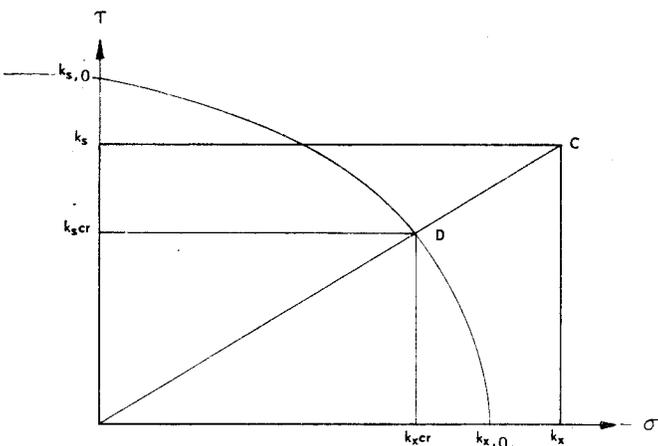


FIGURE 2.2.4-12

Let σ denote the compressive stress that would exist in the cylinder if the sheet did not buckle (i.e., remained fully effective) under the design compression load P . Similarly let τ denote the shear that would exist if the sheet did not buckle under the action of

the design torque T . If these values of σ and τ are used in formulas 16 and 17 the magnitude of k_x and k_s can be found, thus locating point C in Figure 2.2.4-12. The line drawn from C to the origin intersects the interaction curve at point D . The critical stresses σ_{cr} and τ_{cr} can be computed by rearranging formulas 16 and 17 to read

$$\sigma_{cr} = \frac{k_{x,cr} \pi^2 D}{b^2 t} \quad \text{and} \quad \tau_{cr} = \frac{k_{s,cr} \pi^2 D}{b^2 t}$$

These critical stresses are now used in the following steps. For convenience of notation, there are also used the interaction factors

$$R^C = \frac{\sigma_{cr}}{\sigma_{cr,0}} ; \quad R^T = \frac{\tau_{cr}}{\tau_{cr,0}} \quad (18)$$

With the aid of the ratios

$$A = \frac{\tau_{cr,0}}{\sigma_{cr,0}} ; \quad B = \frac{\tau}{\sigma}$$

which can be computed directly from the dimensions of the structure and the specified design loads, the interaction factors can be written in the form

$$R^T = \frac{-A}{2B} + \sqrt{\frac{A^2}{4B^2} + 1} ; \quad R^C = \frac{A}{B} R^T$$

The total stringer stress is the sum of the stringer stress due to the compressive load P and the stringer stress due to the diagonal tension caused by the torque, or

$$\sigma_{ST} = \sigma_{ST}^C + \sigma_{ST}^T \quad (19)$$

The stress σ_{ST}^C is computed by the formula

$$\sigma_{ST}^C = \frac{P}{n(A_{ST} + ht\eta^C)} \quad (20)$$

The load P must be taken as negative because it is compressive; n is the number of stringers, A_{ST} is the area of one stringer, and η^C is the effective-width factor. This factor is taken as the Kármán-Sechler expression for effective width (reference 4), multiplied by the ratio R^C in order to make allowance for the presence of the torque loading; thus

$$\eta^C = R^C 0.89 \sqrt{\frac{\sigma_{cr}}{\sigma_{ST}^C}} \quad (21)$$



If expression (21) is substituted into equation (20), a quadratic equation is obtained which yields

$$\sigma_{ST}^C = \frac{P}{nA_{ST}} - 2D)^2 + 2D \sqrt{D^2 - \frac{P}{nA_{ST}}} \quad (22)$$

where

$$D = 0.445 \frac{ht}{A_{ST}} R^C \sqrt{-\sigma_{cr}} \quad (23)$$

The stress σ_{ST}^T is computed by formula (2), modified by the ratio R^T in order to allow for the presence of the compressive load P ; the modified formula is

$$\sigma_{ST}^T = - \frac{k \tau \cot a}{\frac{A_{ST}}{ht} + 0.5(1 - k) R^T} \quad (24)$$

The interaction factors R^C and R^T , by definition, describe the interaction between compression and torque at the instant of buckling. Their use in formulas (21) and (24) to describe the interaction on the effective width is fundamentally arbitrary. However, in the usual design range, the effect of moderate errors in estimating the effective width is unimportant; any reasonable method for estimating the effect of interaction on effective width is therefore acceptable for the time being.

The stress in a ring is computed, by the unmodified formula (3).

The diagonal strain in the sheet is computed by equation (5), on the implied assumption that it is not modified significantly by the compressive force carried by the sheet. The angle a is computed by formula (4a) or (4b), the strain ϵ_{ST} being computed from the total compressive stress σ_{ST} given by expression (25). The diagonal-tension factor k is obtained from Figure 2.2.4-2 by using τ_{cr} (not $\tau_{cr,0}$).

$$\sigma_{ST} = \sigma_{ST}^C + \sigma_{ST}^T \quad (25)$$

The stress computation for the case of combined loading thus differs from that for the case of pure torque loading in the following items:

- (i) The critical stress is reduced by interaction.
- (ii) The stringer stress due to the load P must be added; this calculation involves an interaction factor.
- (iii) The calculation of the stringer stress due to the torque involves an interaction factor.

Concerning item (i), there is ample theoretical and experimental evidence to justify the belief that

the calculation is sufficiently accurate for design purposes. The factors used in items (ii) and (iii) are arbitrary, but they have only a very minor effect except for low loading ratios. Consequently, the accuracy with which the stresses can be computed under combined loading might be expected to be about the same as for pure torque loading, as long as the ratio τ/τ_{cr} is greater than 2.

The question of allowable stresses for failure is more problematical. The allowable value of skin shear stress is probably not changed significantly by added compression, but there is no experimental evidence on this score. As far as true column failure of the stringers is concerned, it would be immaterial whether the compressive stress in the stringer arises directly from the axial load P , or indirectly (through diagonal-tension action) from the torque; in other words, column failure would be assumed to take place when the total stringer stress given by expression (19) reaches the column allowable value. The condition of true column failure would only exist, however, if the cross section of the stringer were completely immune to forced deformations induced by skin buckles. As mentioned previously, the problem of interaction between forced deformation and column failure is probably more serious in curved than in plane webs, and fragmentary data indicate that no practical stringer section may be completely free from interaction effects.

Since it appears that there will be some interaction in most cases, the investigation of reference 5 was carried out in the region where the interaction is clearly large; namely, on stringers designed to fail by forced crippling in the case of pure-torque loading. Five cylinders of identical construction were built; one was tested in pure compression, one in pure torsion, and the other three in combined compression and torsion. The results were fitted by the interaction formula

$$\left(\frac{T}{T_0}\right)^{1.5} + \frac{P}{P_0} = 1.00 \quad (26)$$

where T and P are the torque and the compressive load that cause stringer failure when acting simultaneously. T_0 is the torque causing stringer failure when acting alone, and P_0 is the compressive load causing stringer failure when acting alone. When this formula is used, it is not necessary to compute the stringer stress by the method described previously for combined loading; a stringer-stress computation is made only for the case of a pure torque to calculate T_0 . Ideally, the load P_0 would also be calculated, but at present it would be safer to obtain this load by a compression test on one bay of the complete cylinder, or on a sector of this bay large enough to contain at least five stringers.



The discussions and formulas for curved diagonal tension have been given on the assumption that the structure considered is a circular cylinder. Evidently, more general types of structure may be analyzed by the same formulas by the usual device of analyzing small regions or individual panels. The questions of detail procedure that will arise must be answered by individual judgment, because more general methods are not available at present. The results will obviously be more uncertain, for instance, if there are large changes in shear flow from one panel to the next. It should be borne in mind that in such cases problems in stress distribution exist even when the skin is not buckled into a diagonal-tension field; the existence of these problems is often overlooked because elementary theories are normally used to compute the shear flows.

As numerical examples of strength analyses of curved diagonal-tension webs, two cylinders will be analyzed that were tested in the investigation of reference 5. The cylinders were of nominally identical construction and differed only in loading conditions. They had 12 stringers of Z-section and rings also of Z-section. The rings were notched to let the stringers pass through them. Clip angles were used to connect the stringers to the rings and at the same time to reinforce the edge of the notch. The analysis will be made for the test loads that produced failure. The third example illustrates the calculation of the angle of twist for the cylinder used in the first example.

Example 1. Pure torsion

The example chosen is cylinder 1 of reference 5. The material is 24S-T3 aluminum alloy.

Basic data:

$$R = 15.0 \text{ in.} \quad t = 0.0253 \text{ in.} \quad d = 15.0 \text{ in.}$$

$$E = 10.6 \times 10^3 \text{ ksi} \quad \mu = 0.32 \quad h = 7.87 \text{ in.}$$

$$\pi R^2 \left(1 - \frac{1}{6} \phi^2\right) = 675 \quad G = 4.0 \times 10^3 \text{ ksi}$$

Stringers: Z-section $\frac{3}{4} \times 1 \times \frac{3}{4} \times 0.040$;

$$A_{ST} = 0.0925 \text{ in.}^2$$

Rings: Z-section $\frac{3}{4} \times 2 \times \frac{3}{4} \times 0.081$;

$$A_{RG} = 0.251 \text{ in.}^2$$

Nominal shear stress

$$\tau = \frac{T}{2 \pi R^2 \left(1 - \frac{1}{6} \phi^2\right)} \quad (27)$$

where ϕ is the angle subtended by two stringers. The quantity $\left(1 - \frac{1}{6} \phi^2\right)$ in the denominator is neglected if its value is small.

$$\tau = \frac{388}{2 \times 675 \times 0.0253} = 11.36 \text{ ksi}$$

Buckling stress

$$Z = \frac{7.87^2}{15.0 \times 0.0253} \sqrt{1 - 0.32^2} = 155$$

From figure 2.2.4-1 $k_s = 35$

$$\tau_{cr} = 35 \frac{\pi^2 \times 10.6 \times 10^3 \times 7.87^2}{12 \times 15^2 \times 155^2} = 3.50 \text{ ksi}$$

Loading ratio

$$\frac{\tau}{\tau_{cr}} = \frac{11.36}{3.50} = 3.24$$

Diagonal-tension factor

$$300 \frac{td}{Rh} = 300 \frac{0.0253 \times 15.0}{15.0 \times 7.87} = 0.965$$

From Figure 2.2.4-2 $k = 0.63$

First approximation for angle of diagonal tension

$$R_R = \frac{dt}{A_{RG}} = \frac{15.0 \times 0.0253}{0.251} = 1.513$$

$$R_S = \frac{ht}{A_{ST}} = \frac{7.87 \times 0.0253}{0.0925} = 2.155$$

$$\frac{1 + R_S}{1 + R_R} = 1.256$$



$$\frac{\frac{h}{R} \sqrt{\frac{E}{\tau}}}{\sqrt{1 + R_R}} = \frac{7.87}{15.0} \sqrt{\frac{10.6 \times 10^3}{11.36}} = 10.1$$

From figure 2.2.4-4: $a_{PDT} = 32.3^\circ$

From figure 2.2.4-5: $\frac{a}{a_{PDT}} = 0.90$

$$a = 32.3^\circ \times 0.90 = 29.0^\circ$$

Stress and strain formulas

From formulas (2) and (3):

$$\sigma_{ST} = -\frac{0.63 \times 11.36}{0.465 + 0.5(1 - 0.63)} \cot a = -11.03 \cot a \text{ ksi}$$

$$\epsilon_{ST} = -1.04 \times 10^{-3} \cot a$$

$$\sigma_{RG} = -\frac{0.63 \times 11.36 \tan a}{0.660 + 0.5(1 - 0.63)} = -8.46 \tan a \text{ ksi}$$

$$\epsilon_{RG} = -0.800 \times 10^{-3} \tan a$$

$$\frac{1}{24} \left(\frac{h}{R}\right)^2 = \frac{1}{24} \left(\frac{7.87}{15.0}\right)^2 = 11.48 \times 10^{-3}$$

$$\frac{\tau}{E} = 1.07 \times 10^{-3}$$

First cycle

$$a = 29^\circ \quad \tan a = 0.554 \quad \cot a = 1.805$$

From figure 2.2.4-3:

$$\frac{\epsilon_E}{\tau} = 1.90; \quad \epsilon = 1.90 \times 1.07 \times 10^{-3} = 2.035 \times 10^{-3}$$

$$\epsilon_{ST} = -1.04 \times 10^{-3} \times 1.805 = -1.875 \times 10^{-3}$$

$$\epsilon_{RG} = -0.800 \times 10^{-3} \times 0.554 = -0.444 \times 10^{-3}$$

According to formula (4a):

$$\tan^2 a = \frac{2.035 + 1.875}{2.035 + 0.440 + 11.48} = 0.280$$

$$\tan a = 0.529$$

Second cycle

The final value of a is closer to the computed value of the preceding cycle than to the initially assumed value; therefore, take as the next approximation

$$\tan a = 0.529 + \frac{1}{4}(0.554 - 0.529) = 0.535$$

$$\cot a = 1.87 \quad a = 28^\circ 10'$$

This slenderness ratio is so low that there is obviously a large margin against column failure at the computed value of stringer stress,

Stringers, forced-crippling failure:

From figure 2.2.4-9:

$$\frac{\sigma_{\max.}}{\sigma} = 1.16$$

$$\sigma_{ST_{\max}} = 20.6 \times 1.16 = -23.85 \text{ ksi}$$

$$\frac{t_{ST}}{t} = \frac{0.0404}{0.0253} = 1.60$$

From figure 2.2.2-10 $\sigma_o = -22.3 \text{ ksi}$

Note: The "design allowable" value of the stringer stress (-22.3 ksi) is 7 percent greater than the calculated value of -23.85 ksi. Therefore, the calculation would have predicted a torque 7 percent lower than the actual failing torque, that is, the calculation is 7 percent conservative. The "best possible estimate" of the allowable stress (based on the middle of the scatter band instead of the lower edge) would be 25 percent higher than the "design allowable" value; a strength prediction based on this value thus would have been 18 percent unconservative.

Example 2 Combined loading

The example chosen to demonstrate the analysis of a cylinder under combined torsion and compression is cylinder 5 of reference 5. In order to simplify the demonstration by making use of partial results obtained in example 1, it will be assumed that the dimensions given for example 1 apply; actually, some of the dimensions differed by as much as 2 percent.

Basic data

Dimensions as in example 1.

$$T = 303 \text{ inch-kips} \quad P = -13.5 \text{ kips}$$



Compression area:

$$12 \text{ stringers} = 12 \times 0.0925 = 1.11 \text{ in.}^2$$

$$\text{Sheet (100\%)} = \pi \times 30 \times 0.0253 = 2.38 \text{ in.}^2$$

$$\text{Total} = 3.49 \text{ in.}^2$$

Basic stresses

$$\sigma_{cr,0} = 3.50 \text{ ksi (see example 1)}$$

$$\sigma_{cr,0} = -5.95 \text{ ksi}$$

The latter value is computed according to the recommendations of section 3.2.

At the design loads, the nominal stresses are

$$\sigma = \frac{-13.5}{3.49} = -3.87 \text{ ksi}$$

$$\tau = \frac{303}{34.2} = 8.86 \text{ ksi}$$

Interaction factors

From formulas 18:

$$A = \frac{3.50}{-5.95} = 0.588 \quad B = \frac{8.86}{-3.87} = -2.29$$

$$R^T = 0.878 \quad R^C = 0.228$$

$$\tau_{cr} = 0.878 \times 3.50 = 3.07 \text{ ksi}$$

$$\sigma_{cr} = -0.228 \times 5.95 = 1.356 \text{ ksi}$$

Compressive stress due to axial load

From formulas (23) and (22), respectively:

$$D = 0.445 \times 2.155 \times 0.228 \times \sqrt{1.356} = 0.254$$

$$\sigma_{ST}^C = -10.55 \text{ ksi} \quad \epsilon_{ST}^C = -0.996 \times 10^{-3}$$

Diagonal-tension factor

$$\frac{\tau}{\tau_{cr}} = \frac{8.86}{3.07} = 2.88 \quad k = 0.59$$

Stress and strain formulae

From formula (24):

$$\sigma_{ST}^T = - \frac{0.59 \times 8.86 \cot \alpha}{0.465 + 0.5(1 - 0.59) \times 0.878}$$

$$= -8.10 \cot \alpha \text{ ksi}$$

$$\epsilon_{ST}^T = -0.764 \times 10^{-3} \cot \alpha$$

From formula (3):

$$\sigma_{RG}^T = - \frac{0.59 \times 8.86 \tan \alpha}{0.660 + 0.5(1 - 0.59)} = -6.05 \tan \alpha \text{ ksi}$$

$$\epsilon_{RG} = -0.570 \times 10^{-3} \tan \alpha$$

Computation cycle

Only the last cycle will be shown here. This computation is essentially the same as for a case of pure torsion (example 1), except that the stringer strain due to axial load (ϵ_{ST}^C) is added to the strain due to the torque (ϵ_{ST}^T).

The first approximation to the angle α may be obtained by disregarding the compression, that is to say, in the same manner as in example 1. An analyst with some experience may improve this approximation by adding a correction for the effect of the axial load (compression load will steepen the angle).

$$\text{Assume } \alpha = 28^\circ 30'; \quad \tan \alpha = 0.543; \quad \cot \alpha = 1.84$$

From figure 2.2.4-3:

$$\frac{\epsilon_E}{\tau} = 1.86;$$

$$\epsilon = 1.86 \times \frac{8.86}{10.6 \times 10^3} = 1.552 \times 10^{-3}$$

From the strain formulas:

$$\epsilon_{ST}^T = -0.764 \times 10^{-3} \times 1.84 = -1.405 \times 10^{-3}$$

$$\epsilon_{RG} = -0.570 \times 10^{-3} \times 0.543 = -0.310 \times 10^{-3}$$

$$\tan^2 \alpha = \frac{1.552 + 1.405 + 0.996}{1.552 + 0.310 + 11.48} = 0.296$$

$$\tan \alpha = 0.544$$

This result agrees with the assumed value within the accuracy of calculation and thus constitutes the final value.

By the stress formulas:

$$\sigma_{ST}^T = \frac{-8.10}{0.544} = -14.90 \text{ ksi}$$



Therefore the total stringer stress is

$$\sigma_{ST} = -14.90 - 10.55 = -25.45 \text{ ksi}$$

The value measured (on a cylinder with slightly different actual dimensions) was -25 ksi.

Failure

Since the torque is much less than in example 1 (pure-torque case), there is a wide margin against web rupture.

The margin against stringer failure is evaluated by formula (24). According to test (reference 5), the cylinder failed under pure compression at $P_o = 42.0$ kips. Under pure torque, the test gave $T_o = 388$ inch-kips: (the calculated value of T_o (example 1) is 7 percent lower). Thus, with the "design loads" $T = 303$ inch-kips and $P = 13.5$ kips

$$\left(\frac{T}{T_o}\right)^{1.5} + \frac{P}{P_o} = \left(\frac{303}{388}\right)^{1.5} + \frac{13.5}{42.0} = 1.01$$

Note: Because the "design loads" T and P used in this example were actually test failing loads and because the interaction curve was based on a series of tests on cylinders of these dimensions, the calculated value of 1.01 indicates that the analytical expression chosen for the interaction curve fits this particular test very well.

Example 3. Angle of twist

In this example, the angle of twist will be calculated for the cylinder of example 1 at the failing torque.

According to example 1:

$$\tan \alpha = 0.535; \cot \alpha = 1.87; \sin 2\alpha = 0.832; k = 0.63$$

$$\frac{E}{G_{DT}} = \frac{4}{\sin^2 2\alpha} + \frac{\tan^2 \alpha}{\frac{A_{ue}}{dt} + 0.5(1-k)} + \frac{\cot^2 \alpha}{\frac{2A_F}{ht} + 0.5(1-k)}$$

$$\frac{E}{G_{DT}} = \frac{4}{0.832^2} + \frac{0.535^2}{0.660 + 0.5(1-0.63)} + \frac{1.872}{0.465 + 0.5(1-0.63)}$$

$$\frac{E}{G_{DT}} = 5.77 + 0.34 + 5.39 = 11.50$$

$$G_{DT} = \frac{E}{11.50} = \frac{10.6 \times 10^3}{11.50} = 0.922 \times 10^3 \text{ ksi}$$

$$\frac{1}{G_{IDT}} = \frac{1-k}{G} + \frac{k}{G_{DT}} \tag{27}$$

$$\frac{1}{G_{IDT}} = \frac{0.37}{4 \times 10^3} + \frac{0.63}{0.922 \times 10^3} =$$

$$0.0925 \times 10^{-3} + 0.683 \times 10^{-3} = 0.775 \times 10^{-3}$$

$$G_{IDT} = 1.29 \times 10^3 \text{ ksi}$$

The torsion constant for the polygon section is

$$J = 2 \times \pi \times 15^3 \times 0.0253 \left(1 - \frac{7}{24} \times 0.524^2\right) = 492$$

For a length of 60 inches, the angle of twist is

$$\frac{TL}{G_{IDT}J} = \frac{388 \times 60}{1.29 \times 10^3 \times 492} = 0.0366 \text{ radian}$$

Interaction Curves

This section provides a solution for curved rectangular panels and cylinders under a combination of shear and direct axial stress. Ref. N.A.C.A. T.N. 1928.

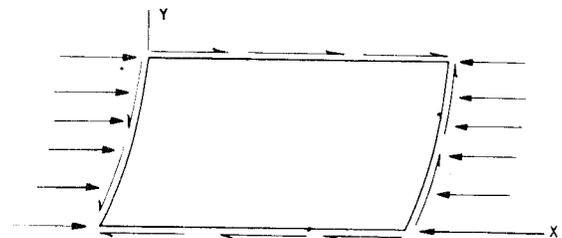


FIGURE 2.2.4-13

The combination of shear and axial stress that causes curve panels to buckle may be obtained from the equations

$$\tau = \frac{k_s \pi^2 D}{b^2 t} \quad \text{and} \quad \sigma_x = \frac{k_x \pi^2 D}{b^2 t}$$

when the stress coefficients k_s and k_x are known.

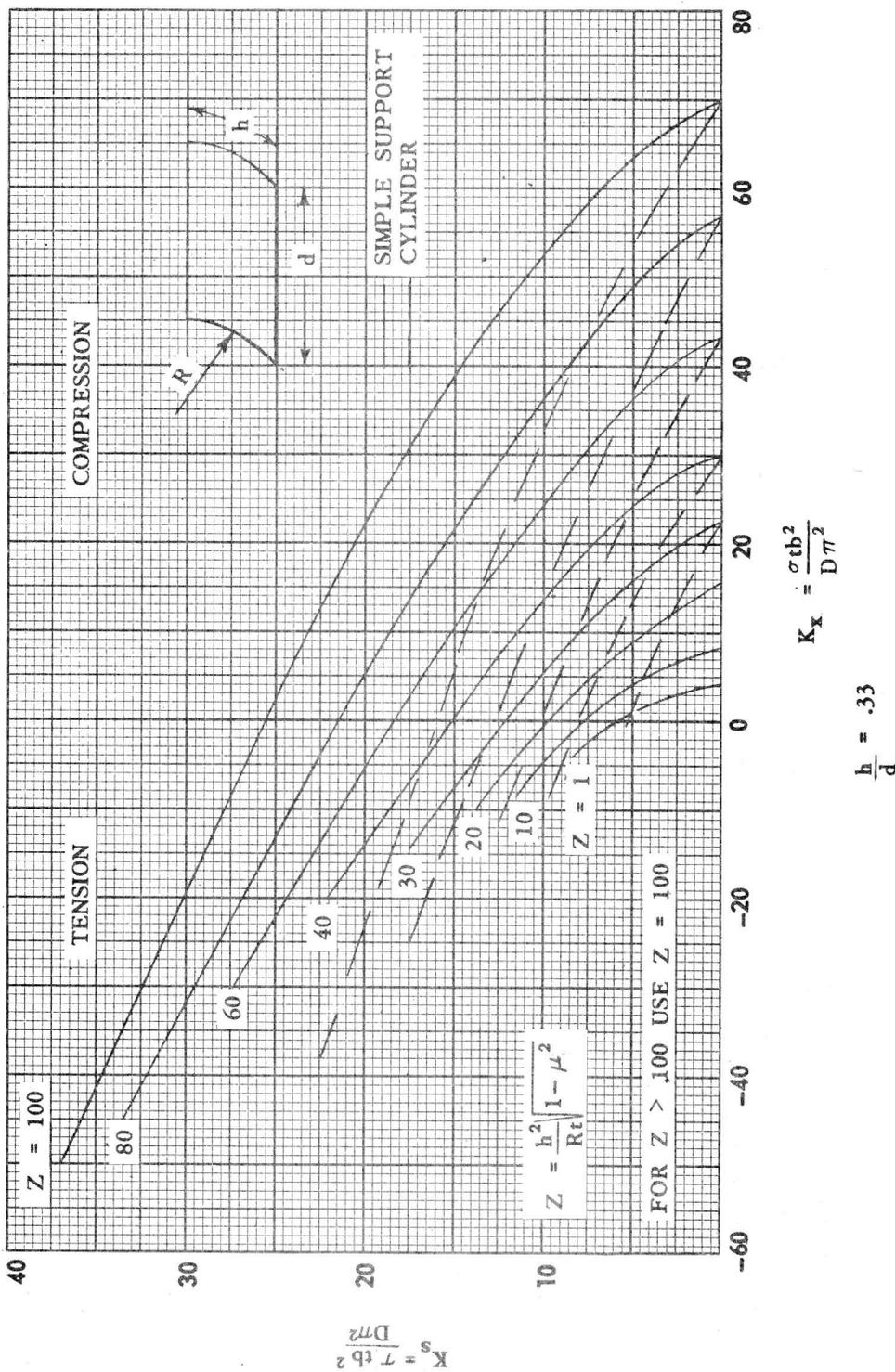
k_s and k_x are determined by use of Figures 2.2.4-14 to 2.2.4-17



b = axial or circumferential dimension of panel, whichever is smaller.

$$D = \frac{E t^3}{12 (1 - \mu^2)}$$

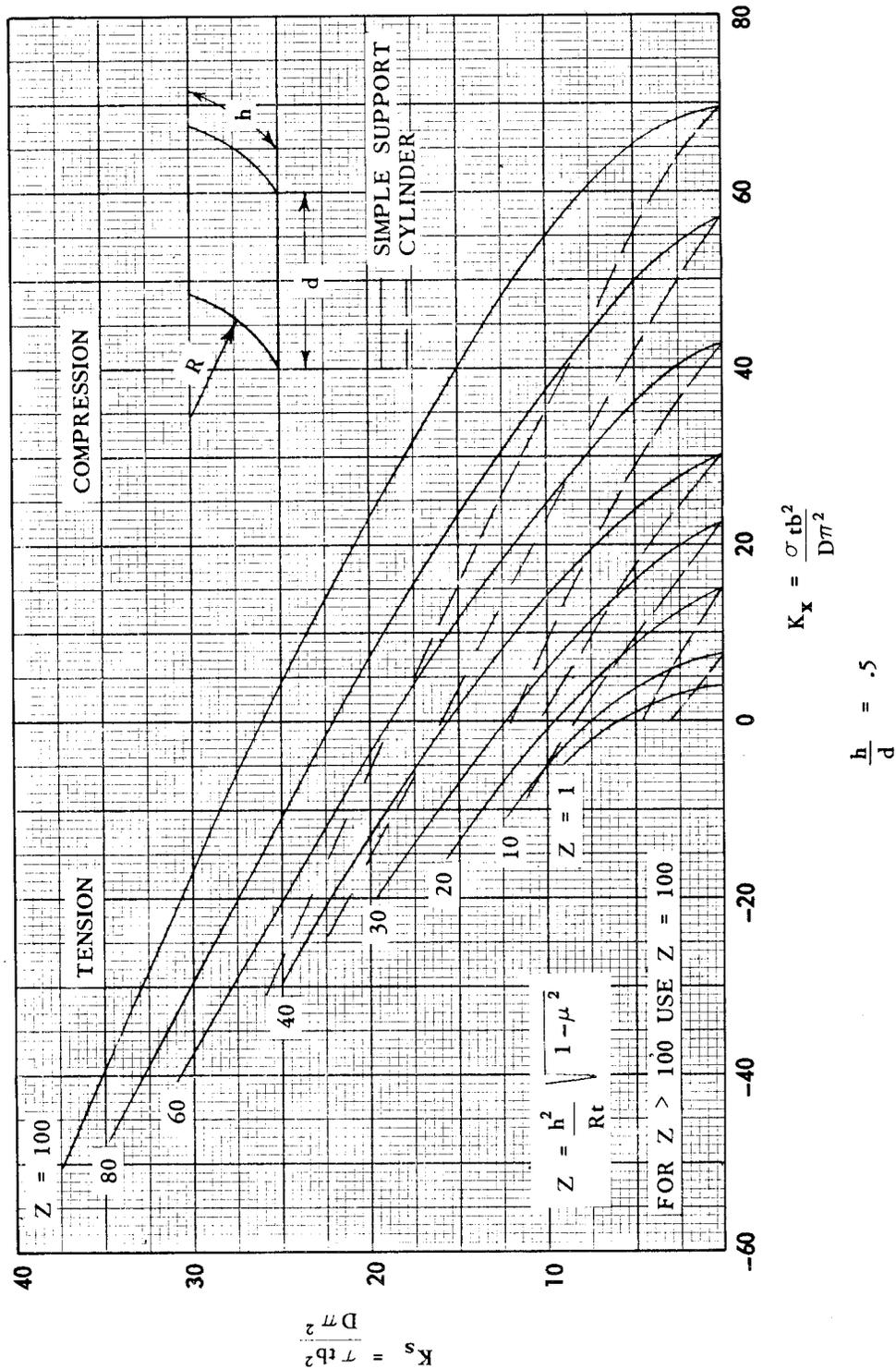
The theoretical combinations of stress coefficients for curved rectangular panels having different ratios of circumferential to axial dimension are given in figures 2.2.4-14 to 2.2.4-17.



BUCKLING INTERACTION OF SHEAR AND NORMAL STRESS FOR CYLINDERS AND CURVED RECTANGULAR PANELS WITH SIMPLY SUPPORTED EDGES

FIGURE 2.2.4-14

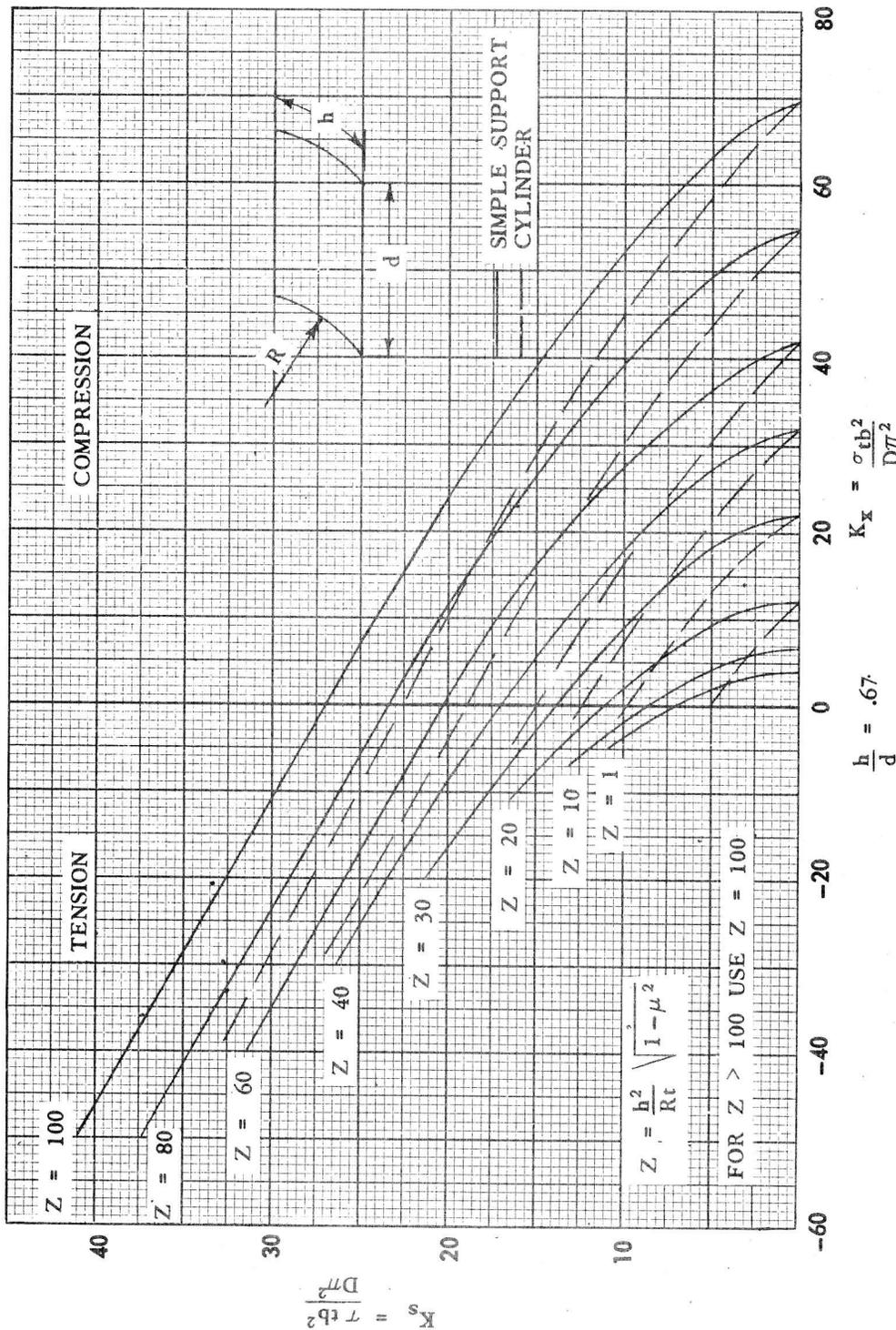
REF: NACA TN 1928



BUCKLING INTERACTION OF SHEAR AND NORMAL STRESS FOR CYLINDERS AND CURVED RECTANGULAR PANELS WITH SIMPLY SUPPORTED EDGES

FIGURE 2.2.4-15

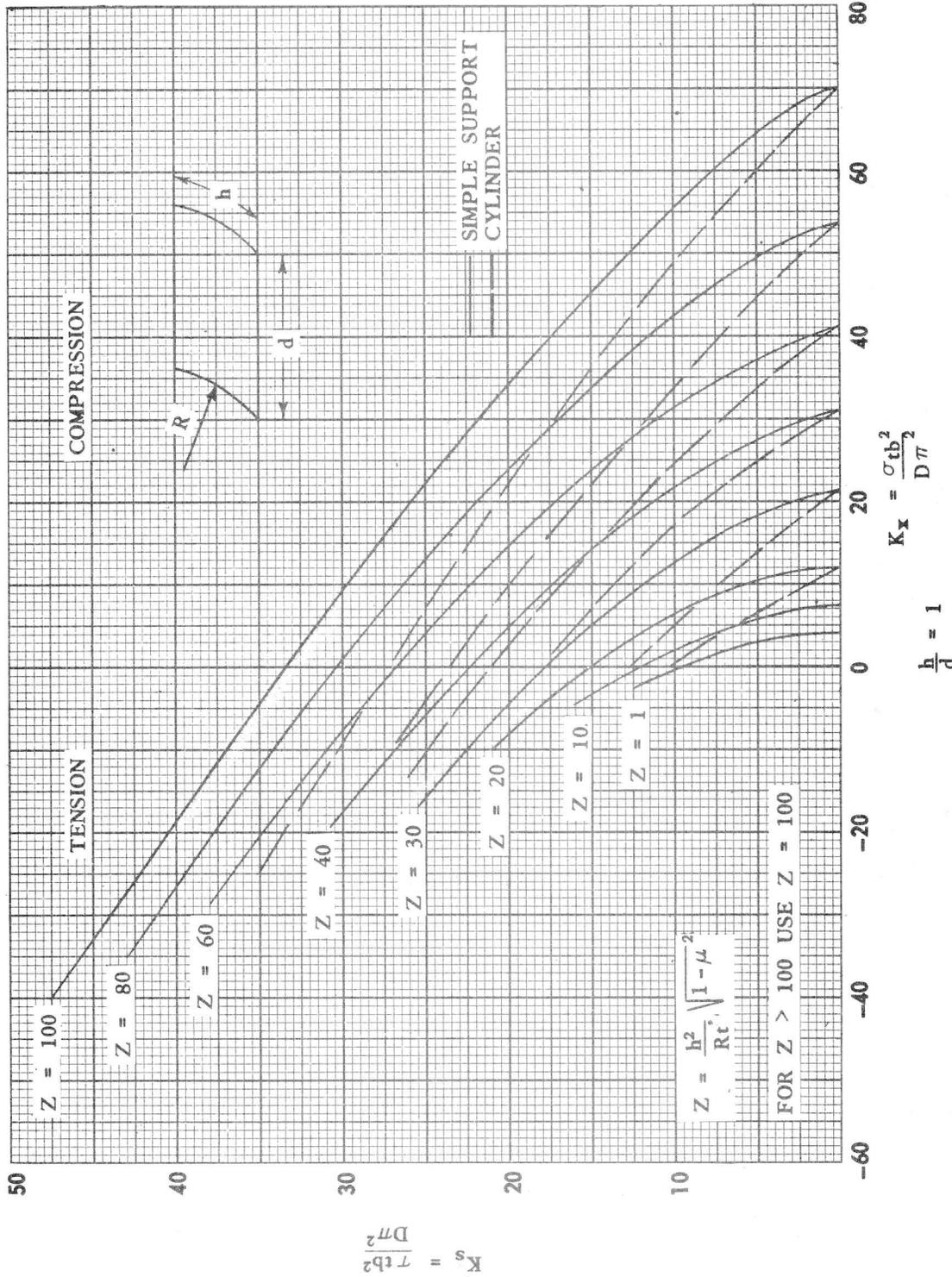
REF.: NACA TN 1928



BUCKLING INTERACTION OF SHEAR AND NORMAL
STRESS FOR CYLINDERS AND CURVED RECTANGULAR
PANELS WITH SIMPLY SUPPORTED EDGES.

FIGURE 2.2.4-16

REF.: NACA TN 1928



BUCKLING INTERACTION OF SHEAR AND NORMAL
STRESS FOR CYLINDERS AND CURVED RECTANGULAR
PANELS WITH SIMPLY SUPPORTED EDGES

FIGURE 2.2.4-17

REF.: NACA TN 1928



BEAMS IN GENERAL

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GEN/1090/852 Notes on Beam Shortness Effect



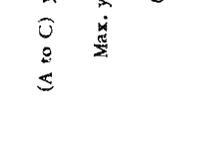
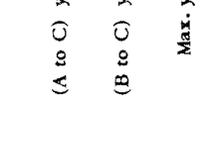
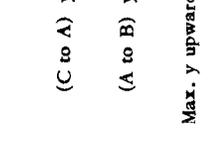
3 Beams In General

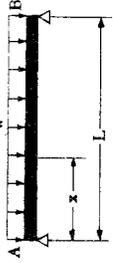
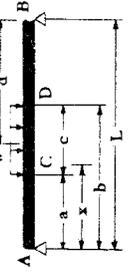
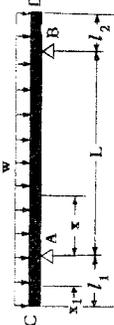
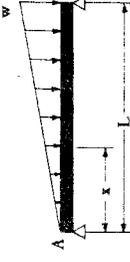
Use the Reference Table below for determining the method of solution of many common beam problems.

REFERENCE TABLE

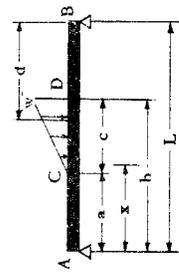
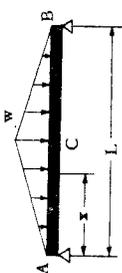
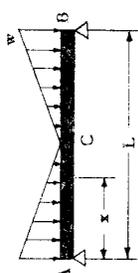
BEAM TYPE	REMARKS
Simply Supported	I Constant } See Section 3.1 for common types of loading. I Varying } Reactions, shear and B.M. are unaffected by varying I.
Cantilever	I Constant } See Section 3.2 for common types of loading. I Varying } Shear and B.M. are unaffected by varying I
Both Ends Fixed (Fixed Beams)	I Constant See Section 3.3.1 for standard loading cases. 3.3.2 for irregular loading cases. 3.3.3 for effect of axial load. 3.3.4 for effect of sinking support. I Varying See Section 3.3.5 for fixed end moments. 3.3.6 for stiffness and carry-over. 3.3.7 for effect of sinking support.
Propped Cantilever	I Constant See Section 3.4.1 for cases with prop at end. For irregular loading and/or prop at intermediate position, see Section 3.4.2. I Varying See Section 3.4.3.
Continuous Beam	I Constant throughout See Section 3.5.1, 3.5.2 and 3.5.3. I Constant per span See Section 3.5.4, 3.5.5 and 3.5.6. I Varying per span See Section 3.5.7.
Curved Beams	For circumferential stresses see 3.6

R→

SIMPLY SUPPORTED BEAM	REACTIONS R_A AND R_B (L.B.) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADJANS)
<p>(1)</p> <p style="text-align: center;">CENTRE LOAD</p> 	$R_A = R_B = \frac{W}{2}$	$(A \text{ to } C) M = \frac{Wx}{2}$ $(C \text{ to } B) M = \frac{W}{2}(L-x)$ $\text{Max. } M = \frac{WL}{4} \text{ at } C$	<p style="text-align: center;">POSITIVE THUS</p>  $(A \text{ to } C) y = -\frac{Wx}{48EI} (3L^2 - 4x^2)$ $\text{Max. } y = -\frac{WL^3}{48EI} \text{ at } C$ $\theta = -\frac{WL^2}{16EI} \text{ at } A$
<p>(2)</p> <p style="text-align: center;">INTERMEDIATE LOAD</p>  <p style="text-align: right;">$a > b$</p>	$R_A = \frac{Wb}{L}$ $R_B = \frac{Wa}{L}$	$(A \text{ to } C) M = \frac{Wb}{L} x$ $(B \text{ to } C) M = \frac{Wa}{L} (L-x)$ $\text{Max. } M = \frac{Wab}{L} \text{ at } C$	$(A \text{ to } C) y = -\frac{Wbx}{6EI} [a(L+b) - x^2]$ $(B \text{ to } C) y = -\frac{Wa(L-x)}{6EI} [b(L+a) - (L-x)^2]$ $\text{Max. } y = -\frac{Wb(L^2 - b^2)^{3/2}}{9\sqrt{3}EI}$ $\text{at } x = \sqrt{\frac{L^2 - b^2}{3}}$ $\text{Approx. } y \text{ at centre } (x = L/2) = -\frac{Wb}{48EI} [3L^2 - 4b^2]$ $\theta = -\frac{Wab}{6EI} (L+b) \text{ at } A$ $\theta = \frac{Wab}{6EI} (L+a) \text{ at } B$
<p>(3)</p> <p style="text-align: center;">OVERHANGING LOAD</p> 	$R_A = \frac{W}{L} (L+l)$ $R_B = -\frac{Wl}{L}$	$(C \text{ to } A) M = -W(l-x_1)$ $(A \text{ to } B) M = -\frac{Wl}{L} (L-x)$ $\text{Max. } M = -Wl \text{ at } A$	$(C \text{ to } A) y = -\frac{Wl}{6EI} [3lx_1 - x_1^2 + 2Ll]$ $(A \text{ to } B) y = +\frac{Wlx}{6EI} (L-x) (2L-x)$ $\text{Max. } y \text{ upward} = \frac{WlL^2}{15.59EI}$ $\text{at } x = 0.42265L$ $\text{Max. } y \text{ downward} = -\frac{Wl^2}{3EI} (L+l) \text{ at } C$

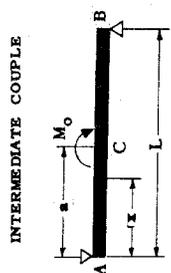
SIMPLY SUPPORTED BEAM	REACTIONS R_A AND R_B (LB) [POSITIVE UPWARD]	BENDING MOMENT M (LB. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN) 
<p>(4)</p> <p>UNIFORM LOADING</p> 	$R_A = R_B = \frac{wL}{2}$ <p>where $W = wL$</p>	$M = \frac{1}{2} w \left(x - \frac{x^2}{L} \right)$ $\text{Max. } M = \frac{wL^2}{8} \text{ at } x = \frac{L}{2}$	$y = -\frac{1}{24} \frac{wx}{EI} [L^3 - 2Lx^2 + x^3]$ $\text{Max. } y = -\frac{5}{384} \frac{wL^3}{EI} \text{ at } x = \frac{L}{2}$ $\theta = -\frac{1}{24} \frac{wL^2}{EI} \text{ at A}$
<p>(5)</p> <p>PARTIAL UNIFORM LOADING</p> 	$R_A = \frac{wd}{L}$ $R_B = \frac{w}{L} \left(a + \frac{c}{2} \right)$ <p>where $W = wc$ and $d = L - \left(a + \frac{c}{2} \right)$</p>	$(A \text{ to } C) \quad M = R_A x$ $(C \text{ to } D) \quad M = R_A x - \frac{w(x-a)^2}{2c}$ $(D \text{ to } B) \quad M = R_A x - w \left[x - \frac{1}{2} (a+b) \right]$ $\text{Max. } M = \frac{wd}{L} \left(a + \frac{cd}{2L} \right)$ <p>at $x = \left(a + \frac{cd}{L} \right)$</p>	$(A \text{ to } C) \quad y = \frac{wx}{48EI} [8d(x^2 - L^2 + d^2) + c^2(2L - 2b + c)]$ $(C \text{ to } D) \quad y = \frac{w}{48EI} [8dx(x^2 - L^2 + d^2) + c^2x(2L - 2b + c) - \frac{2L}{c}(x-a)^4]$ $(D \text{ to } B) \quad y = \frac{wx}{48EI} [8d(x^2 - L^2 + d^2) + c^2(c - 2b)]$ $\theta = \frac{w}{48EI} [8d(d^2 - L^2) + c^2(2L + c - 2b)] \text{ at A}$ $\theta = \frac{w}{48EI} [8d(L - d)(2L - d) + c^2(c - 2b)] \text{ at B}$
<p>(6)</p> <p>OVERHUNG AT BOTH ENDS UNIFORM LOAD</p> 	$R_A = \frac{w}{2L} [L + l_1 - l_2]$ $R_B = \frac{w}{2L} [L + l_2 - l_1]$ <p>where $W = w(L + l_1 + l_2)$</p>	$(C \text{ to } A) \quad M = -\frac{w}{2} (l_1 - x_1)^2$ $(A \text{ to } B) \quad M = -\frac{w}{2} (l_1 + x)^2 + R_A x$	$\text{At C } y = -\frac{wl_1^3}{24EI} [2L(l_2^2 + 2l_1^2) + 3l_1^2 - L^3]$ <p>where $L^3 = (L + l_1 + l_2)^3$</p> $\text{At D } y = -\frac{wl_2^3}{24EI} [2L(l_1^2 + 2l_2^2) + 3l_2^2 - L^3]$
<p>(7)</p> <p>TRIANGULAR LOADING</p> 	$R_A = \frac{1}{3} W$ $R_B = \frac{2}{3} W$ <p>where $W = \frac{1}{2} wL$</p>	$M = \frac{w}{3} \left(x - \frac{x^3}{L^2} \right)$ $\text{Max. } M = 0.128 \frac{wL^2}{3}$ <p>at $x = \frac{L\sqrt{3}}{3} = 0.5774L$</p>	$y = -\frac{1}{180} \frac{wx}{EI} (3x^4 - 10L^2x^2 + 7L^4)$ $\text{Max. } y = -0.01304 \frac{wL^3}{EI} \text{ at } x = 0.519L$ $\theta = -\frac{7}{180} \frac{wL^2}{EI} \text{ at A}$ $\theta = +\frac{8}{180} \frac{wL^2}{EI} \text{ at B}$

3.1 Simply Supported Beams: Tabular Summary (Cont'd)

SIMPLY SUPPORTED BEAM	REACTIONS R_A AND R_B (LB) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN) POSITIVE THUS 
<p>(8)</p> <p>PARTIAL TRIANGULAR LOADING</p> 	$R_A = \frac{Wd}{L}$ $R_B = \frac{W}{L}(L-d)$ <p>where $W = \frac{1}{2}wc$ $d = L - (a + \frac{2}{3}c)$</p>	<p>(A to C) $M = R_A x$</p> <p>(C to D) $M = R_A x - \frac{W(x-a)^3}{3c^2}$</p> <p>(D to B) $M = R_A x - \frac{W}{3}(3x-a-2b)$</p> <p>Max. $M = \frac{Wd}{L} (a + \frac{2}{3}c\sqrt{\frac{d}{L}})$ at $x = a + c\sqrt{\frac{d}{L}}$</p>	<p>(A to C) $y = \frac{Wx}{6EIL} [d(x^2 - L^2 - d^2 + \frac{c^2}{6}) + \frac{c^3}{135}]$</p> <p>(C to D) $y = \frac{Wx}{6EIL} [d(x^2 - L^2 + d^2 + \frac{c^2}{6}) + \frac{c^3}{135} - \frac{L}{10x} - \frac{(x-a)^5}{c^2}]$</p> <p>(D to B) $y = \frac{W(L-x)}{6EIL} [c^2(\frac{17}{270}c - \frac{b}{6}) - d(x^2 + Lx + d^2)]$</p> <p>$\theta = \frac{W}{6EIL} [\frac{c^2}{6}(L-b + \frac{17}{270}c) - d(L^2 - d^2)]$ at A</p> <p>$\theta = \frac{W}{6EIL} [d(L-d)(2L-d) + \frac{c^2}{6}(\frac{17}{45} - b)]$ at B</p>
<p>(9)</p> <p>TRIANGULAR LOADING</p> 	$R_A = R_B = \frac{W}{2}$ <p>where $W = \frac{1}{2}wL$</p>	<p>(A to C) $M = \frac{W}{6}(3x - \frac{4x^3}{L^2})$</p> <p>(C to B) $M = \frac{W}{6}[3(L-x) - 4\frac{(L-x)^3}{L^2}]$</p> <p>Max. $M = \frac{WL}{6}$ at C</p>	<p>(A to C) $y = -\frac{Wx}{30EIL^2}(\frac{5}{4}L^2 - x^2)^2$</p> <p>Max. $y = -\frac{1}{60}\frac{WL^3}{EI}$ at C</p> <p>$\theta = -\frac{5}{96}\frac{WL^2}{EI}$ at A</p>
<p>(10)</p> <p>TRIANGULAR LOADING</p> 	$R_A = R_B = \frac{W}{2}$ <p>where $W = \frac{1}{2}wL$</p>	<p>(A to C) $M = \frac{1}{2}W(x - \frac{2x^2}{L} + \frac{4x^3}{3L^2})$</p> <p>(C to B) $M = \frac{1}{2}W[(L-x) - 2\frac{(L-x)^2}{L} + \frac{4}{3}\frac{(L-x)^3}{L^2}]$</p> <p>Max. $M = \frac{WL}{12}$ at C</p>	<p>(A to C) $y = \frac{1}{12}\frac{W}{EI}(x^3 - \frac{x^4}{L} + \frac{2}{5}\frac{x^5}{L^2} - \frac{3}{8}L^2x)$</p> <p>Max. $y = -\frac{3}{320}\frac{WL^3}{EI}$ at C</p> <p>$\theta = -\frac{1}{32}\frac{WL^2}{EI}$ at A</p>
<p>(11)</p> <p>END COUPLE</p> 	$R_A = -\frac{M_0}{L}$ $R_B = +\frac{M_0}{L}$	<p>$M = M_0 + R_A x$</p> <p>Max. $M = M_0$ at A</p>	<p>$y = \frac{1}{6}\frac{M_0}{EI}(3x^2 - \frac{x^3}{L} - 2Lx)$</p> <p>Max. $y = -0.0642\frac{M_0 L^2}{EI}$ at $x = 0.422L$</p> <p>$\theta = -\frac{1}{3}\frac{M_0 L}{EI}$ at A</p>

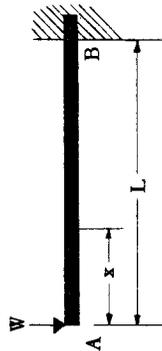
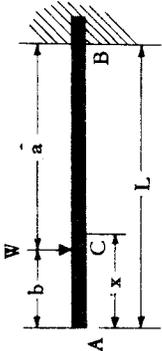
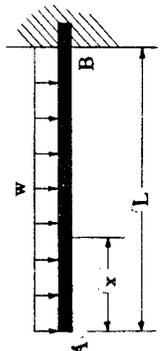
For numerical example, see Report Gen/1090/80L4

3.1 Simply Supported Beams: Tabular Summary (Cont'd)

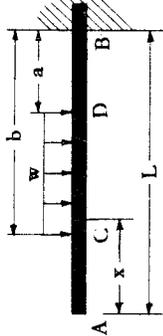
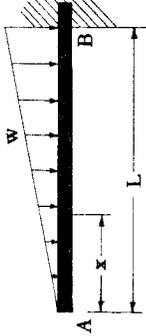
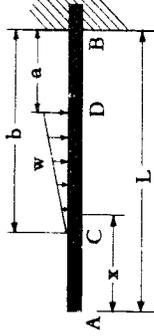
SIMPLY SUPPORTED BEAM	REACTIONS R_A AND R_B (LB) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN) POSITIVE THUS 
<p>(12)</p> <p>INTERMEDIATE COUPLE</p> 	$R_A = -\frac{M_0}{L}$ $R_B = +\frac{M_0}{L}$	<p>(A to C) $M = R_A x$</p> <p>(C to B) $M = R_A x + M_0$</p> <p>Max. - $M = R_A a$ just to the left of C</p> <p>Max. + $M = R_A a + M_0$ just to the right of C</p>	<p>(A to C) $y = \frac{1}{6} \frac{M_0}{EI} \left[(6a - \frac{3a^2}{L} - 2L) x - \frac{x^3}{L} \right]$</p> <p>(C to B) $y = \frac{1}{6} \frac{M_0}{EI} \left[3a^2 + 3x^2 - \frac{x^3}{L} - (2L + \frac{3a^2}{L}) x \right]$</p> <p>$\theta = -\frac{1}{6} \frac{M_0}{EI} (2L - 6a + \frac{3a^2}{L})$ at A</p> <p>$\theta = +\frac{1}{6} \frac{M_0}{EI} (L - \frac{3a^2}{L})$ at B</p> <p>$\theta = \frac{M_0}{EI} (a - \frac{a^2}{L} - \frac{L}{3})$ at C</p> <p><i>For numerical example, see Report Gen/1090/801.5</i></p>

3.1 Simply Supported Beams: Tabular Summary (Cont'd)



CANTILEVER	BENDING MOMENT M (LB. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(1)</p> <p style="text-align: center;">END LOAD</p> 	$M = -Wx$ $\text{Max. } M = -WL \text{ at B}$	$y = -\frac{1}{6} \frac{W}{EI} (x^3 - 3L^2x + 2L^3)$ $\text{Max. } y = -\frac{1}{3} \frac{WL^3}{EI} \text{ at A}$ $\theta = \frac{1}{2} \frac{WL^2}{EI} \text{ at A}$
<p>(2)</p> <p style="text-align: center;">INTERMEDIATE LOAD</p> 	$(A \text{ to } C) \quad M = 0$ $(C \text{ to } B) \quad M = -W(x - b)$ $\text{Max. } M = -Wa \text{ at B}$	$(A \text{ to } C) \quad y = -\frac{1}{6} \frac{W}{EI} (3a^2L - a^3 - 3a^2x)$ $(C \text{ to } B) \quad y = -\frac{1}{6} \frac{W}{EI} [(x - b)^3 - 3a^2(x - b) + 2a^3]$ $\text{Max. } y = -\frac{1}{6} \frac{W}{EI} (3a^2L - a^3)$ $\theta = \frac{1}{2} \frac{Wa^2}{EI} \text{ from A to C}$
<p>(3)</p> <p style="text-align: center;">UNIFORM LOADING</p> 	$M = -\frac{w}{2L} x^2$ <p style="text-align: center;">where $W = wL$</p>	$y = -\frac{1}{24} \frac{w}{EI} (x^4 - 4L^3x + 3L^4)$ $\text{Max. } y = -\frac{wL^3}{8EI}$ $\theta = \frac{1}{6} \frac{wL^2}{EI} \text{ at A}$

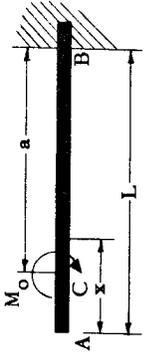


CANTILEVER	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(4)</p> <p>PARTIAL UNIFORM LOADING</p> 	<p>(A to C) $M = 0$</p> <p>(C to D) $M = -\frac{1}{2} \frac{W}{(b-a)} (x-L+b)^2$</p> <p>(D to B) $M = -\frac{1}{2} W (2x-2L+a+b)$</p> <p>Max. $M = -\frac{1}{2} W (a+b)$ at B</p> <p>where $W = w(b-a)$</p>	<p>For y and θ, see Roark, p. 98, Case 4.</p>
<p>(5)</p> <p>TRIANGULAR LOADING</p> 	<p>$M = -\frac{1}{3} \frac{W}{L^2} x^3$</p> <p>Max. $M = -\frac{WL}{3}$ at B</p> <p>where $W = \frac{1}{2} wL$</p>	<p>$y = -\frac{1}{60} \frac{W}{EI} L^2 (x^5 - 5L^4x + 4L^5)$</p> <p>Max. $y = -\frac{1}{15} \frac{WL^3}{EI}$ at A</p> <p>$\theta = \frac{1}{12} \frac{WL^2}{EI}$ at A</p>
<p>(6)</p> <p>PARTIAL TRIANGULAR LOADING</p> 	<p>(A to C) $M = 0$</p> <p>(C to D) $M = -\frac{1}{3} \frac{W(x-L+b)^3}{(b-a)^2}$</p> <p>(D to B) $M = -\frac{1}{3} W(3x-3L+b+2a)$</p> <p>Max. $M = -\frac{1}{3} W(b+2a)$ at B</p> <p>where $W = \frac{1}{2} w(b-a)$</p>	<p>For y and θ see Roark, p. 99, Case 6.</p>

3.2 Cantilevers: Tabular Summary (Cont'd)

CANTILEVER	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(7)</p> <p style="text-align: center;">TRIANGULAR LOADING</p>	$M = -\frac{1}{3} W L^2 (3Lx^2 - x^3)$ <p style="text-align: center;">Max. M = $-\frac{2}{3} WL$ at B</p> <p style="text-align: center;">where $W = \frac{1}{2} wL$</p>	$y = -\frac{1}{60} \frac{W}{EI} L^2 (5Lx^4 + 11L^5 - x^5 - 15L^4x)$ <p style="text-align: center;">Max. y = $-\frac{11}{60} \frac{WL^3}{EI}$ at A</p> <p style="text-align: center;">$\theta = \frac{1}{4} \frac{WL^2}{EI}$ at A</p>
<p>(8)</p> <p style="text-align: center;">PARTIAL TRIANGULAR LOADING</p>	<p>(A to C) $M = 0$</p> <p>(C to D) $M = -\frac{1}{3} W \left[\frac{3(x-L+b)^2}{(b-a)} - \frac{(x-L+b)^3}{(b-a)^2} \right]$</p> <p>(D to B) $M = -\frac{1}{3} W (3x - 3L + 2b + a)$</p> <p style="text-align: center;">Max. M = $-\frac{1}{3} W (2b + a)$ at B</p> <p style="text-align: center;">where $W = \frac{1}{2} w(b-a)$</p>	<p style="text-align: center;">For y and θ see Roark, p. 99, Case 8.</p>
<p>(9)</p> <p style="text-align: center;">END COUPLE</p>	<p style="text-align: center;">$M = M_0$</p> <p style="text-align: center;">Max. M = M_0 (throughout)</p>	$y = \frac{1}{2} \frac{M_0}{EI} (L^2 - 2Lx + x^2)$ <p style="text-align: center;">Max. y = $\frac{1}{2} \frac{M_0 L^2}{EI}$ at A</p> <p style="text-align: center;">$\theta = -\frac{M_0 L}{EI}$ at A</p>



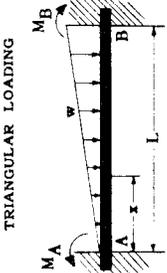
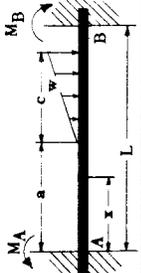
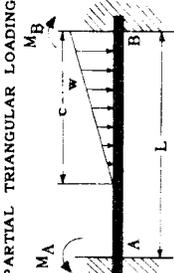
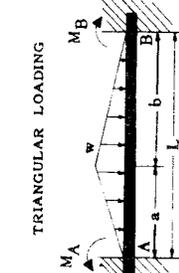
CANTILEVER	BENDING MOMENT M (LB. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(10)</p> <p>INTERMEDIATE COUPLE</p> 	<p>(A to C) $M = 0$</p> <p>(C to B) $M = M_0$</p> <p>Max. $M = M_0$ (C to B)</p>	<p>(A to C) $y = \frac{M_0 a^2}{EI} (L - \frac{1}{2} a - x)$</p> <p>(C to B) $y = \frac{1}{2} \frac{M_0}{EI} (L - x)^2$</p> <p>Max. $y = \frac{M_0 a}{EI} (L - \frac{1}{2} a)$ at A</p> <p>$\theta = - \frac{M_0 a}{EI}$ (A to C)</p>

3.2 Cantilevers: Tabular Summary (Cont'd)

FIXED BEAM	REACTIONS R_A AND R_B (L.B.) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD]
<p>(1)</p> <p style="text-align: center;">CENTRE LOAD</p>	$R_A = R_B = \frac{W}{2}$	$M_A = -\frac{WL}{8}$ $M_B = -\frac{WL}{8}$ <p>(A to C) $M = \frac{1}{8} W (4x - L)$</p> <p>(C to B) $M = \frac{1}{8} W (3L - 4x)$</p> <p style="text-align: center;">Max. $+ M = \frac{1}{8} WL$ at C</p>	$(A \text{ to } C) \ y = -\frac{1}{48} \frac{W}{EI} (3Lx^2 - 4x^3)$ $\text{Max. } y = -\frac{1}{192} \frac{WL^3}{EI} \text{ at C}$
<p>(2)</p> <p style="text-align: center;">INTERMEDIATE LOAD</p>	$R_A = \frac{Wb^2}{L^3} (3a + b)$ $R_B = \frac{Wa^2}{L^3} (3b + a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wab^2}{L^2} a$ <p>(A to C) $M = -\frac{Wab^2}{L^2} + R_A x$</p> <p>(C to B) $M = -\frac{Wab^2}{L^2} + R_A x - W(x - a)$</p> <p style="text-align: center;">Max. $+ M$ (at C) $= -\frac{Wab^2}{L^2} + R_A a$</p>	<p>(A to C) $y = \frac{1}{6} \frac{Wb^2 x^2}{EIL^3} (3ax + bx - 3aL)$</p> <p>(C to B) $y = \frac{1}{6} \frac{Wa^2 (L - x)^2}{EIL^3} [(3b + a)(L - x) - 3bL]$</p> <p style="text-align: center;">Max. $y = -\frac{2}{3} \frac{W}{EI} \frac{a^3 b^2}{(3a + b)^2}$</p> <p style="text-align: center;">at $x = \frac{2aL}{(3a + b)}$ for $a > b$</p>
<p>(3)</p> <p style="text-align: center;">UNIFORM LOADING</p>	$R_A = R_B = \frac{W}{2}$ <p style="text-align: center;">where $W = wL$</p>	$M_A = -\frac{WL}{12}$ $M_B = -\frac{WL}{12}$ $M = \frac{1}{2} W \left(x - \frac{x^2}{L} - \frac{L}{6} \right)$ <p style="text-align: center;">Max. $+ M = \frac{1}{24} WL$</p> <p style="text-align: center;">at $x = \frac{L}{2}$</p>	$y = \frac{1}{24} \frac{Wx^2}{EIL} (2Lx - L^2 - x^2)$ $\text{Max. } y = -\frac{1}{384} \frac{WL^3}{EI} \text{ at } x = \frac{L}{2}$
<p>(4)</p> <p style="text-align: center;">PARTIAL UNIFORM LOADING</p>			

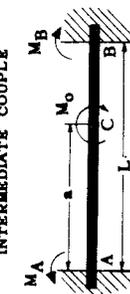
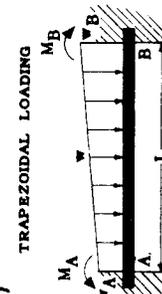
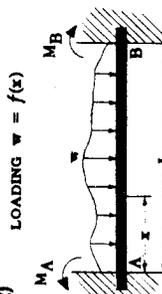
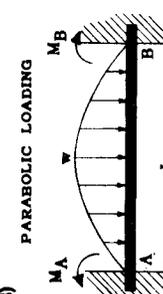
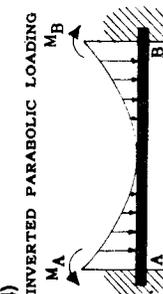
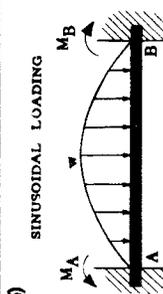
See Roark, p. 106, Case 34.



FIXED BEAM	REACTIONS R_A AND R_B (L.B.) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD]
(5) TRIANGULAR LOADING 	$R_A = \frac{3}{10} W$ $R_B = \frac{7}{10} W$ $W = \frac{1}{2} wL$	$M_A = -\frac{wL}{15}$ $M_B = -\frac{wL}{10}$ $M = W\left(\frac{3}{10}x - \frac{1}{15} - \frac{1}{3}x^2\right)$ <p>Max. $M = 0.043 WL$ at $x = 0.548 L$</p>	$y = \frac{1}{60} \frac{w}{EI} (3x^3 - 2Lx^2 - \frac{x^5}{2})$ <p>Max. $y = -0.002617 \frac{wL^3}{EI}$ at $x = 0.525 L$</p>
See Roark, p. 107, Case 36			
(6) PARTIAL TRIANGULAR LOADING 	$W = \frac{1}{2} wC$ $R_B = W \frac{c^2}{L^2} \left(\frac{3}{2} - \frac{4c}{5L} \right)$ $R_A = W - R_B$	$M_A = -\frac{Wc}{15L^2} (10L^2 - 15cL + 6c^2)$ $M_B = -\frac{Wc^2}{10L^2} (5L - 4c)$	A special case of (6)
(7) PARTIAL TRIANGULAR LOADING 	$R_A = \frac{Wc^2}{10L^3} (5L - 2c)$ $R_B = W - R_A$ $W = \frac{1}{2} wc$	$M_A = -\frac{Wc^2}{30L^2} (5L - 3c)$ $M_B = -\frac{Wc}{30L^2} (10L^2 - 10cL + 3c^2)$	A special case of (6)
(8) PARTIAL TRIANGULAR LOADING 	$W = \frac{1}{2} wL$	$M_A = -\frac{W}{30L^3} [2a^2(a+4b) + 3b^2(4a+b)]$ $M_B = -\frac{W}{30L^2} [3a^2(a+4b) + 2b^2(4a+b)]$ <p>For derivation, see Report GEN/1090/801.2</p>	

3.3.1 Fixed Beams of Constant I (Cont'd)



FIXED BEAM	REACTIONS R_A AND R_B (LB.) [POSITIVE UPWARD]	BENDING MOMENT M (LB. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD]
(10) INTERMEDIATE COUPLE 	$R_A = -\frac{M_0}{L^3}(aL - a^2)$ $R_B = \frac{M_0}{L^3}(aL - a^2)$	$M_A = -\frac{M_0}{L^2}(4La - 3a^2 - L^2)$ $M_B = -\frac{M_0}{L^2}(2La - 3a^2)$	See Roark, p. 107, Case 37.
(11) TRAPEZOIDAL LOADING 	$W = \frac{W_A + W_B}{2} L$	$M_A = -\frac{L^2}{60} [5W_A + 2(W_B - W_A)]$ $M_B = -\frac{L^2}{60} [5W_A + 3(W_B - W_A)]$	
(12) LOADING $w = f(x)$ 		$M_A = -\frac{1}{L^2} \int_0^L wx(L-x)^2 dx$ $M_B = -\frac{1}{L^2} \int_0^L wx^2(L-x) dx$	
(13) PARABOLIC LOADING 	$W = \frac{2}{3} wL$	$M_A = -\frac{WL}{10}$ $M_B = -\frac{WL}{10}$	For derivation, see Report GEN/1090/801.3
(14) INVERTED PARABOLIC LOADING 	$W = \frac{1}{3} wL$	$M_A = -\frac{WL}{20}$ $M_B = -\frac{WL}{20}$	
(15) SINUSOIDAL LOADING 	$W = \frac{2}{\pi} wL$	$M_A = -\frac{WL}{\pi^2}$ $M_B = -\frac{WL}{\pi^2}$	For derivation, see Report GEN/1090/801.1

3.3.1 Fixed Beams of Constant I (Cont'd)



3.3.2 Fixed Beam of Constant I: Fixed end moments for irregular loading.

When it is not possible to determine the fixed end moments by means of one or more of the standard cases of 3.3.1, use the following method employing slope deflection formulas. (See Report GEN/1090/803 for derivation).

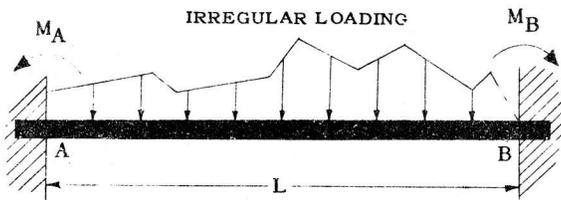
PROCEDURE:

By graphical integration (or otherwise) find the area of the free bending moment diagram and the span-wise position of its centroid. The fixed end moments, acting as shown in Fig. 3.3.2-1, are then:-

$$M_A = -\frac{2A}{L^2} (\bar{x}_A - 2\bar{x}_B)$$

$$M_B = \frac{2A}{L^2} (2\bar{x}_A - \bar{x}_B)$$

where L = beam span,
 A = area under free B.M. curve,
 \bar{x}_A = distance of its centroid from A
and \bar{x}_B = distance of its centroid from B



There is zero slope at A and B due to building in.

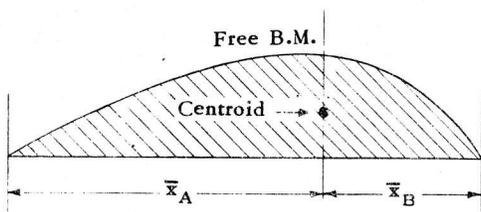


Figure 3.3.2-1

NUMERICAL EXAMPLE:

The loading on a fixed beam of span $L = 20$ in. is shown in Curve I of Fig. 3.3.2-2.

(1) Integrate Curve I graphically to give Curve II. The ordinate at B of this curve represents the total load on the beam, viz 30.62 lb.

(2) Integrate Curve II graphically to find the moment of the external loads about B. Moment about B = 330.5 lb. in.

$$\text{Free reaction at A, } R_A^1 = \frac{330.5}{20} = 16.53 \text{ lb.}$$

(3) Free B.M. on beam at any station can now be obtained and the free B.M. curve drawn (Curve III).

(4) Integrate Curve III to give Curve IV, the ordinate of which at B represents the area under the free B.M. curve, viz 1224. Integration of Curve IV gives the moment of the free B.M. about B, viz 12530 and hence:

$$\bar{x}_B = \frac{12530}{1224} = 10.22 \text{ in.}$$

$$\bar{x}_A = 9.78 \text{ in.}$$

$$(5) L = 20 \quad L^2 = 400 \quad \bar{x}_A = 9.78 \quad \bar{x}_B = 10.22$$

$$\frac{2A}{L^2} = \frac{2 \times 1224}{400} = 6.12$$

$$M_A = -6.12(9.78 - 20.44) = +6.12 \times 10.66 = 65.3 \text{ lb.in.}$$

$$M_B = 6.12(19.56 - 10.22) = 6.12 \times 9.34 = 57.2 \text{ lb.in.}$$

The resultant B.M. is shown shaded in Fig. 3.3.2-3.

Check: For the same total load, uniformly distributed:

$$M_A = M_B = \frac{WL}{12} = 30.62 \times \frac{20}{12} = 51.0 \text{ lb. in.}$$

3.3.3 Fixed Beam of Constant I: Effect of axial load.

(a) The end moments on a fixed beam of constant I due to the combined effect of axial load and lateral load are given under Beam Columns (6.5) for common loading cases.

For moment distribution calculations, however, stiffness and carry-over factors, as well as the fixed-end moments, are required. These are given in Niles and Newell, Vol. II, in Tables 14:8 and 14:9 on pp. 122 - 126, for certain standard loading cases.

NUMERICAL EXAMPLE 1

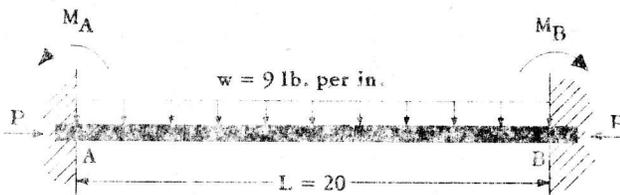


Figure 3.3.3-1

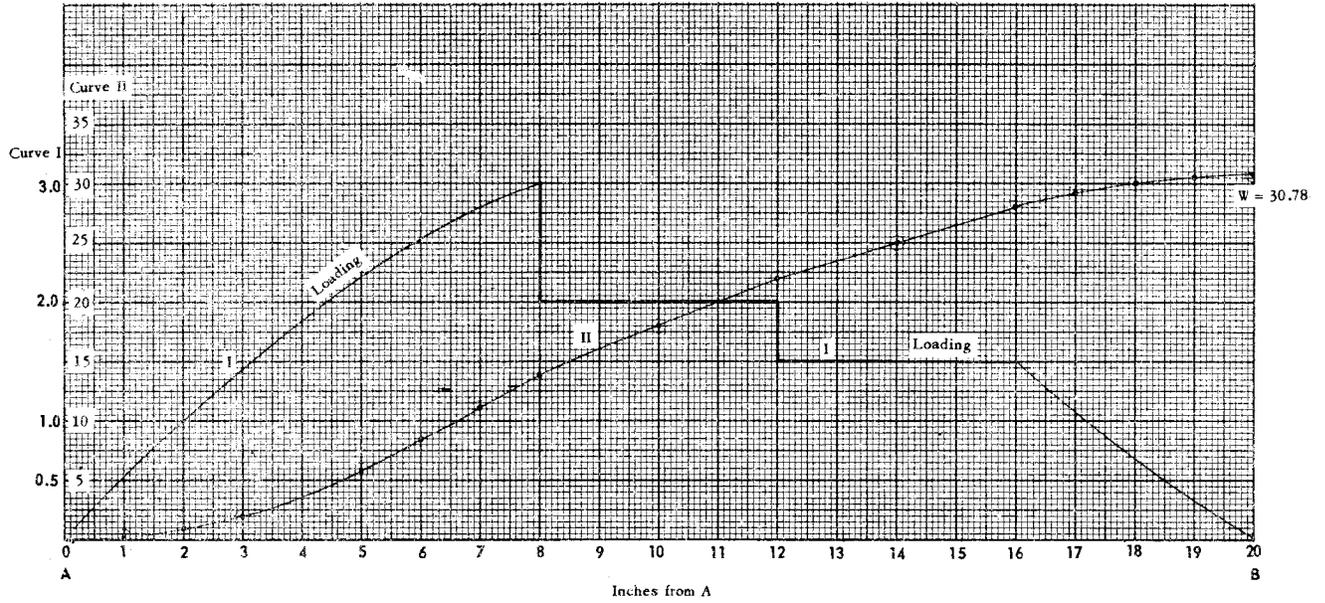


Figure 3.3.2-2

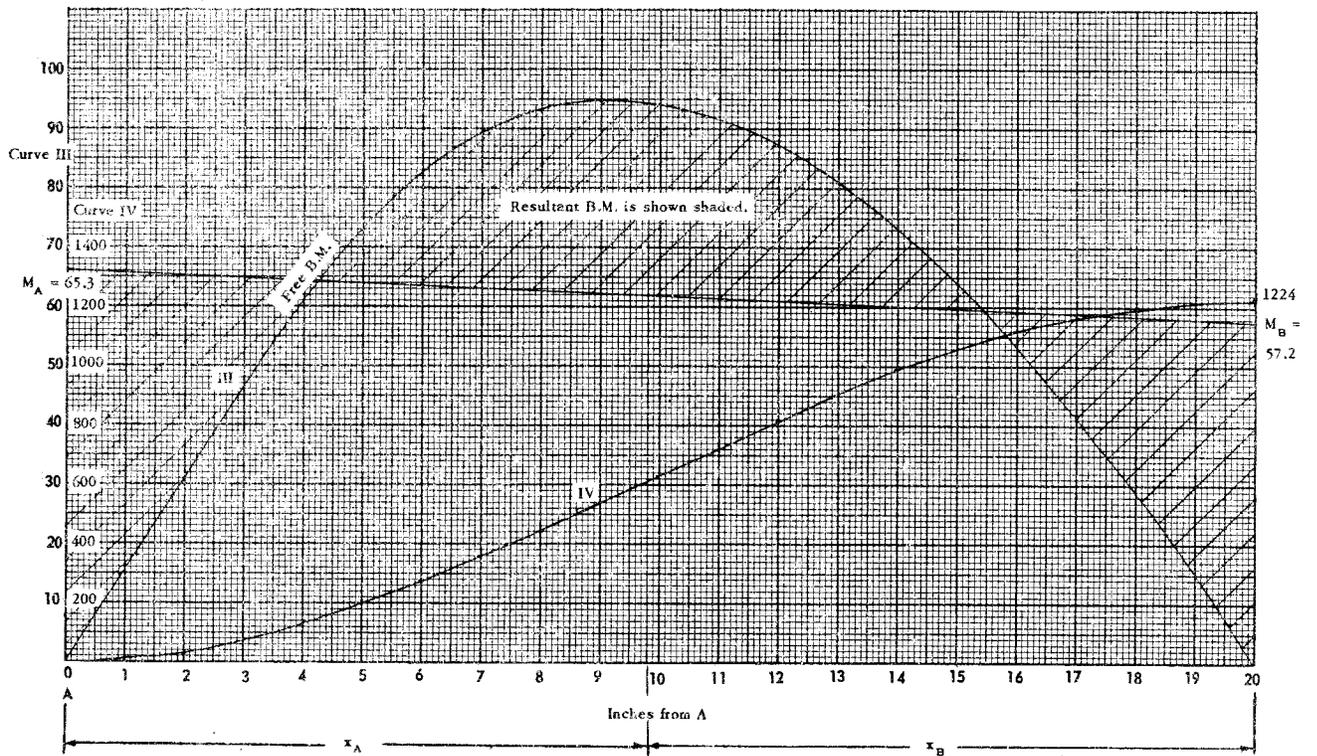


Figure 3.3.2-3



Fixed end beam shown in Fig. 3.3.3-1 carrying a uniformly distributed lateral load $w = 9$ lb. per in. and a compressive end load $P = 10,000$ lb. Take $EI = 10^6$

$$j^2 = \frac{EI}{P} = 100 \quad j = 10 \quad L/j = \frac{20}{10} = 2.0$$

From Niles and Newell, Table 14:8, fixed end moment coefficient $C = 11.176$

$$M_A = \frac{WL}{C} = \frac{9 \times 20 \times 20}{11.176} = \underline{322 \text{ lb. in.}}$$

This value checks with that obtained in Beam Columns, (6.5).

$$\begin{aligned} \text{Carry-over factor} &= .6263 \\ \text{Stiffness coefficient} &= .8590 \end{aligned}$$

$$\text{Absolute stiffness} = .8590 \times \frac{4EI}{L} = 3.44 \frac{EI}{L}$$

This is because the absolute stiffness of a non-tapered beam with fixed ends, without end load, is $\frac{4EI}{L}$, as shown in Report GEN/1090/804.

3.3.4 Fixed Beam of Constant I: Effect of sinking support.

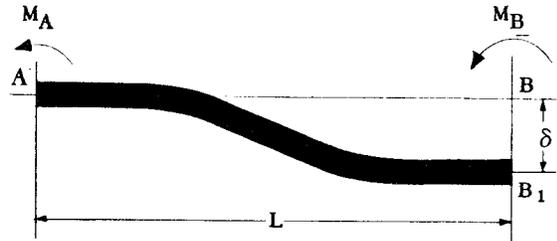


Figure 3.3.4-1

It is shown in Report GEN/1090/803, Equation 6 that, for a fixed beam carrying no lateral load, if end B sinks an amount δ relative to A, the end moments induced are:-

$$M_A = M_B = -\frac{6EI}{L} \cdot \frac{\delta}{L} \quad (\text{See Fig. 3.3.4-1})$$

This moment must be superimposed on the moment due to lateral load and incorporated in the moment distribution calculations for a continuous beam, (see 3.5.5).



3.3.5 Fixed Beam of Varying I: End moment determination.

A method is given in Report GEN/1090/805 using the Column Analogy.

Briefly, this method entails the replacement of the original beam by an analogous column having a width equal to $\frac{1}{EI}$ (or $\frac{1}{I}$ if more convenient), a depth equal to the span L and a length l which is taken to be very small.

However, in the case of linear tapered beams the following design charts can be used for the particular loadings given.

END MOMENTS FOR LINEAR TAPERED BEAMS

Figs. 3.3.5-1 to 5 give design charts for linearly tapered beams under common loading conditions, from which the fixed end moments can be found.

$$W = wL$$

$$M_A = \frac{WL}{K_A}$$

$$M_B = \frac{WL}{K_B}$$

When $d_A = d_B$,

$$\frac{d_B}{d_A} = 1.0$$

and $M_A = M_B = \frac{WL}{12}$

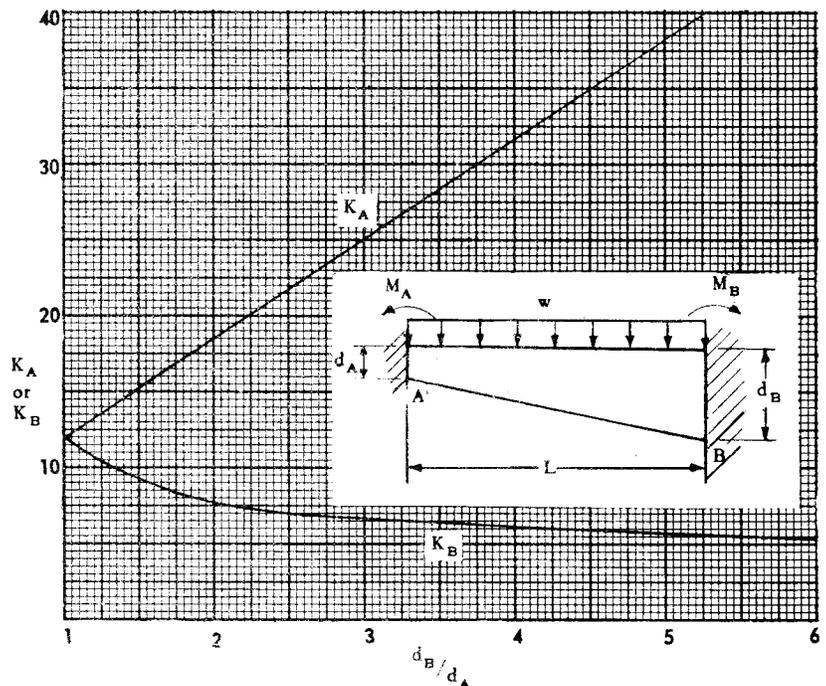


Figure 3.3.5-1 Uniform Load



End Moments for Linear Tapered Beams (Cont'd)

W = total load on beam

$$M_A = \frac{WL}{K_A}$$

$$M_B = \frac{WL}{K_B}$$

When $d_A = d_B$, $d_B/d_A = 1.0$

and $M_A = \frac{WL}{15}$

$$M_B = \frac{WL}{10}$$

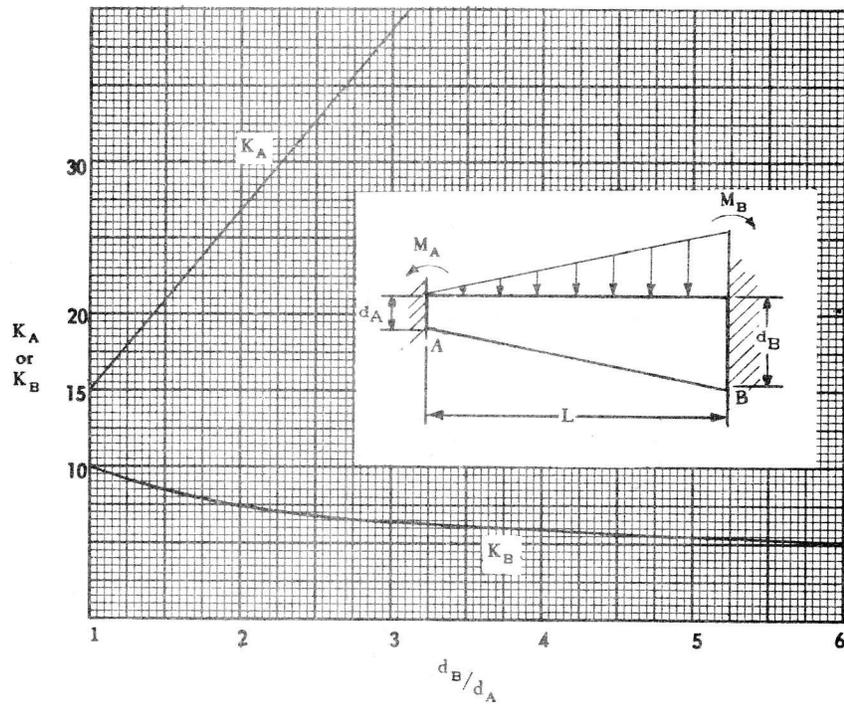


Figure 3.3.5-2 Triangular Load

W = total load on beam

$$M_A = \frac{WL}{K_A}$$

$$M_B = \frac{WL}{K_B}$$

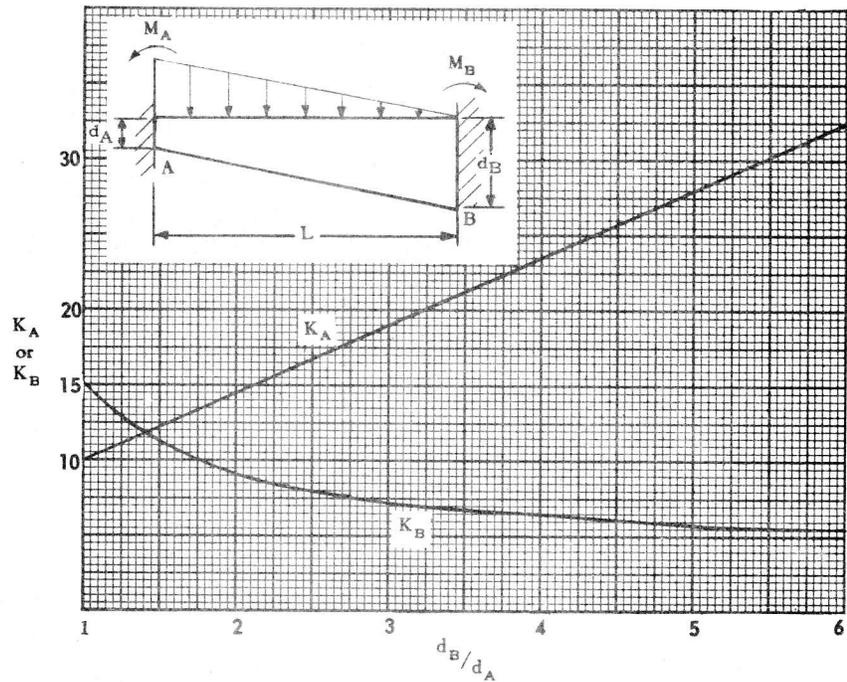


Figure 3.3.5-3 Triangular Load



End Moments for Linear Tapered Beams (Cont'd)

When $d^1 = 0$, beam is prismatic
 For central load, $K = .5$
 and $M_A = .125 WL = \frac{WL}{8}$

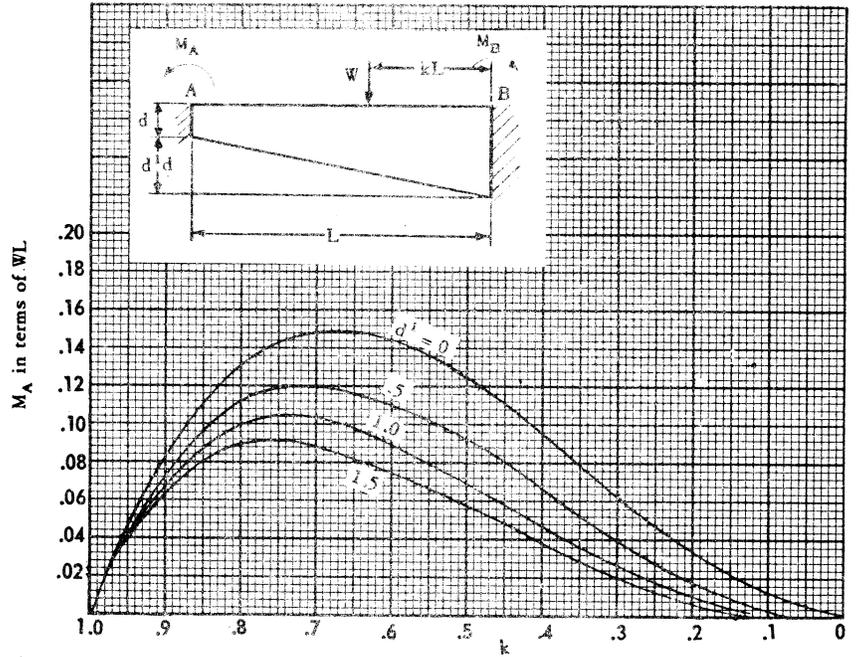


Figure 3.3.5-4

When $d^1 = 0$
 and $K = .5$
 $M_B = .125 WL = \frac{WL}{8}$

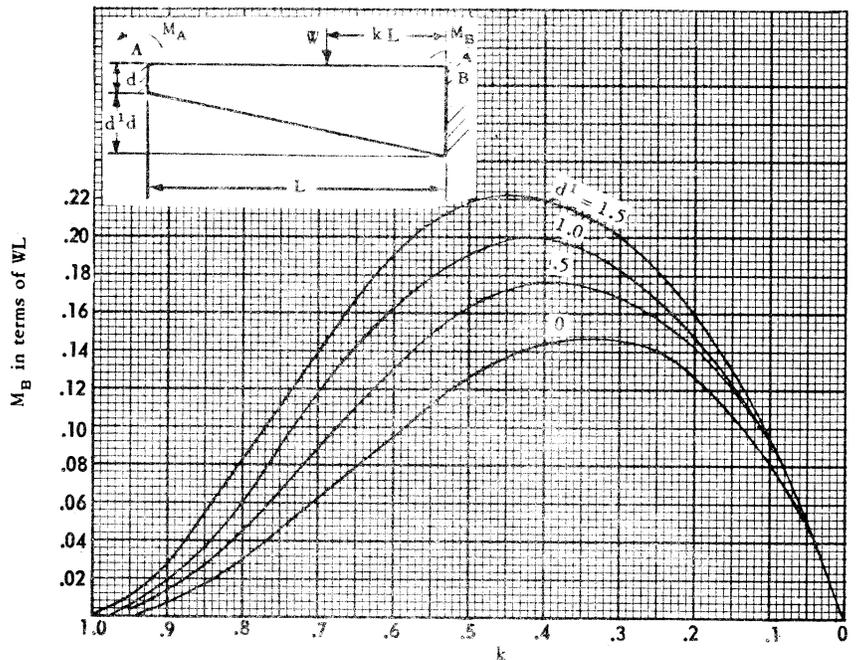


Figure 3.3.5-5



3.3.6 Fixed Beams of Varying I: Stiffness and Carry-over

See Report GEN/1090/805 for the solution of a typical problem by Column Analogy methods.

Design charts giving stiffness and carry-over for straight-tapered beams appear in Figs. 3.3.6-1 and 2.

When $d^1 = 0$

$$\text{Stiffness at A} = \frac{4EI}{L}$$

$$B = \frac{4EI}{L}$$

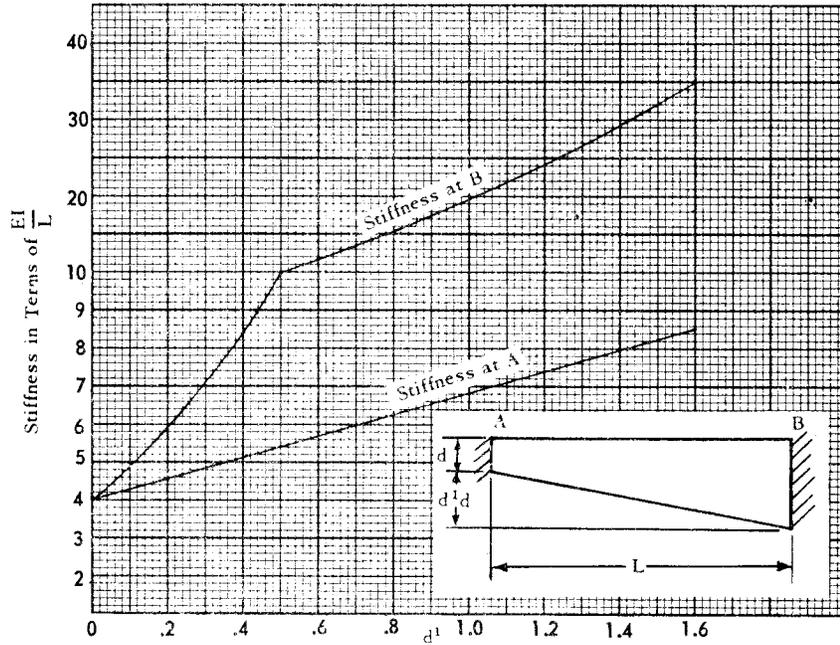


Figure 3.3.6-1

When $d_A = d_B$ (non-tapered beam),

carry-over factor. A to B = 0.5
 B to A = 0.5

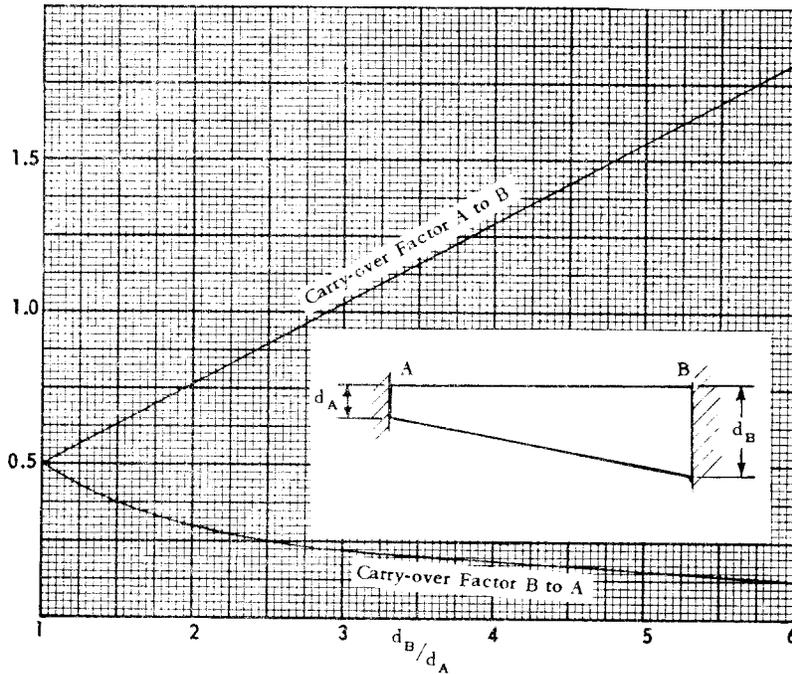


Figure 3.3.6-2

3.3.7 Fixed Beam of Varying I: Effect of Sinking Support.

The end moments resulting from a vertical deflection at one support equal to δ can be obtained by column analogy methods in the manner shown in the numerical example below.

NUMERICAL EXAMPLE:

To determine the end moments induced by a deflection $\delta = 1.0$ of support B relative to support A for the tapered beam shown in Fig. 3.3.7-1. The moment of inertia of the beam varies linearly from $I_A = 1.0$ at A to $I_B = 2.0$ at B.

$$A_{COL} = \frac{1.0 + 2.0}{2} \times 10 = 15$$

From Fig. 4.1.5-1, Section Properties of Trapezoid,

$$\text{for } a/b = .50, \quad \rho_x/d = .2838$$

$$\rho_x = 2.838 \quad \rho_x^2 = 8.0$$

$$I_{na} = A\rho_x^2 = 15 \times 8 = 120$$

$$I_{COL} = \frac{120}{E}$$

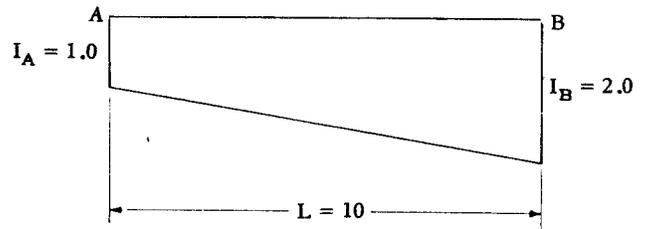
$$\bar{y}/d = .445 \quad \bar{y} = 4.45$$

For $\delta = 1.0$,

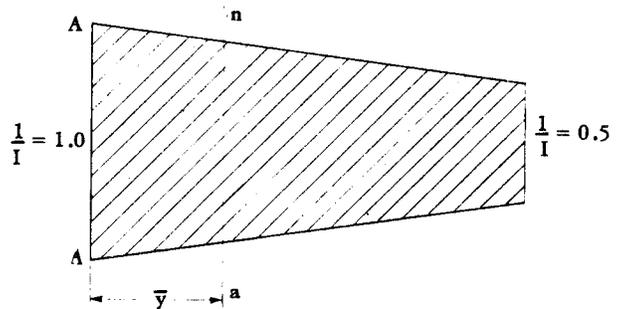
$$M_A = 1.0 \times 4.45 \frac{E}{60} = .0741 E$$

$$M_B = 1.0 \times 5.55 \frac{E}{60} = .0924 E$$

as explained in GEN/1090/805.5



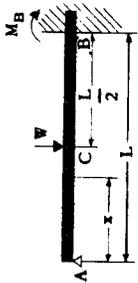
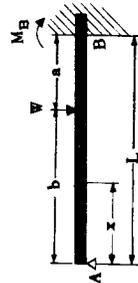
ORIGINAL BEAM



ANALOGOUS COLUMN

Figure 3.3.7-1



PROPPED CANTILEVER	REACTIONS R_A AND R_B (LB.) [POSITIVE UPWARD]	BENDING MOMENT M (LB. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(1)</p> <p>CENTRE LOAD</p> 	$R_A = \frac{5}{16} W$ $R_B = \frac{11}{16} W$	$M_A = 0$ $M_B = -\frac{3}{16} WL$ <p>(A to C) $M = \frac{5}{16} Wx$</p> <p>(C to B) $M = W(\frac{L}{2} - \frac{11}{16} x)$</p> <p>Max. +M = $\frac{5}{32} WL$ at C</p>	<p>(A to C) $y = \frac{1}{96} \frac{W}{EI} (5x^3 - 3L^2x)$</p> <p>(C to B) $y = \frac{1}{96} \frac{W}{EI} [5x^3 - 16(x - \frac{L}{2})^3 - 3L^2x]$</p> <p>Max. $y = -0.00932 \frac{WL^3}{EI}$ at $x = 0.4472L$</p> <p>$\theta = -\frac{1}{32} \frac{WL^2}{EI}$ at A</p>
<p>(2)</p> <p>INTERMEDIATE LOAD</p> 	$R_A = \frac{W}{2} (\frac{3a^2L - a^3}{L^3})$ $R_B = W - R_A$	$M_A = 0$ $M_B = -\frac{W}{2} (\frac{a^3 + 2aL^2 - 3a^2L}{L^2})$ <p>(A to C) $M = R_A x$</p> <p>(C to B) $M = R_A x - W(x - L + a)$</p> <p>Max. +M = $R_A(L - a)$ at C</p> <p>Max. possible value is $M = 0.174WL$ when $a = 0.634L$</p>	<p>(A to C) $y = \frac{1}{6EI} [R_A(x^3 - 3L^2x) + 3Wa^2x]$</p> <p>(C to B) $y = \frac{1}{6EI} [R_A(x^3 - 3L^2x) + W[3a^2x - (x - b)^3]]$</p> <p>$\theta = \frac{1}{4} \frac{W a^3}{EI} (\frac{L}{a} - a^2)$ at A</p>
<p>(3)</p> <p>UNIFORM LOADING</p> 	$R_A = \frac{3}{8} W$ $R_B = \frac{5}{8} W$ $W = wL$	$M_A = 0$ $M_B = -\frac{WL}{8}$ $M = W(\frac{3}{8}x - \frac{1}{2} \frac{x^2}{L})$ <p>Max. +M = $\frac{9}{128} WL$ at $x = \frac{3}{8} L$</p>	$y = \frac{1}{48} \frac{W}{EIL} (3Lx^3 - 2x^4 - L^3x)$ <p>Max. $y = -0.0054 \frac{WL^3}{EI}$ at $x = 0.4215L$</p> <p>$\theta = -\frac{1}{48} \frac{WL^2}{EI}$ at A</p>

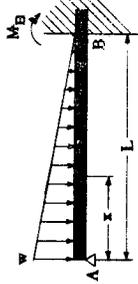
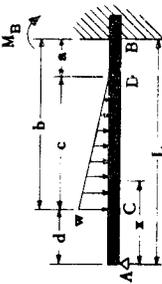
3.4.1 Propped Cantilevers of Constant I



PROPPED CANTILEVER	REACTIONS R_A AND R_B (L.B.) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(4)</p> <p style="text-align: center;">PARTIAL UNIFORM LOADING</p>	$R_A = \frac{W}{8L^3} [(a^2 + ab + b^2)(4L - a) - b^3]$ $R_B = W - R_A$ $W = wc$	$M_A = 0$ $M_B = R_A L - \frac{1}{2} W(a + b)$ <p style="text-align: center;">(A to C) $M = R_A x$</p> <p style="text-align: center;">(C to D) $M = R_A x - \frac{W(x-d)^2}{2c}$</p> <p style="text-align: center;">(D to B) $M = R_A x - W(x-d - \frac{c}{2})$</p> $\text{Max. } + M = R_A(d + \frac{1}{2} \frac{R_A}{W} c)$ $\text{at } x = (d + \frac{R_A}{W} c)$	<p>For y and θ, see Roark, p. 104, Case 24.</p>
<p>(5)</p> <p style="text-align: center;">TRIANGULAR LOADING</p>	$R_A = \frac{1}{5} W$ $R_B = \frac{4}{5} W$ $W = \frac{1}{2} wL$	$M_A = 0$ $M_B = -\frac{2}{15} WL$ $M = W(\frac{1}{5}x - \frac{1}{3}L^2)$ $\text{Max. } + M = 0.06 WL$ $\text{at } x = 0.4474 L$	$y = -\frac{Wx}{60EI L^2} (L^2 - x^2)^2$ $\text{Max. } y = -0.00477 \frac{WL^3}{EI}$ $\text{at } x = L\sqrt{\frac{1}{5}}$ $\theta = -\frac{1}{60} \frac{WL^2}{EI} \text{ at } A$
<p>(6)</p> <p style="text-align: center;">PARTIAL TRIANGULAR LOADING</p>	$R_A = \frac{W}{20L^3} [(3a^2 + 2ab + b^2)(5L - b) - 4a^3]$ $R_B = W - R_A$ $W = \frac{1}{2} wc$	$M_A = 0$ $M_B = R_A L - \frac{W}{3} (2a + b)$ <p style="text-align: center;">(A to C) $M = R_A x$</p> <p style="text-align: center;">(C to D) $M = R_A x - \frac{W(x-d)^3}{c^2}$</p> <p style="text-align: center;">(D to B) $M = R_A x - W(x-d - \frac{2}{3}c)$</p> $\text{Max. } + M = R_A(d + \frac{2}{3}c) \sqrt{\frac{R_A}{W}}$ $\text{at } x = d + c \sqrt{\frac{R_A}{W}}$	<p>For y and θ, see Roark p. 104, Case 26.</p>

3.4.1 Propped Cantilevers of Constant I (Cont'd)



PROPPED CANTILEVER	REACTIONS R_A AND R_B (LB) [POSITIVE UPWARD]	BENDING MOMENT M (L.B. IN.) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION y (IN.) [POSITIVE UPWARD] AND SLOPE θ (RADIAN)
<p>(7)</p> <p>TRIANGULAR LOADING</p> 	$R_A = \frac{11}{20} W$ $R_B = \frac{9}{20} W$ $W = \frac{1}{2} wL$	$M_A = 0$ $M_B = -\frac{7}{60} WL$ $M = W\left(\frac{11}{20}x - \frac{x^2}{L} + \frac{1}{3}\frac{x^3}{L^2}\right)$ <p>Max. $M = 0.0846 WL$ at $x = 0.329 L$</p>	$y = \frac{1}{120} \frac{W}{EI} (11Lx^3 - 3L^3x - 10x^4 + \frac{2x^5}{L})$ <p>Max. $y = -0.00609 \frac{WL^3}{EI}$ at $x = 0.402 L$</p> $\theta = -\frac{1}{40} \frac{WL^2}{EI}$ at A
<p>(8)</p> <p>PARTIAL TRIANGULAR LOADING</p> 	$R_A = \frac{W}{20L^3} [(a^2 + 2ab + 3b^2)(5L - a) - 4b^3]$ $R_B = W - R_A$ $W = \frac{1}{2} wc$	$M_A = 0$ $M_B = R_A L - \frac{W}{3}(a + 2b)$ <p>(A to C) $M = R_A x$</p> <p>(C to D) $M = R_A x - \frac{(x-d)^2}{c} W + \frac{(x-d)^3}{3c^2} W$</p> <p>(D to B) $M = R_A x - W(x-d - \frac{c}{3})$</p>	<p>For y and θ, see Roark p. 105, Case 28.</p>
<p>(9)</p> <p>END COUPLE</p> 	$R_A = -\frac{3}{2} \frac{M_0}{L}$ $R_B = \frac{3}{2} \frac{M_0}{L}$	$M_A = M_0$ $M_B = -\frac{M_0}{2}$ $M = \frac{1}{2} M_0 (2 - \frac{x}{L})$ <p>Max. $M = M_0$ at A</p>	$y = -\frac{1}{4} \frac{M_0}{EI} \frac{x}{L} (L - x)^2$ <p>Max. $y = -\frac{1}{27} \frac{M_0 L^2}{EI}$ at $x = \frac{L}{3}$</p> $\theta = -\frac{1}{4} \frac{M_0 L}{EI}$ at A

3.4.1 Propped Cantilevers of Constant I (Cont'd)



3.4.2 Propped Cantilever of Constant I: Irregular loading and prop at intermediate position.

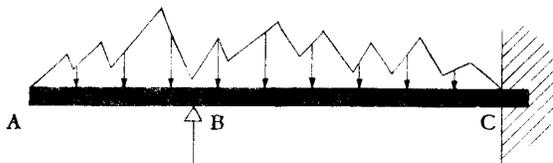


Figure 3.4.2-1

The general case of a propped cantilever with the prop at some intermediate position, such as B in Fig. 3.4.2-1, carrying an irregular load can be solved as follows:-

- (a) Find the bending moment at the support B due to the overhung portion AB.
- (b) Find the fixed end moments for span BC, by the method of 3.3.2 for irregular load.
- (c) Balance Joint B by moment distribution methods.

NUMERICAL EXAMPLE:

By way of illustration, take the fairly straightforward case of Fig. 3.4.2-2 wherein the load varies linearly from zero at the tip A to 30 lb. per in. at the root C.

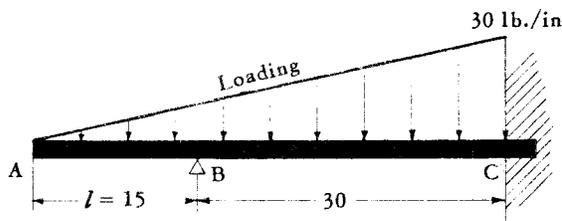


Figure 3.4.2-2

(a) B.M. at B due to overhang = $\frac{wl}{3} = \frac{1}{2} 15 \cdot 10 \times 5 = +375$ lb. in. (Section 3.2, Case 5).

(b) Fixed end moments on span BC assuming that it is an isolated fixed beam are, from Section 3.3.1, Case 11,

$$M_B = \frac{L^2}{60} [5w_A + 2(w_B - w_A)]$$

$$= -15(50 + 40) = -1350 \text{ lb. in.}$$

$$M_C = \frac{L^2}{60} [5w_A + 3(w_B - w_A)]$$

$$= 15(50 + 60) = +1650 \text{ lb. in.}$$

(c) Moment Distribution	B	C
Initial moments	+ 375	- 1350 + 1650
Balance Joint B	+	+ 975 + 488
Final moments	+ 375	- 375 + 2138

The B.M. diagram is shown in Fig. 3.4.2-3.

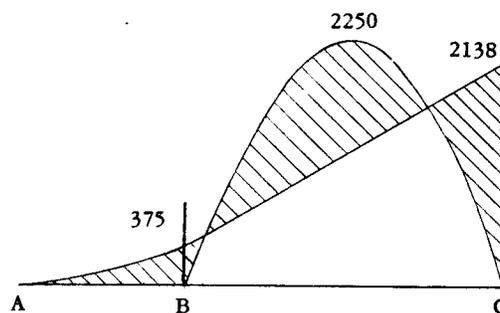


Figure 3.4.2-3

3.4.3 Propped Cantilever of Varying I: Irregular loading and prop at intermediate position.

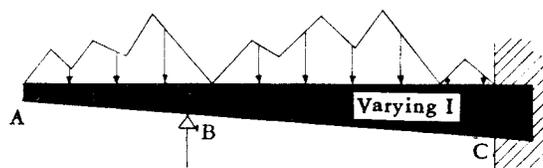


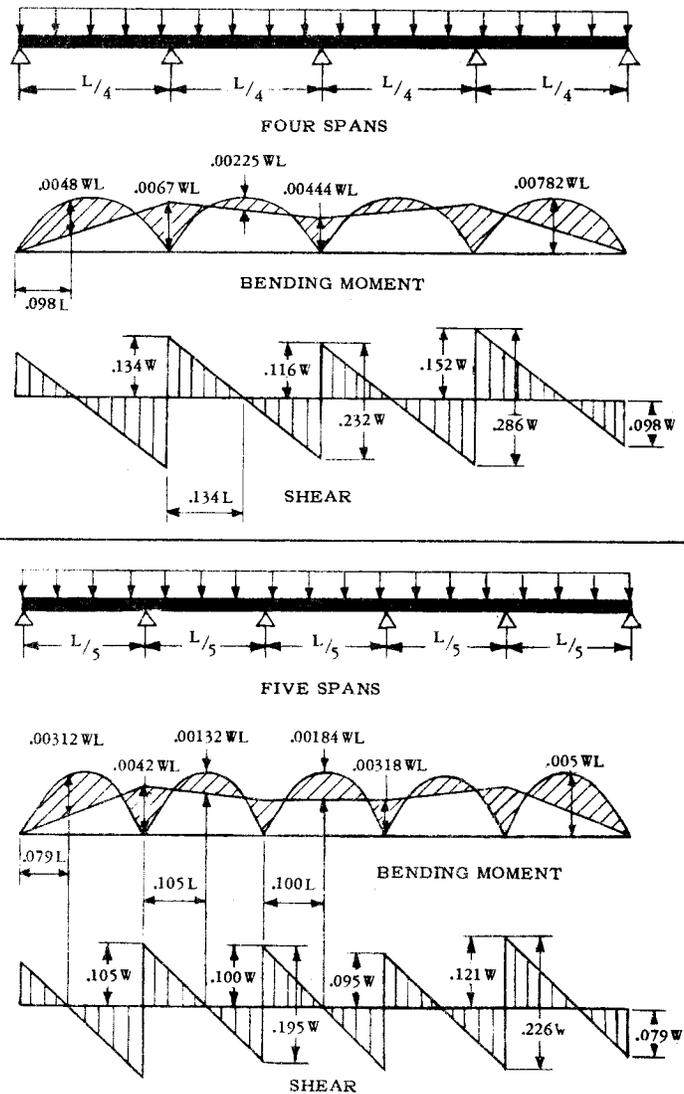
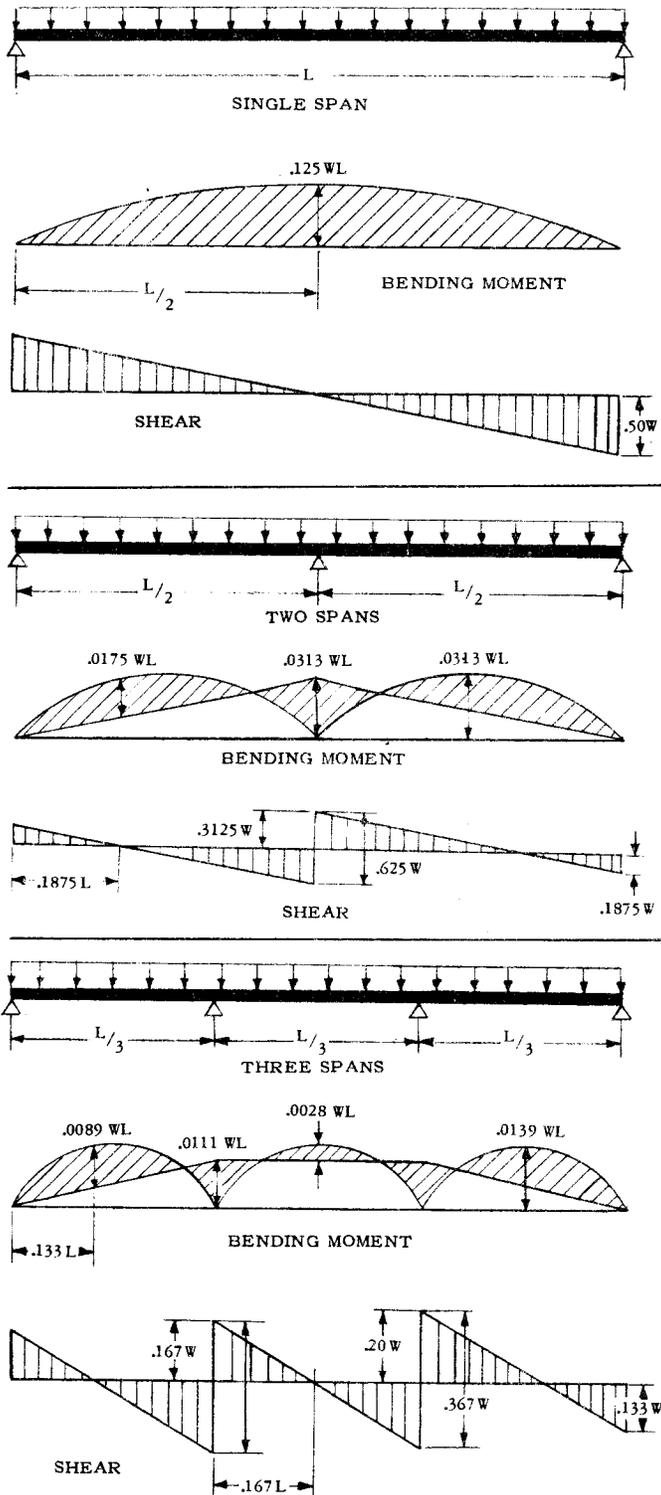
Figure 3.4.3-1

The method is basically the same as that of 3.4.2 and is as follows:-

- (a) Find the bending moment at support B due to the overhung portion AB.
- (b) Find the fixed end moments, stiffness and carry-over for span BC as an isolated fixed beam by the methods of 3.3.5, 3.3.6 and 3.3.7.
- (c) Balance Joint B by moment distribution methods.



3.5.1 Continuous Beam: I Constant throughout. Tabular Summary for Uniform Loading on Beams up to 5 Spans.



W = total load on beam
 L = overall span

Figure 3.5.1-1



3.5.2 Continuous Beam: 1 Constant Throughout. Design chart for uniform load on 2 spans.
(Simply supported at ends)

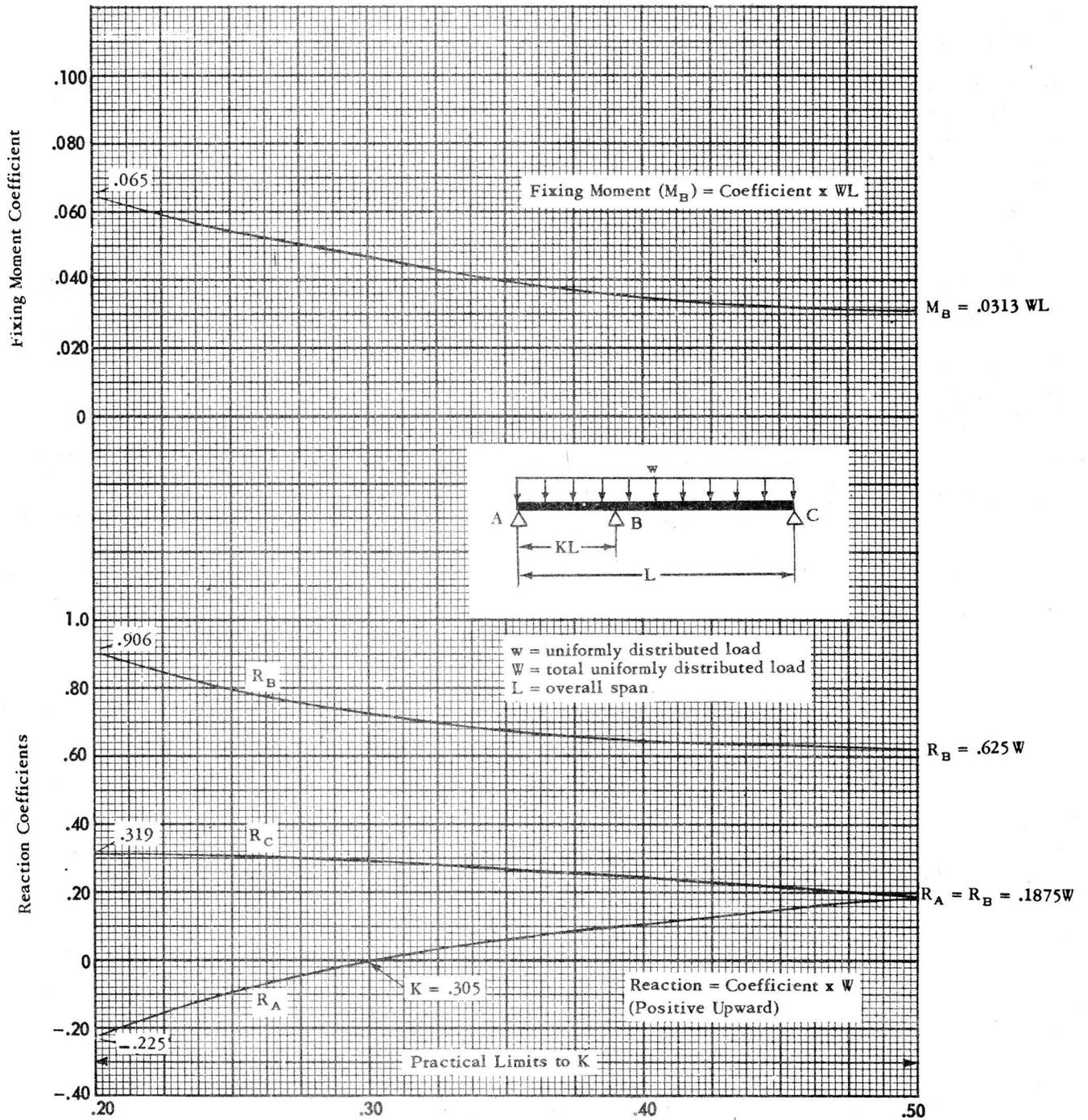


Figure 3.5.2-1



**3.5.3 Continuous Beam: I Constant throughout.
 Tabular Summary for Various Loadings by Moment
 Distribution.**

The moment distribution (Hardy Cross) method for solving continuous beam problems is a rapid and essentially practical method wherein each span is first treated as an independent fixed beam and the resulting end moments determined.

These end moments are easy to find for a beam of constant I, because they are governed entirely by span and loading and so usually resolve themselves into one or more of the standard cases given in 3.3.1.

Any resulting out-of-balance at the supports is overcome by applying an equal and opposite releasing moment and distributing this between adjacent spans with due regard to their stiffness.

The following rules must be followed:-

RULE 1 - Carry-over: For a beam AB of constant I the carry-over moment to B when A is released is $\frac{1}{2}$ the releasing moment at A. That is, the carry-over factor is $\frac{1}{2}$ (See Report GEN/1090/804).

RULE 2 - Stiffness: When considering the relative stiffness of two adjacent spans AB and BC of constant I and span L_1 and L_2 respectively, the following conditions apply. (See Report GEN/1090/804).

CONDITION	STIFFNESS	
	of Span AB = C_1	of Span BC = C_2
A and C both fixed	$4 \frac{EI}{L_1}$	$4 \frac{EI}{L_2}$
A and C both pinned	$3 \frac{EI}{L_1}$	$3 \frac{EI}{L_2}$
A fixed, C pinned	$4 \frac{EI}{L_1}$	$3 \frac{EI}{L_2}$
A pinned, C fixed	$3 \frac{EI}{L_1}$	$4 \frac{EI}{L_2}$

NOTE

For convenience when E and I are constant it is customary to consider the stiffness in terms of $\frac{1}{L}$ or $\frac{I}{L}$

Stiffness Ratio:

$$\text{The stiffness ratio of span AB} = \frac{C_1}{C_1 + C_2}$$

$$\text{The stiffness ratio of span BC} = \frac{C_2}{C_1 + C_2}$$



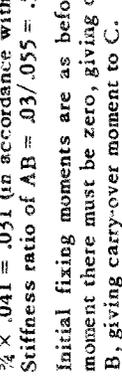
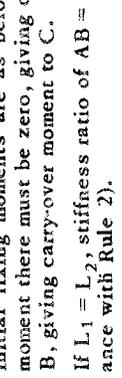
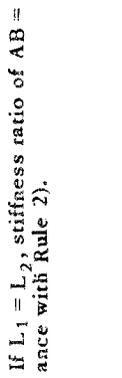
E and I Assumed Constant Throughout - Clockwise Moments are Positive		COMMENTS																	
CONTINUOUS BEAM																			
<p>(1)</p> <p>2 SPANS, FREELY SUPPORTED AT A, B AND C, DISTRIBUTED LOAD ON AB AND BC.</p> <p style="text-align: center;">$w = 10 \text{ lb. per in.}$</p> <p style="text-align: center;">$L_1 = 25 \quad L_2 = 40$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">A</td> <td style="text-align: center;">B</td> <td style="text-align: center;">C</td> </tr> <tr> <td style="text-align: center;">-521</td> <td style="text-align: center;">-385</td> <td style="text-align: center;">+1333</td> </tr> <tr> <td style="text-align: center;">+521 →</td> <td style="text-align: center;">← 666</td> <td style="text-align: center;">← 1333</td> </tr> <tr> <td style="text-align: center;">+781</td> <td style="text-align: center;">+1999</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+749</td> <td style="text-align: center;">+469</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+1530</td> <td style="text-align: center;">-1530</td> <td style="text-align: center;">0</td> </tr> </table> <p>Stiffness ratio Fixing moments Release A and C Nett moments Distribution Final moments</p>	A	B	C	-521	-385	+1333	+521 →	← 666	← 1333	+781	+1999	0	+749	+469	0	+1530	-1530	0	<p>End moments on AB as a fixed beam = $\pm \frac{wL_1^2}{12} = \pm \frac{10 \times 625}{12} = \pm 521$.</p> <p>Corresponding end moments on BC are ± 1330. Stiffness of AB = $I/25 = .04I$ and of BC = $.025I$. Stiffness ratio of AB = $.04/.065 = .615$ and of BC = $.385$. Since beam is freely supported at A and C, final moments there must be zero. Hence release A and C by applying equal and opposite cancelling moments. This gives carry-over moments - as arrowed - to each side of B. Nett out of balance at B, viz., -1999 + 781 = -1218, is divided between AB and BC in proportion to their stiffness ratios. If $L_1 = L_2$, the stiffness ratio of each span = $.50$</p>
A	B	C																	
-521	-385	+1333																	
+521 →	← 666	← 1333																	
+781	+1999	0																	
+749	+469	0																	
+1530	-1530	0																	
<p>(2)</p> <p>2 SPANS, FREELY SUPPORTED AT A, B AND C, DISTRIBUTED LOAD ON AB, CONCENTRATED LOAD ON BC.</p> <p style="text-align: center;">$w = 10 \text{ lb. per in.} \quad W = 200 \text{ lb.}$</p> <p style="text-align: center;">$L_1 = 25 \quad L_2 = 40$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">A</td> <td style="text-align: center;">B</td> <td style="text-align: center;">C</td> </tr> <tr> <td style="text-align: center;">-521</td> <td style="text-align: center;">-385</td> <td style="text-align: center;">+1176</td> </tr> <tr> <td style="text-align: center;">+521 →</td> <td style="text-align: center;">← 588</td> <td style="text-align: center;">← 1176</td> </tr> <tr> <td style="text-align: center;">+781</td> <td style="text-align: center;">+1092</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+191</td> <td style="text-align: center;">+120</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+972</td> <td style="text-align: center;">-972</td> <td style="text-align: center;">0</td> </tr> </table> <p>Stiffness ratio Fixing moments Release A and C Nett moments Distribution Final moments</p>	A	B	C	-521	-385	+1176	+521 →	← 588	← 1176	+781	+1092	0	+191	+120	0	+972	-972	0	<p>The procedure is exactly the same as that in Case (1). End moments on AB are ± 521 and on BC are $M_{BC} = \frac{W}{L^2} ab^2 = \frac{200}{1600} \times 28 \times 144 = -504$, $M_{CB} = \frac{W}{L^2} a^2 b = \frac{1}{8} 784 \times 12 = +1176$. Stiffness values are as before.</p>
A	B	C																	
-521	-385	+1176																	
+521 →	← 588	← 1176																	
+781	+1092	0																	
+191	+120	0																	
+972	-972	0																	
<p>(3)</p> <p>2 SPANS, FIXED AT A AND C, DISTRIBUTED LOAD ON AB, CONCENTRATED LOAD ON BC.</p> <p style="text-align: center;">$w = 10 \text{ lb. per in.} \quad W = 200 \text{ lb.}$</p> <p style="text-align: center;">$L_1 = 25 \quad L_2 = 40$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">A</td> <td style="text-align: center;">B</td> <td style="text-align: center;">C</td> </tr> <tr> <td style="text-align: center;">-521</td> <td style="text-align: center;">-385</td> <td style="text-align: center;">+1176</td> </tr> <tr> <td style="text-align: center;">+521 →</td> <td style="text-align: center;">← 588</td> <td style="text-align: center;">← 1176</td> </tr> <tr> <td style="text-align: center;">+781</td> <td style="text-align: center;">+1092</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+191</td> <td style="text-align: center;">+120</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">+972</td> <td style="text-align: center;">-972</td> <td style="text-align: center;">0</td> </tr> </table> <p>Stiffness ratio Fixing moments Release B Final moments</p>	A	B	C	-521	-385	+1176	+521 →	← 588	← 1176	+781	+1092	0	+191	+120	0	+972	-972	0	<p>Initial end moments and stiffness ratios will be as for Case (2). Since beam is freely supported at B the final moments on each side of it must be equal but opposite. Therefore apply there a releasing moment equal and opposite to the out-of-balance moment, viz., +521 - 504 = +17 and divide this between AB and BC in proportion to their stiffness ratios. There are carry-over values to A and C, as arrowed. If $L_1 = L_2$, the stiffness ratio of each span = $.50$</p>
A	B	C																	
-521	-385	+1176																	
+521 →	← 588	← 1176																	
+781	+1092	0																	
+191	+120	0																	
+972	-972	0																	



E and I Assumed Constant Throughout - Clockwise Moments are Positive

CONTINUOUS BEAM

COMMENTS

(4)	<p>2 SPANS, FREELY SUPPORTED AT A AND B, FIXED AT C. DISTRIBUTED LOAD ON AB, CONCENTRATED LOAD ON BC.</p> <p>$w = 10 \text{ lb. per in.}$ $W = 200 \text{ lb.}$</p>  <p>$L_1 = 25$ $L_2 = 40$</p>	<p>Since AB is freely supported but BC is fixed at C, stiffness of AB = $\frac{3}{4} \times .041 = .031$ (in accordance with Rule 2) and of BC = .025I, as before. Stiffness ratio of AB = .03/.055 = .546 and of BC = .454.</p> <p>Initial fixing moments are as before. First release A, since the final moment there must be zero, giving carry-over moment to B. Then release B, giving carry-over moment to C.</p> <p>If $L_1 = L_2$, stiffness ratio of AB = .429 and of BC = .571 (still in accordance with Rule 2).</p>																																				
	<table border="1" data-bbox="267 714 389 1092"> <tr><td>A</td><td></td><td>B</td><td>C</td></tr> <tr><td>Stiffness ratio</td><td></td><td>.546</td><td>.454</td></tr> <tr><td>Down load moments</td><td>- 521</td><td>+ 521</td><td>- 504</td></tr> <tr><td>Up load moments</td><td>+ 521</td><td>- 260</td><td>+ 1176</td></tr> <tr><td>Nett moments</td><td>+ 326</td><td>+ 781</td><td>- 504</td></tr> <tr><td>Release A</td><td>- 326</td><td></td><td></td></tr> <tr><td>Release B</td><td></td><td>- 151</td><td>- 126</td></tr> <tr><td>Release C</td><td></td><td></td><td>- 63</td></tr> <tr><td>Final moments</td><td>0</td><td>+ 630</td><td>+ 1113</td></tr> </table>	A		B	C	Stiffness ratio		.546	.454	Down load moments	- 521	+ 521	- 504	Up load moments	+ 521	- 260	+ 1176	Nett moments	+ 326	+ 781	- 504	Release A	- 326			Release B		- 151	- 126	Release C			- 63	Final moments	0	+ 630	+ 1113	
A		B	C																																			
Stiffness ratio		.546	.454																																			
Down load moments	- 521	+ 521	- 504																																			
Up load moments	+ 521	- 260	+ 1176																																			
Nett moments	+ 326	+ 781	- 504																																			
Release A	- 326																																					
Release B		- 151	- 126																																			
Release C			- 63																																			
Final moments	0	+ 630	+ 1113																																			
(5)	<p>2 SPANS, WITH OVERHANG AT A. FREELY SUPPORTED AT A AND B, FIXED AT C. DISTRIBUTED LOAD ON AB, CONCENTRATED LOAD ON BC.</p> <p>$w = 10 \text{ lb. per in.}$ $W = 200 \text{ lb.}$</p>  <p>$L_1 = 10$ $L_2 = 25$ $L_3 = 40$</p>	<p>Stiffness ratios and initial fixing moments on AB and BC are as for Case (4). Fixing moment to left of A due to overhang = $\frac{1}{2}wl^2 = \frac{1}{2} \times 10 \times 100 = +500 \text{ lb. in.}$</p> <p>Release A by adding to M_{AB} a moment of +21; this gives a carry-over of +10.5 to B.</p> <p>Release B and add carry-over to C.</p> <p>If $L_1 = L_2$ stiffness ratio of AB = .429 and of BC = .571.</p>																																				
	<table border="1" data-bbox="332 714 454 1092"> <tr><td>A</td><td></td><td>B</td><td>C</td></tr> <tr><td>Stiffness ratio</td><td></td><td>.546</td><td>.454</td></tr> <tr><td>Down load moments</td><td>- 521</td><td>+ 521</td><td>- 504</td></tr> <tr><td>Up load moments</td><td>+ 21</td><td>+ 10.5</td><td>+ 1176</td></tr> <tr><td>Nett moments</td><td>+ 531.5</td><td>- 504</td><td></td></tr> <tr><td>Release A</td><td>- 15</td><td>- 12.5</td><td>- 6.3</td></tr> <tr><td>Release B</td><td></td><td>+ 516.5</td><td>- 516.5</td></tr> <tr><td>Release C</td><td></td><td></td><td>+ 1170</td></tr> <tr><td>Final moments</td><td>+ 500</td><td>+ 516.5</td><td>+ 1170</td></tr> </table>	A		B	C	Stiffness ratio		.546	.454	Down load moments	- 521	+ 521	- 504	Up load moments	+ 21	+ 10.5	+ 1176	Nett moments	+ 531.5	- 504		Release A	- 15	- 12.5	- 6.3	Release B		+ 516.5	- 516.5	Release C			+ 1170	Final moments	+ 500	+ 516.5	+ 1170	
A		B	C																																			
Stiffness ratio		.546	.454																																			
Down load moments	- 521	+ 521	- 504																																			
Up load moments	+ 21	+ 10.5	+ 1176																																			
Nett moments	+ 531.5	- 504																																				
Release A	- 15	- 12.5	- 6.3																																			
Release B		+ 516.5	- 516.5																																			
Release C			+ 1170																																			
Final moments	+ 500	+ 516.5	+ 1170																																			
(6)	<p>2 SPANS, WITH OVERHANG AT C. FREELY SUPPORTED AT A, B AND C. IRREGULAR LOAD ON AB, DISTRIBUTED LOAD ON BC.</p> <p>$w = 10 \text{ lb per in.}$</p>  <p>Loading 5 lb. per in.</p>	<p>Initial fixing moments on AB due to up load are $M_{AB} = 384$ and $M_{BA} = -116$; due to down load are $M_{AB} = -58$ and $M_{BA} = +192$. Initial end moments on BC are ± 667; fixing moment to right of C due to overhang = -1000. Stiffness ratio = $\frac{1}{2}$ since A, B and C are all simply supported.</p> <p>After finding the nett initial fixing moments, the procedure is similar to that in earlier examples.</p>																																				
	<table border="1" data-bbox="397 714 519 1092"> <tr><td>A</td><td></td><td>B</td><td>C</td></tr> <tr><td>Stiffness ratio</td><td></td><td>.50</td><td>.50</td></tr> <tr><td>Down load moments</td><td>- 58</td><td>+ 192</td><td>- 667</td></tr> <tr><td>Up load moments</td><td>+ 384</td><td>- 116</td><td>+ 667</td></tr> <tr><td>Nett moments</td><td>+ 326</td><td>+ 76</td><td>+ 667</td></tr> <tr><td>Release A</td><td>- 326</td><td></td><td></td></tr> <tr><td>Release B</td><td></td><td>+ 167</td><td>+ 333</td></tr> <tr><td>Release C</td><td></td><td>+ 294</td><td>+ 294</td></tr> <tr><td>Final moments</td><td>0</td><td>+ 207</td><td>+ 1000</td></tr> </table>	A		B	C	Stiffness ratio		.50	.50	Down load moments	- 58	+ 192	- 667	Up load moments	+ 384	- 116	+ 667	Nett moments	+ 326	+ 76	+ 667	Release A	- 326			Release B		+ 167	+ 333	Release C		+ 294	+ 294	Final moments	0	+ 207	+ 1000	
A		B	C																																			
Stiffness ratio		.50	.50																																			
Down load moments	- 58	+ 192	- 667																																			
Up load moments	+ 384	- 116	+ 667																																			
Nett moments	+ 326	+ 76	+ 667																																			
Release A	- 326																																					
Release B		+ 167	+ 333																																			
Release C		+ 294	+ 294																																			
Final moments	0	+ 207	+ 1000																																			

3.5.3 Continuous Beam: Tabular Summary (Cont'd)



E and I Assumed Constant Throughout -- Clockwise Moments are Positive		COMMENTS																																																																											
(7)	<p style="text-align: center;">CONTINUOUS BEAM</p> <p style="text-align: center;">$W = 200 \text{ lb.}$</p> <p style="text-align: center;">10 lb. per in. 10 lb. per in.</p> <p style="text-align: center;">A B C D</p> <p style="text-align: center;">25 12 40</p>	<p>Stiffness ratios AB : BC and CD : BC will be .615 : .385 as earlier. First release B, giving carry-over to A and C : then release C, giving carry-overs to D and B, which will now be unbalanced again. Continue to release B and C until the resultant out of balance moment is so small as to be negligible. If the spans are all equal, the stiffness ratios = .50.</p>																																																																											
	<table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> </tr> </thead> <tbody> <tr> <td>Stiffness ratio</td> <td>.615</td> <td>.385</td> <td>.385</td> <td>.615</td> </tr> <tr> <td>Fixing moments</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>Release B</td> <td>- 521</td> <td>+ 521</td> <td>+ 1176</td> <td>- 521</td> </tr> <tr> <td></td> <td>- 5.2 ←</td> <td>10.4</td> <td>6.6 →</td> <td>3.3</td> </tr> <tr> <td>Release C</td> <td>- 526.2</td> <td>+ 510.6</td> <td>+ 1172.7</td> <td>- 521</td> </tr> <tr> <td></td> <td></td> <td>- 125 ←</td> <td>249.7</td> <td>- 402 →</td> </tr> <tr> <td>Release B</td> <td>- 526.2</td> <td>+ 510.6</td> <td>+ 923</td> <td>- 923</td> </tr> <tr> <td></td> <td>+ 38.4 ←</td> <td>76.8</td> <td>48.2 →</td> <td>24.1</td> </tr> <tr> <td>Release C</td> <td>- 487.8</td> <td>+ 587.4</td> <td>+ 947.1</td> <td>- 923</td> </tr> <tr> <td></td> <td></td> <td>- 4.7 ←</td> <td>9.3</td> <td>14.8 →</td> </tr> <tr> <td>Release B</td> <td>- 487.8</td> <td>+ 587.4</td> <td>+ 937.8</td> <td>- 937.8</td> </tr> <tr> <td></td> <td>+ 1.4 ←</td> <td>2.9</td> <td>1.8 →</td> <td>.9</td> </tr> <tr> <td>Final moments</td> <td>- 486.4</td> <td>+ 590.3</td> <td>+ 938.7</td> <td>- 937.8</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>+ 312.6</td> </tr> </tbody> </table>		A	B	C	D	Stiffness ratio	.615	.385	.385	.615	Fixing moments					Release B	- 521	+ 521	+ 1176	- 521		- 5.2 ←	10.4	6.6 →	3.3	Release C	- 526.2	+ 510.6	+ 1172.7	- 521			- 125 ←	249.7	- 402 →	Release B	- 526.2	+ 510.6	+ 923	- 923		+ 38.4 ←	76.8	48.2 →	24.1	Release C	- 487.8	+ 587.4	+ 947.1	- 923			- 4.7 ←	9.3	14.8 →	Release B	- 487.8	+ 587.4	+ 937.8	- 937.8		+ 1.4 ←	2.9	1.8 →	.9	Final moments	- 486.4	+ 590.3	+ 938.7	- 937.8					+ 312.6	
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3.5.4 Continuous Beams: I Constant Per Span.
 Numerical example of 4 span beam.

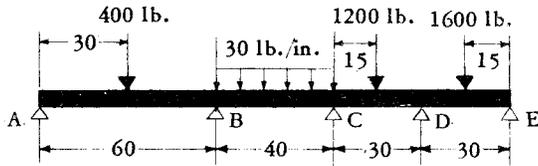


Figure 3.5.4-1

MEMBER	STIFFNESS	STIFFNESS RATIO
AB	$\frac{3}{4} \frac{I}{L} = .75 \times \frac{60}{60} = .75$	$\frac{.75}{2.25} = .33$
BC	$\frac{I}{L} = \frac{60}{40} = 1.5$	
CD	$\frac{I}{L} = \frac{45}{30} = 1.5$	$\frac{1.5}{3.0} = .50$
DE	$\frac{3}{4} \frac{I}{L} = .75 \times \frac{45}{30} = 1.125$	
		$\frac{1.5}{2.625} = .571$

$I_{AB} = I_{BC} = 60 \text{ in.}^4$
 $I_{CD} = I_{DE} = 45 \text{ in.}^4$

E constant

	A	B	C	D	E			
Stiffness ratio		.33	.67	.50	.50	.571	.429	
Initial moments	- 3000	+ 3000	- 4000	+ 4000	- 4500	+ 4500	- 6000	+ 6000
Release A & E	+ 3000 →	+ 1500					- 3000	← - 6000
Release B		+ 4500	- 4000	+ 4000	- 4500	+ 4500	- 9000	
Release D		- 167	- 334 →	- 167				
Release C		+ 4333	- 4334	+ 3833	- 4500	+ 4500	- 9000	
Release B				+ 3833	+ 1285 ←	+ 2570	+ 1930	
Release D					- 3215	+ 7070	- 7070	
Release C			- 154 ←	← - 309	- 309 →	- 154		
Release B		+ 4333	- 4488	+ 3524	- 3524	+ 6916	- 7070	
Release D		+ 51	+ 102 →	+ 51				
Release C		+ 4384	- 4384	+ 3575	- 3524	+ 6916	- 7070	
Release B				+ 3575	+ 44 ←	← + 88	+ 66	
Release D					- 3480	+ 7004	- 7004	
Release C			- 24 ←	← - 48	- 48 →	- 24		
Release B		+ 4384	- 4408	+ 3527	- 3528	+ 6980	- 7004	
Release D		+ 8	+ 16 →	+ 8				
Release C		+ 4392	- 4392	+ 3535	- 3528			
Release B				+ 3535	+ 7 ←	← + 14	+ 10	
Release D					- 3521	+ 6994	- 6994	
Release C			- 2 ←	← - 3.5	- 3.5 →	- 2		
Final moments	0	+ 4392	- 4394	+ 3531	- 3535	+ 6992	- 6994	0

Note that A and E released first and kept released throughout.

Further accuracy in the final moments will be achieved by continuing with the distribution calculations.



3.5.5 Continuous Beams: I Constant Per Span. Numerical Example on Effect of Axial Load.

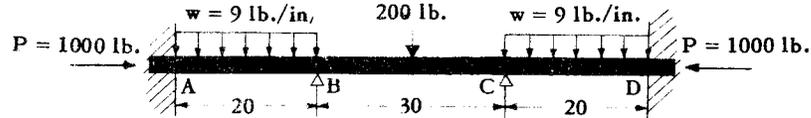


Figure 3.5.5-1

Given that $EI = 10^6$

	$j^2 = \frac{EI}{P}$	j	L	$\frac{L}{j}$	FIXED END MOMENT COEFFICIENT C	WL.	FIXED END MOMENT $\frac{WL}{C}$	CARRY- OVER FACTOR	STIFFNESS COEFFICIENT	ABSOLUTE STIFFNESS	STIFFNESS RATIO
AB	100	10	20	2.0	11.176	3600	322	.6263	.859	$\frac{.859}{20} \times 4EI = .171 EI$	$\frac{.171}{.2585} = .66$
BC	100	10	30	3.0	6.44	6000	931	.9189	.656	$\frac{.656}{30} \times 4EI = .0875 EI$	
CD	100	10	20	2.0	11.176	3600	322	.6263	.859	$= .171 EI$	$\frac{.0875}{.2585} = .34$

The fixed-end moments, stiffness and carry-over are obtained by the method of 3.3.3. Proceed now as for a normal moment distribution calculation. If the end load P in each bay is different, the procedure is basically the same, because the effect of P is automatically taken care of in finding $j^2 = \frac{EI}{P}$.

The above example is taken from "Aircraft Structures", Peery, p.552. There is an error in the value of his fixed-end moment coefficient for span BC: 6.9 should be 6.44.

This affects the moment distribution in Peery, Fig. 18.39, p. 553.



3.5.6 Continuous Beams: I Constant Per Span. Effect of Sinking Supports

To determine the moments induced when the supports sink, each joint of the *unloaded* beam is allowed to sink without angular rotation and the end moments found as though each span were an isolated fixed beam.

Moment distribution methods are then used for balancing.

NUMERICAL EXAMPLE:

The same beam as in 3.5.4. Take $E = 10^7$

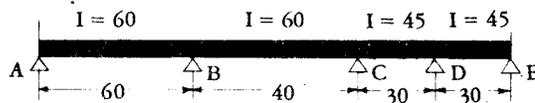


Figure 3.5.6-1

Let the supports sink as follows:-

A	B	C	D	E
.01"	.02"	.025"	.02"	.015"

MEMBER	AB	BC	CD	DE
NETT DEFLECTION δ	.01	.005	.005	.005
L	60	40	30	30
L^2	3600	1600	900	900
I	60	60	45	45
$\frac{6EI}{L^2}$	1.0×10^6	2.25×10^6	3.0×10^6	3.0×10^6
$M = \frac{6EI}{L^2} \delta$	-10,000	-11,250	+15,000	+15,000

Proceed now by ordinary moment distribution methods.

3.5.7 Continuous Beams: I Varying Per Span. General Notes

Treat each span as an isolated fixed beam and determine the end moments, stiffness and carry-over by the methods of 3.3.5, 3.3.6 and 3.3.7.

Proceed then by the ordinary technique of moment distribution.

3.6 Curved Beams: Circumferential Stresses

Ordinary bending theory cannot be used for a beam which is initially curved to such an extent that the radius of curvature is small compared with the dimensions of the beam.

In such a case the circumferential bending stress (for sections other than thin-walled) can be obtained by the method outlined below.

Consider the beam shown in Fig. 3.6-1 which has an initial curvature R measured to the centroidal axis.

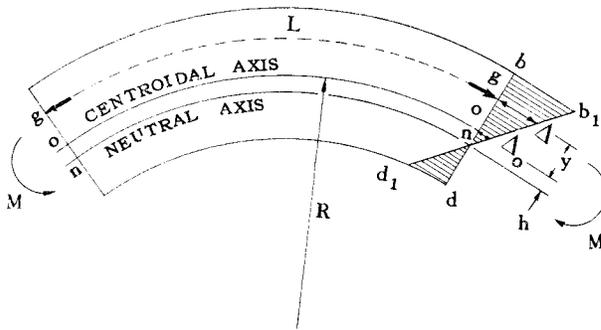


Fig. 3.6-1

Assume that a plane cross-section such as bnd remains plane after bending and, under the action of an applied bending moment M , tending to increase the curvature of the beam, moves to the position b_1nd_1 .

Note that in Fig. 3.6-1 the position of the neutral axis nn , that is, the position of zero bending stress, is not coincident with the centroidal axis oo . Later it will be shown that this is because the stress distribution across the section is hyperbolic and not linear, as in ordinary beam theory.

For the time being, however, it will be necessary to accept the fact that n is below o in this particular case:

3.6.1 Development of Formulas -- Let a longitudinal fibre gg of the beam of original length L situated at a distance y above the centroidal axis undergo an extension Δ . Then, if L_o is the original length of a fibre on the centroidal axis,

$$\frac{L}{R+y} = \frac{L_o}{R}$$

$$\text{or } L = \frac{L_o}{R} (R+y) = K_1(R+y) \text{ say.}$$

Also, if Δ_o is the extension of a longitudinal fibre on the centroidal axis and h is the distance of the neutral axis below the centroidal axis,

$$\frac{\Delta}{h+y} = \frac{\Delta_o}{h}$$

$$\Delta = \Delta_o + \frac{\Delta_o}{h} y$$

$$= K_2 + K_3 y, \text{ say.}$$

The normal or circumferential bending stress f developed in fibre gg is clearly $E \times \text{strain}$ or

$$f = E \frac{\Delta}{L} = E \frac{(K_2 + K_3 y)}{K_1(R+y)}$$

By regrouping, $f = a + \frac{K}{R+y}$ Eqn. (1)

If dA (Fig. 3.6-2) is the area of an element of cross-section, $A = \sum dA$ is the total area of the section and it is clear that $\sum f dA$ is the total axial load P applied. That is, $P = \sum f dA$

$$= aA + K \sum \frac{dA}{R+y} \quad \text{Eqn. (2)}$$

In addition,

$$M = \sum f y dA$$

$$= a \sum y dA + K \sum \frac{y dA}{R+y}$$

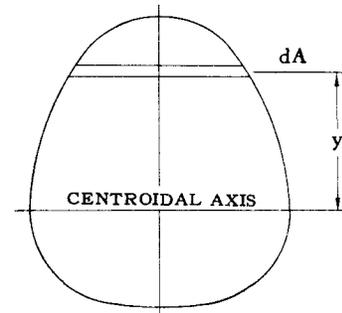


Fig. 3.6-2

But $\sum y dA = 0$ because it is the sum of the moments of all elements above the centroidal axis.

$$M = K \sum \frac{y dA}{R+y}$$

$$= K \sum \left(1 - \frac{R}{R+y}\right) dA$$

$$= K \left[A - R \sum \frac{dA}{R+y} \right] \quad \text{Eqn. (3)}$$



3.6.2 Procedure in Practice - When the section is such that the curves of 3.6.3 cannot be used, proceed as follows:-

- (1) For a given value of M calculate A and $\sum \frac{dA}{R+y}$ and hence K from Equation (3).
- (2) Find 'a' from Equation (2), assuming P = zero, as it is for pure bending.
- (3) Find the circumferential bending stress f from Equation (1) and hence
- (4) The position of the neutral axis given by y = h when f = 0.

3.6.3 Numerical Example - The method will be demonstrated by reference to a solid circular section of diameter d = 2.0 in. having an initial radius of curvature R = 3.0 in. measured to the centroidal axis, under the action of a bending moment M.

- (1) The integral $\sum \frac{dA}{R+y}$ can be re-written as:-

$$\sum_{-\frac{d}{2}}^{+\frac{d}{2}} \frac{2w dy}{R+y}$$

where w is the half width of an elemental strip of thickness dy (Fig. 3.6-3). That is dA = 2w dy.

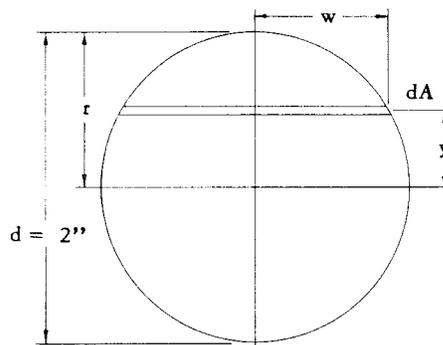


Fig. 3.6-3

The mathematical solution of this is:-

$$2\pi [R - \sqrt{R^2 - (\frac{d}{2})^2}]$$

which for R = 3.0, d = 2.0 and R/d = 1.5 gives

$$\sum \frac{dA}{R+y} = 1.079.$$

This result could have been obtained by graphical integration by tabulating values of 2w and y as in Tables 3.6-1 and 3.6-2. In fact, this procedure would have to be adopted in the general case of a section which did not lend itself to a mathematical approach.

TABLE 3.6-1

Clearly $w = \sqrt{r^2 - y^2}$

y	0	0.2	0.4	0.6	0.8	0.9	1.0
w	1.0	.98	.916	.80	.60	.435	0
2w	2.0	1.96	1.832	1.60	1.20	.87	0
R + y	3.0	3.2	3.4	3.6	3.8	3.9	4.0
$\frac{2w}{R+y}$.66	.612	.538	.444	.316	.223	0

TABLE 3.6-2

y	0	-0.2	-0.4	-0.6	-0.8	-0.9	-1.0
R + y	3.0	2.8	2.6	2.4	2.2	2.1	2.0
$\frac{2w}{R+y}$.66	.70	.704	.66	.545	.413	0



$$R \sum \frac{dA}{R+y} = 3.0 \times 1.079 = 3.237$$

$$[A - R \sum \frac{dA}{R+y}] = \pi - 3.237 = -0.0954$$

$$K = \frac{M}{-0.0954} = -10.48 M$$

(2) For $P = 0$,

$$aA = -K \sum \frac{dA}{R+y} = 10.48 M \times 1.079 = 11.31 M$$

$$a = \frac{11.31 M}{\pi} = 3.606 M$$

$$(3) f = a + \frac{K}{R+y} = M [3.606 - \frac{10.48}{3+y}]$$

When $y = +1.0$,
stress at extreme outer fibre $s_o = .985 M$

When $y = -1.0$,
stress at extreme inner fibre $s_i = -1.632 M$

$$s = \frac{M}{Z} = \frac{32M}{\pi d^3} = 1.272 M$$

$$\frac{s_o}{s} = \frac{.985}{1.272} = .773$$

$$\frac{s_i}{s} = \frac{1.632}{1.272} = 1.28$$

Check from Roark, p.144, Case 2.

$$\frac{R}{c} = \frac{3.0}{1.0} = 3.0 \quad (\text{In Roark's notation } c = \frac{d}{2})$$

$$K_o = \frac{s_o}{s} = 0.79$$

$$K_i = \frac{s_i}{s} = 1.33$$

$$(4) \frac{f}{M} = 3.606 - \frac{10.48}{(3+y)}$$

When $f = 0$, $y = h = -0.094$

CHECK. From Roark, p.144, Case 2, for $\frac{R}{c} = 3.0$,

$$\frac{h}{R} = .03, \quad h = .09$$

DISTRIBUTION OF BENDING STRESS

y	+1.0	+0.5	0	-.09	-0.5	-1.0
3+y	4.0	3.5	3.0	2.91	2.5	2.0
$\frac{10.48}{3+y}$	2.62	2.99	3.49	3.6	4.17	5.24
f/M	.985	.616	.12	0	-.56	-1.632

The distribution of bending stress is hyperbolic, as in Fig. 3.6-4.

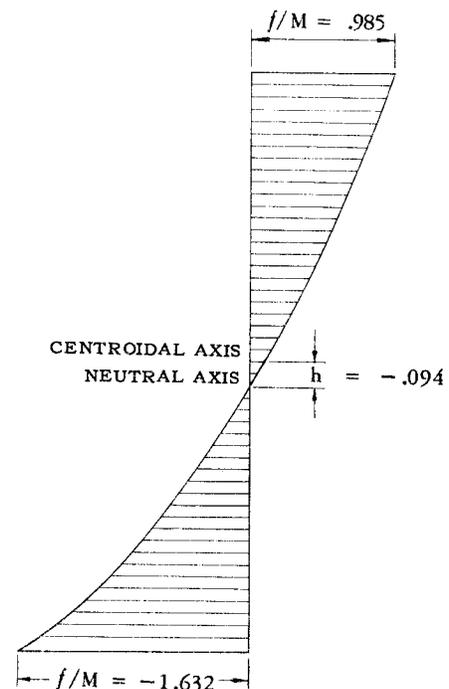


Fig. 3.6-4

3.6.4 Data Sheets – Curves for rectangular and solid circular sections appear in Fig. 3.6-5 and I, T and trapezoidal sections in Fig. 3.6-6.

When there is an applied axial load, in addition to the bending moment, the effect of the resulting direct stress and the bending moment due to the offset of this axial load from the neutral axis – because of its non-coincidence with the centroidal axis – must be included.

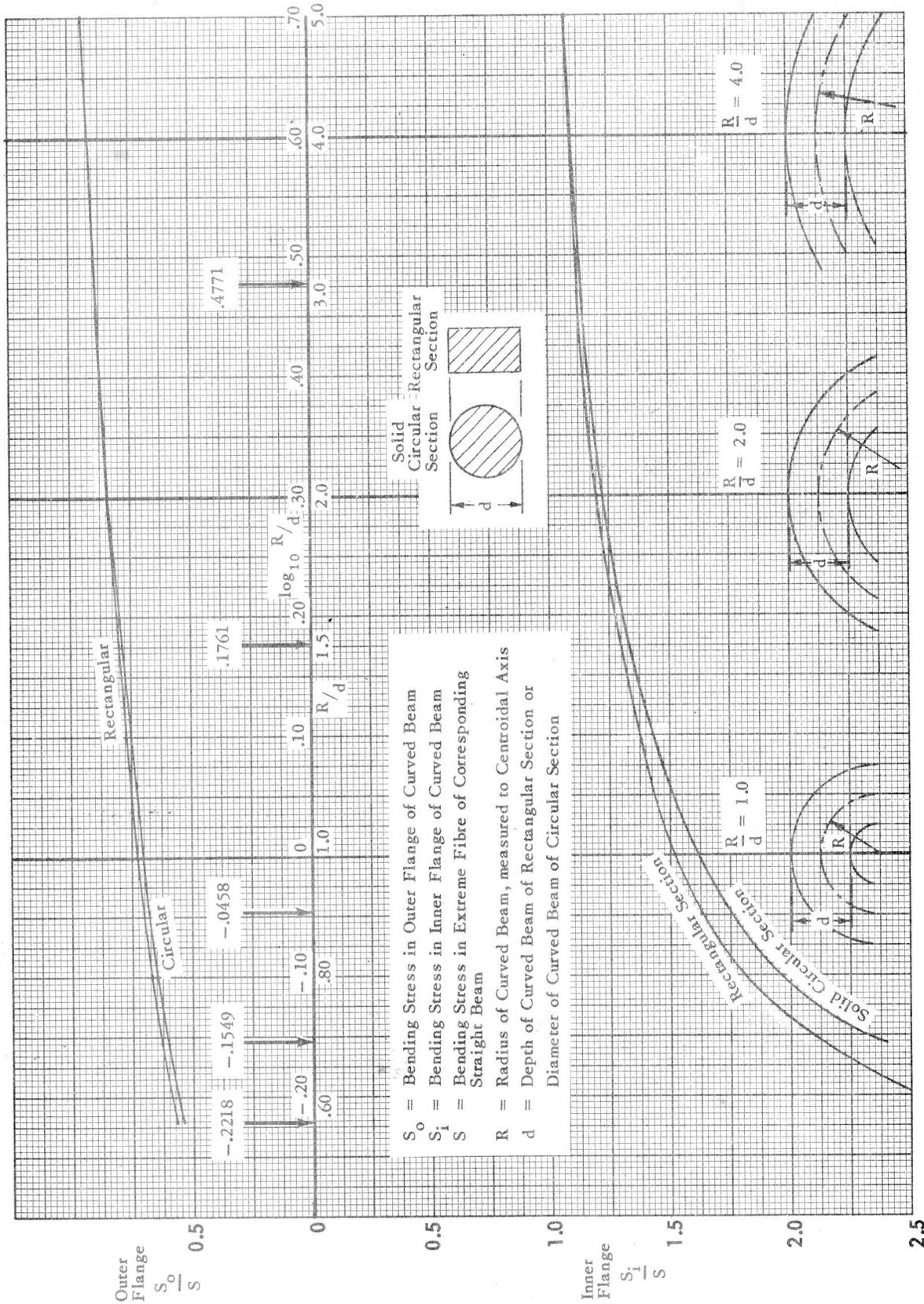


Fig. 3.6-5 Circumferential Bending Stresses in Curved Beams

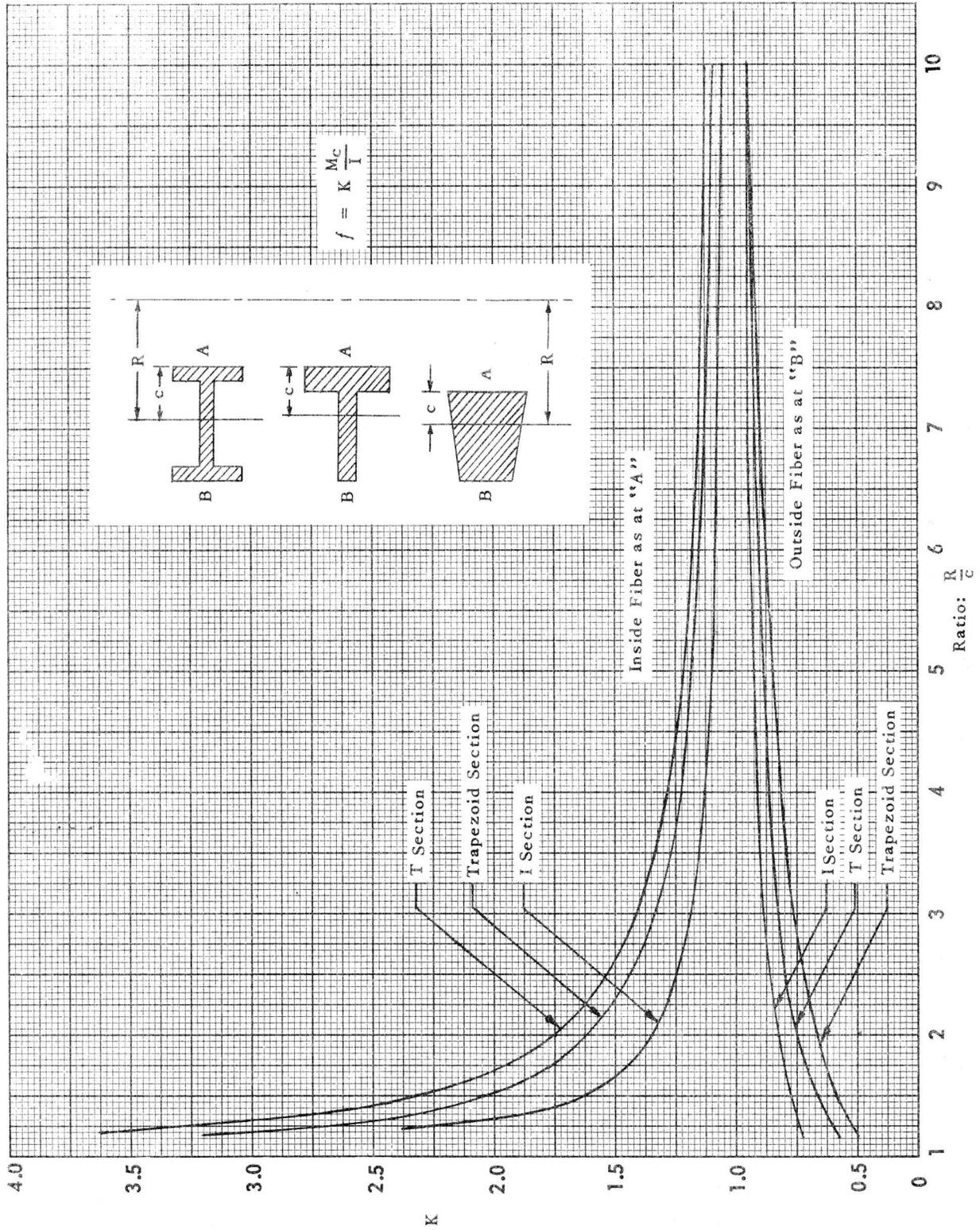


Fig. 3.6-6 I, T and Trapezoidal Sections



3.7

UNSYMMETRICAL BENDING

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3.7.2 Notation
3.7.3 Formulas
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3.7.7 Numerical Example Principal Axes

3.7.1 Introduction

The bending stresses on a section subjected simultaneously to bending moments in two planes can be found by the formulas given in 3.7.3, without having to determine the principal axes, even when the section is completely unsymmetrical.

The theory is developed in Report GEN/1090/807.

3.7.2 Notation

XX YY	}	Any two convenient rectangular axes through the centroid O of the section. When the section has an axis of symmetry, XX or YY would naturally be chosen to coincide with this axis.
NA		
x	IN.	Co-ordinate measured along the XX axis.
y	IN.	Co-ordinate measured along the YY axis. (x and y are positive in the first quadrant.)
M_x	LB.IN.	Component of bending moment about XX.

M_y	LB.IN.	Component of bending moment about YY. (M_x and M_y are positive when acting as shown in 3.7.3.)		
I_x	IN. ⁴	Moment of Inertia of effective section about XX.		
I_y	IN. ⁴	Moment of Inertia of effective section about YY.		
I_{xy}	IN. ⁴	Product of Inertia of effective section about axes XX and YY. (The "effective section" is the area effective in bending)		
A_1 A_2 B m n	}	See 3.7.3 for these coefficients.		
β			DEG.	angle between the neutral axis of the section and axis XX such that $\tan \beta = \frac{m}{n}$
σ			P.S.I.	bending stress at station whose co-ordinates are x and y.



3.7.3 Formulas

Bending stress at station having co-ordinates x, y is: $\sigma = mx + ny$ where m and n are as below. Positive σ is a compressive stress.

M_x and M_y are positive when acting as shown.

When $\tan \beta = \frac{m}{n}$ is positive, β is as shown.

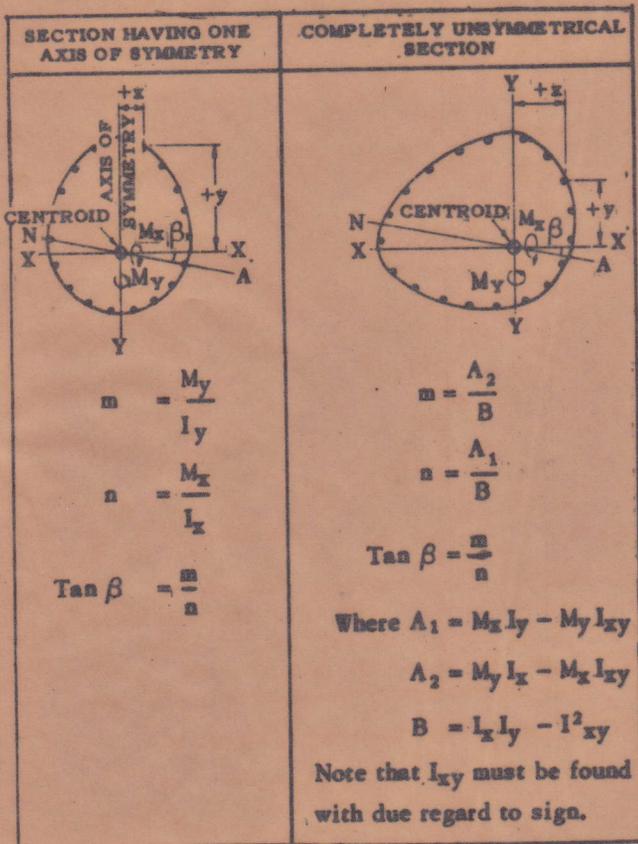


Fig. 3.7-1

3.7.4 Procedure

- Find the position of the centroid of the effective section, that is, the section effective in bending, which in many cases will consist of stringers plus effskin.
- Decide on any convenient rectangular axes XX and YY through the centroid.
- Find I_x, I_y and I_{xy} , the last-mentioned with due regard to sign.
- Find the coefficients A_1, A_2 and B and calculate m, n and the ratio $\frac{m}{n}$.
- Calculate the bending stress $\sigma = mx + ny$.

A numerical example appears below.

3.7.5 Numerical Example

Fig. 3.7-2 shows a completely unsymmetrical wing section for which the stringer positions relative to axes XX and YY through the centroid are as in the Table below. (The example is taken from BRUHN, Fig. A 13-4). It is required to find the distribution of bending stress with the following given data:

$$I_x = 186.5 \text{ IN}^4.$$

$$I_y = 431.7 \text{ IN}^4.$$

$$I_{xy} = -36.41 \text{ IN}^4.$$

$$M_x = 71.3 \times 10^4 \text{ LB. IN.}$$

$$M_y = -3.8 \times 10^4 \text{ LB. IN.}$$

Steps (a), (b), and (c) of Procedure have been covered.

$$(d) A_1 = M_x I_y - M_y I_{xy}$$

$$= 71.3 \times 10^4 \times 431.7 - 3.8 \times 10^4 \times 36.41$$

$$= 308 \times 10^6 - 1.383 \times 10^6 = + 306.617 \times 10^6$$

$$A_2 = M_y I_x - M_x I_{xy}$$

$$= -3.8 \times 10^4 \times 186.5 + 71.3 \times 10^4 \times 36.41$$

$$= -7.1 \times 10^6 + 25.95 \times 10^6 = + 18.85 \times 10^6$$

$$B = I_x I_y - I_{xy}^2$$

$$= 186.5 \times 431.7 - (36.41)^2$$

$$= 8.06 \times 10^4 - 1320$$

$$= 7.928 \times 10^4$$

$$m = \frac{A_2}{B} = \frac{18.85 \times 10^6}{7.928 \times 10^4} = + 237.5$$

$$n = \frac{A_1}{B} = \frac{306.617 \times 10^6}{7.928 \times 10^4} = + 3870$$

$$\tan \beta = \frac{m}{n} = \frac{237.5}{3870} = 0.0613$$

$$\beta = 3^\circ 30'$$



3.7.5 (cont'd)

(e) $\sigma = mx + ny$
 $= 237.5x + 3870y$ (+ Compression)

Values of σ for various values of x and y are given.

The distribution of σ is shown in Fig. 3.7-2.

+ COMPRESSION

ITEM	x (IN.)	y (IN.)	237.5x	3870y	σ (P.S.I.)
1	-17.41	4.40	-4120	17,000	12,880
2	-13.54	6.45	-3210	25,000	21,790
3	-9.11	7.40	-2165	28,600	26,430
4	-5.44	7.77	-1290	30,000	28,710
5	-0.86	7.95	-205	30,800	28,750
6	3.14	7.90	745	30,600	31,350
7	7.14	7.70	1700	29,800	31,500
8	11.74	7.30	2790	28,300	31,090
9	15.39	6.90	3650	26,700	30,350
10	-17.51	-2.90	-4160	-11,220	-15,380
11	-13.54	-4.50	-3210	-17,420	-20,630
12	-9.11	-5.55	-2165	-21,500	-23,670
13	-2.96	-7.00	-700	-27,100	-27,800
14	3.32	-7.73	790	-29,900	-29,110
15	9.64	-8.22	2290	-31,800	-29,510
16	15.39	-8.47	3650	-32,800	-29,150

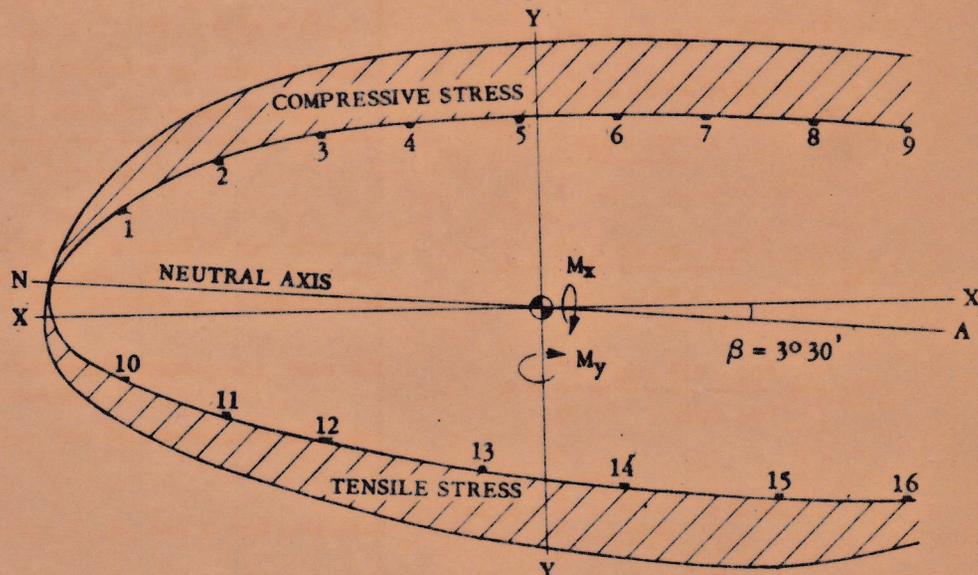


Fig. 3.7-2



3.7.6 Numerical Example No. 2 (Fig. 3.7-3)

Same Example but $M_y = 0$

$$A_1 = M_x I_y = 308 \times 10^6$$

$$A_2 = -M_x I_{xy} = 25.95 \times 10^6$$

$$B = I_x I_y - I_{xy}^2 = 7.928 \times 10^4 \text{ as before}$$

$$m_1 = \frac{A_2}{B} = \frac{25.95 \times 10^6}{7.928 \times 10^4} = +327$$

$$n_1 = \frac{A_1}{B} = \frac{308 \times 10^6}{7.928 \times 10^4} = +3885$$

$$\tan \beta_1 = \frac{m_1}{n_1} = \frac{327}{3885} = 0.084$$

$$\beta_1 = 4^\circ 48'$$

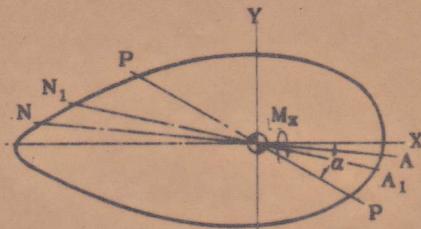


Fig. 3.7-3

Hence neutral axis moves clockwise from NOA to $N_1O_1A_1$. This is as would be expected since removal of M_y will reduce the compressive stress, and increase the tensile stress, at N and hence zero value could only be obtained by having N_1 clockwise relative to N.

3.7.7 Principal Axes

As a matter of interest the Principal axis PP for this section is at an angle $\alpha = -8^\circ 16.25'$ to XX (See Bruhn, A13.6).

PP is thus located as in Fig. 3.7-3 (not to scale).

TABLE OF CONTENTS

3.8.1	Introduction
3.8.2	Section Factor
3.8.3	Design Charts
3.8.4	Bending Stress at Extreme Fibre
3.8.5	Bending Stress at Intermediate Fibre
3.8.6	Shear Flow

3.8.1 Introduction

Classic bending theory is based on the assumption that plane sections before bending remain plane after bending. Since, within the elastic limit, stress is proportional to strain, it follows that the stress distribution over the cross-section will be linear, varying from zero at the neutral axis to a maximum at the extreme fibre, as in Fig. 3.8-1 (b). This distribution is expressed mathematically for a symmetrical cross-section subjected to a bending moment

$$m \text{ by: } \frac{my}{I}$$

When the extreme fibre stress exceeds the proportional limit stress of the material, the stress distribution on the cross-section is no longer linear but tends to follow the stress-strain characteristics of the material (Fig. 3.8-1 (c)). Consequently, the conventional formula for linear bending stress distribution no longer applies.

It has been standard aircraft practice, in such cases, to use a FORM FACTOR to indicate the amount by which the allowable ultimate stress of the material (or the allowable bending moment) could be considered to be increased in order to apply the conventional formula given above. To use this method it is necessary to have form factors for all the cross-sectional shapes commonly used, information which is not readily available.

A procedure has therefore been developed for symmetrical sections, (see "Bending Strength in the Plastic Range" Cozzone, J. Ae. Sc., May 1943, Page 137) whereby the engineers' bending stress is reduced by using a SECTION FACTOR in conjunction with a series of Design Charts for various materials; this reduced value is then compared with the allowable bending stress of the material.

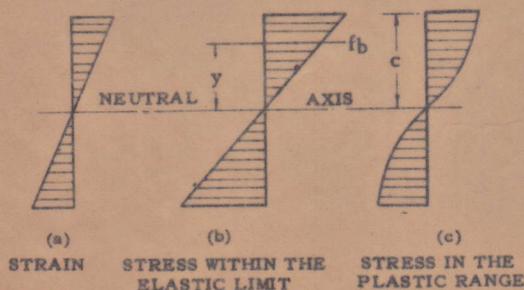


Fig. 3.8-1



3.8.2 Section Factor

The section factor K is given by the formula:

$$K = \frac{2Q_m}{I/c}$$

where Q_m = the static (first) moment about the neutral axis of the area between the neutral axis and the extreme fibre.

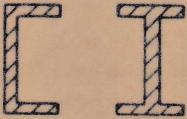
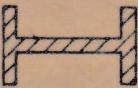
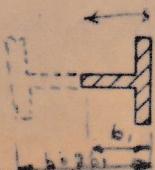
I = moment of inertia about the neutral axis.

c = distance from the neutral axis to the extreme fibre of the section.

Values of K for various common symmetrical sections appear in Table 3.8-1 and in Figs. 3.8-2, 3 and 4.

If K by calculation exceeds 2.0, use 2.0.

TABLE 3.8-1

(1)		$K = 1.5$
(2)		$K = 1.7$
(3)		See Fig. 3.8-2
(4)		See Fig. 3.8-3
(5)		See Fig. 3.8-4
(6)		Value determined in Chart

Do not use a value of K greater than 2.0.



3.8.2 Section Factor (cont'd)

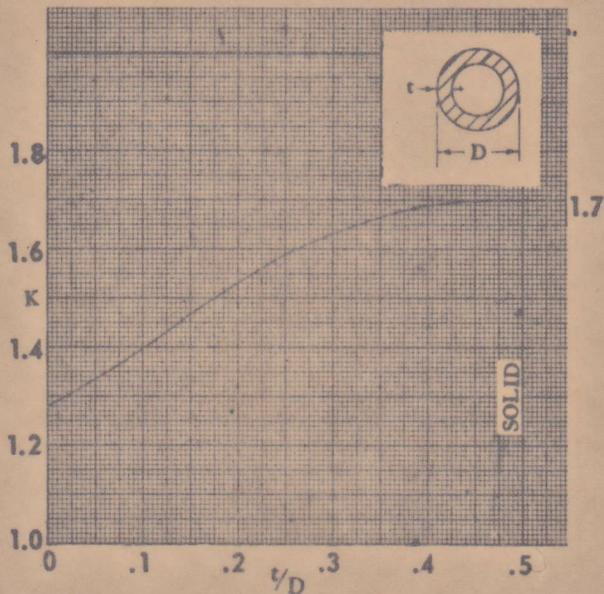


Fig. 3.8-2 K value for Round Tube

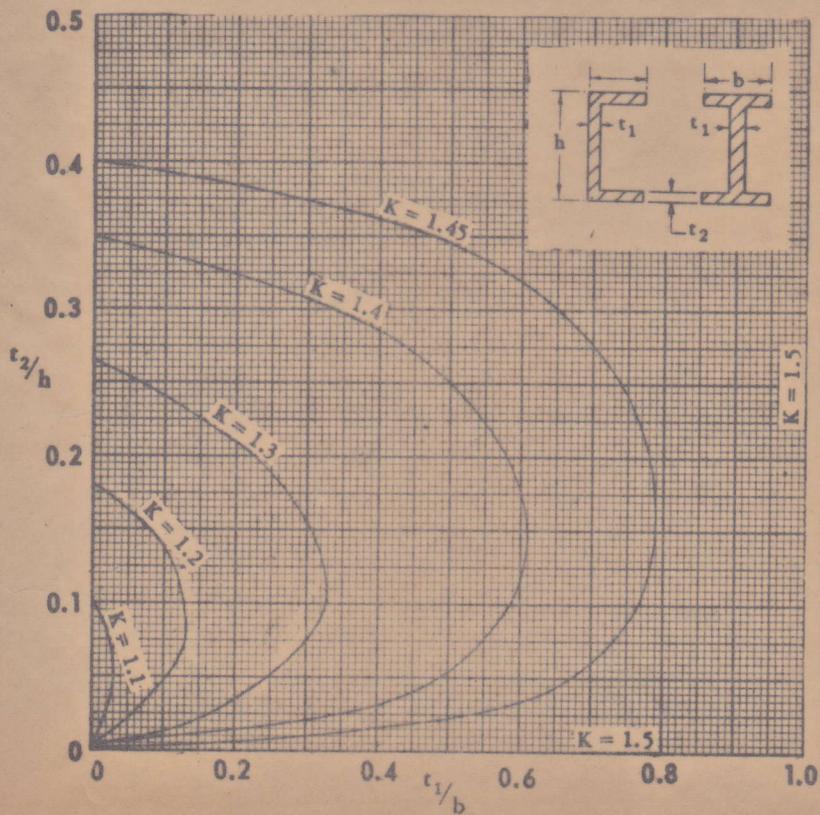


Fig. 3.8-3 K value for I or Channel Section



3.8.2 Section Factor (cont'd)

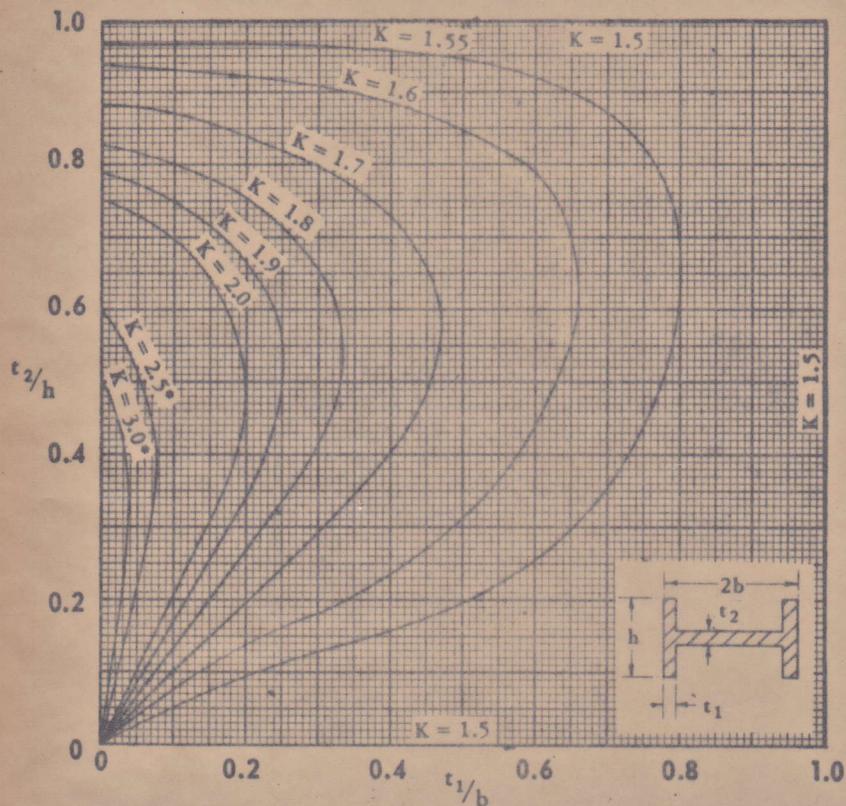


Fig. 3.8-4 Kvalue for I or T section

* DO NOT USE VALUES OF
K GREATER THAN 2.0.

3.8.3 Design Charts

The following Design Charts are given:

- Fig. 3.8- 5 14S-T Aluminum Alloy Forging
- Fig. 3.8- 6 220-T4 Aluminum Alloy Sand Casting
- Fig. 3.8- 7 195-T6 Aluminum Alloy Sand Casting
- Fig. 3.8- 8 75S-T Aluminum Bar
- Fig. 3.8- 9 Alloy Steel H.T. 125,000 p.s.i.
- Fig. 3.8-10 Alloy Steel H.T. 150,000 p.s.i.
- Fig. 3.8-11 Alloy Steel H.T. 180,000 p.s.i.
- Fig. 3.8-12 HTA Magnesium Sand Castings



3.8.3 Design Charts (cont'd)

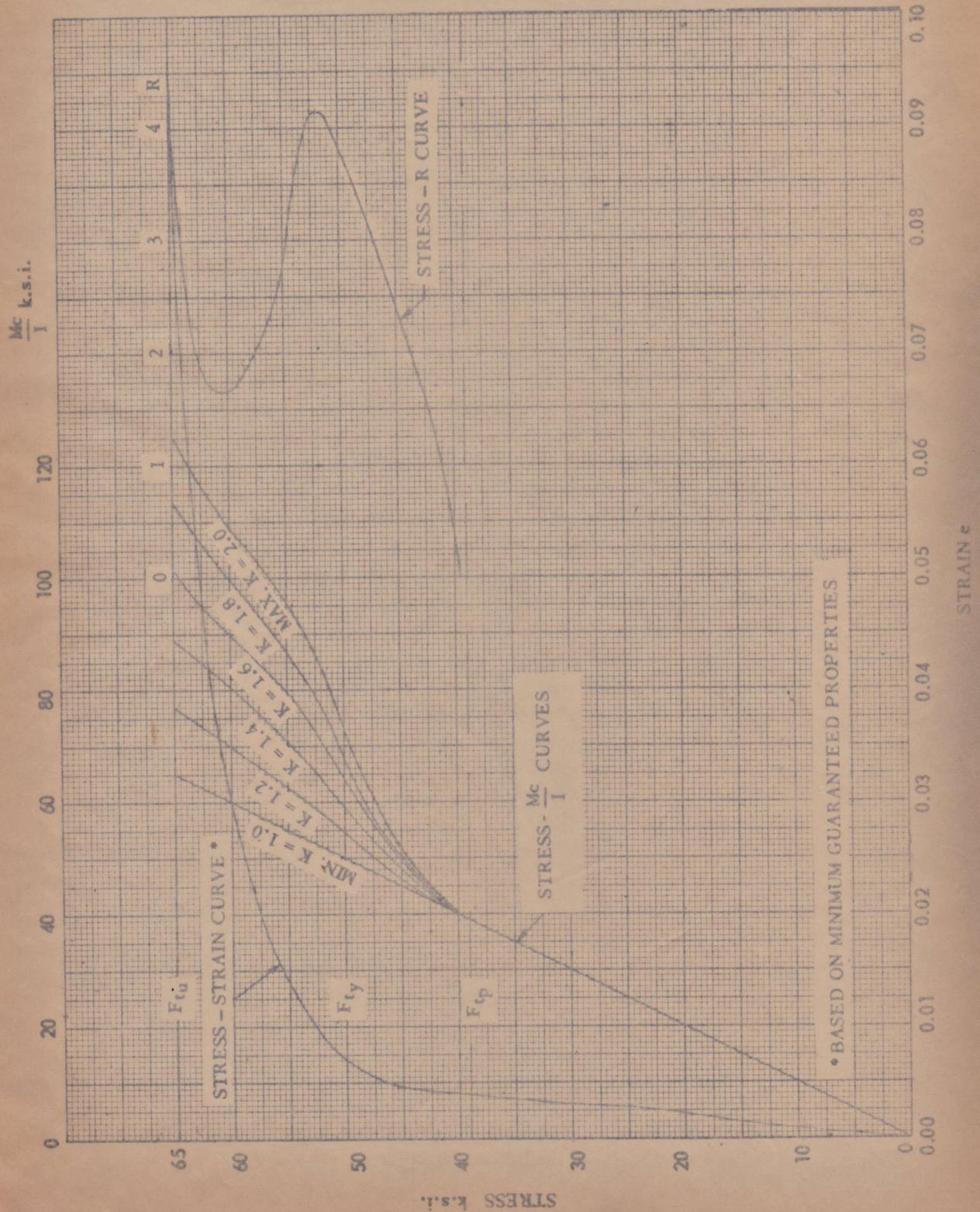


Fig. 3.8-5 14S-T Aluminum Alloy Forging The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

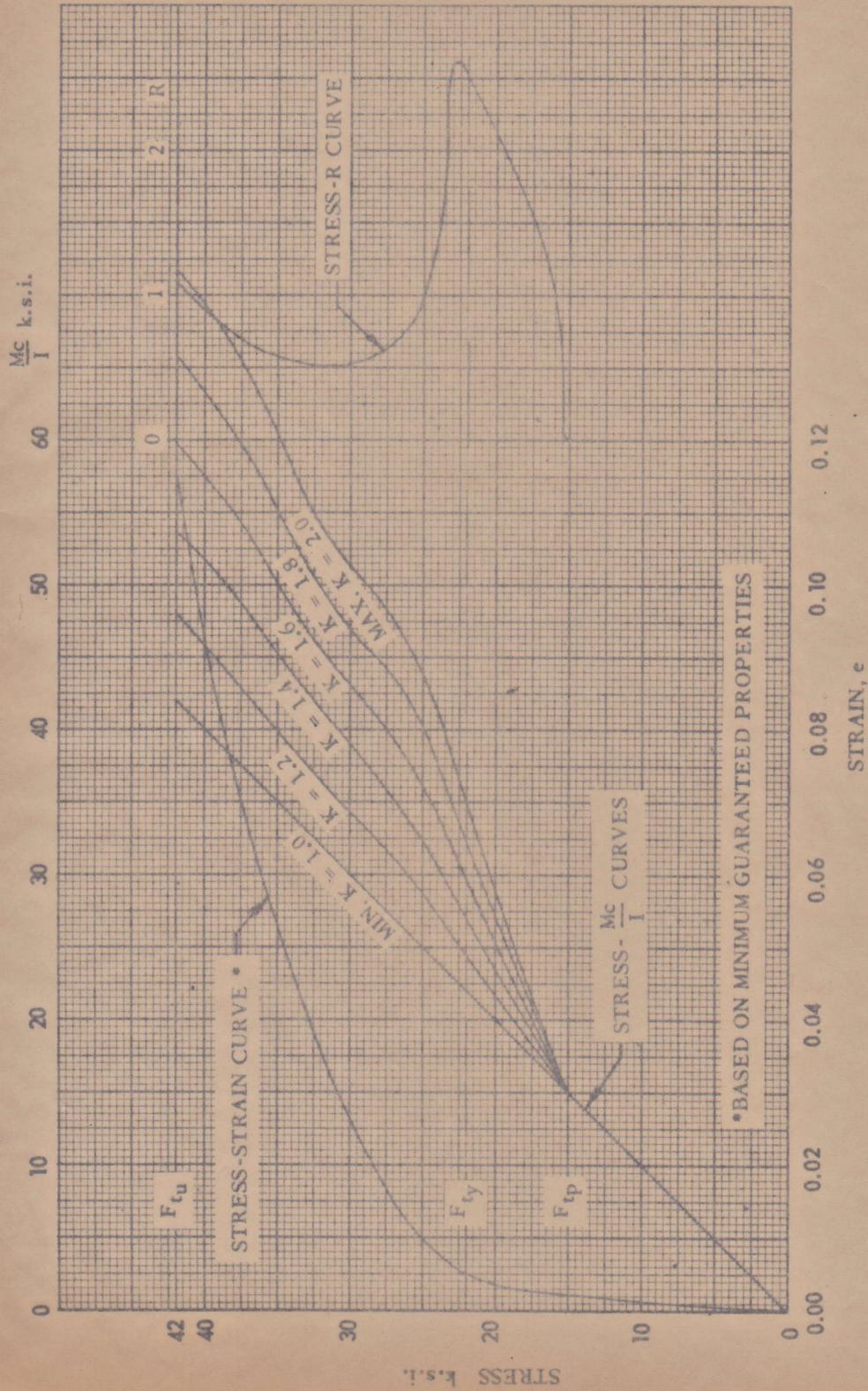


Fig. 3.8-6 220-T4 Aluminum Alloy Sand Casting

The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

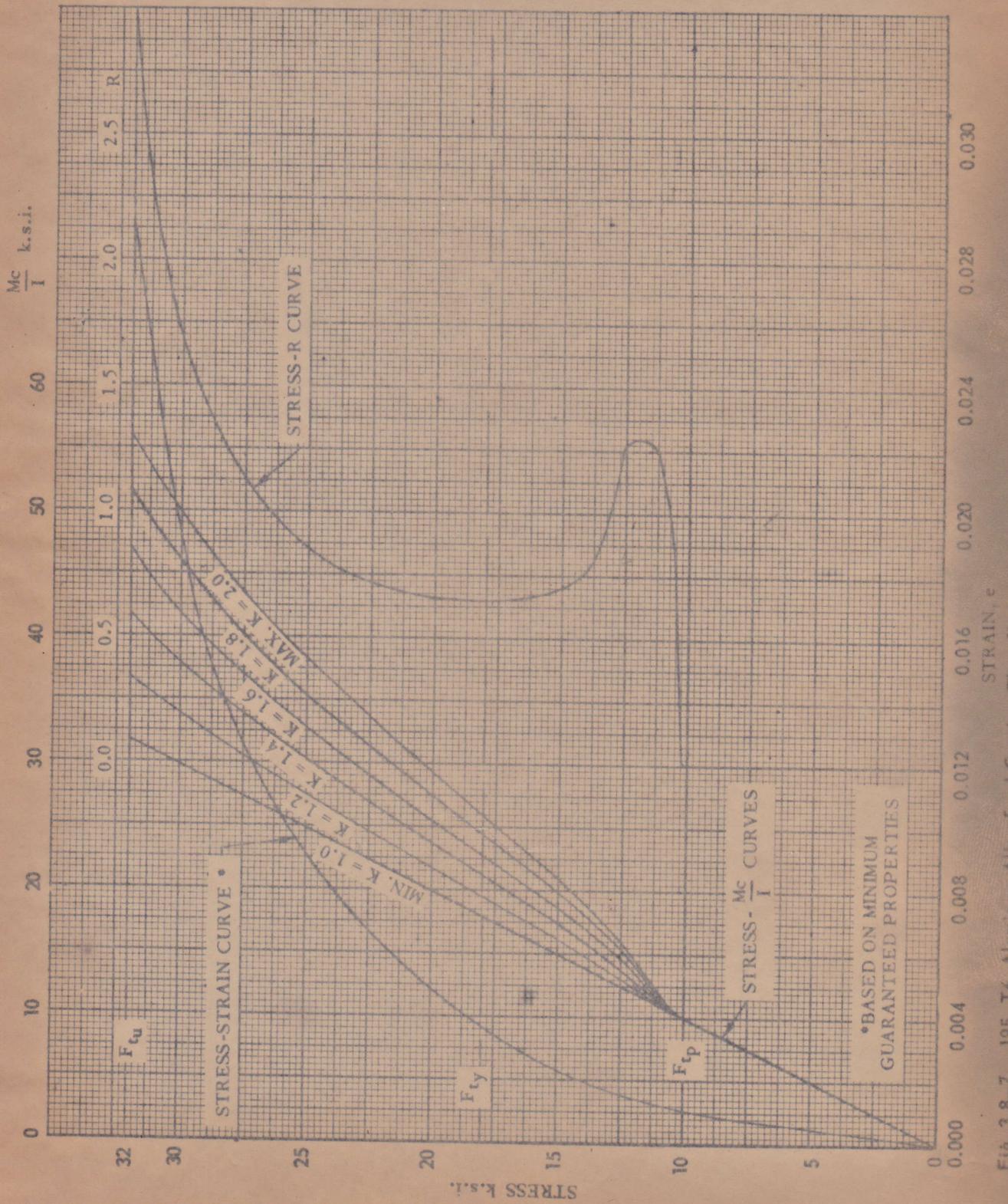


Fig. 3.8-7 195-T6 Aluminum Alloy Sand Casting. The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

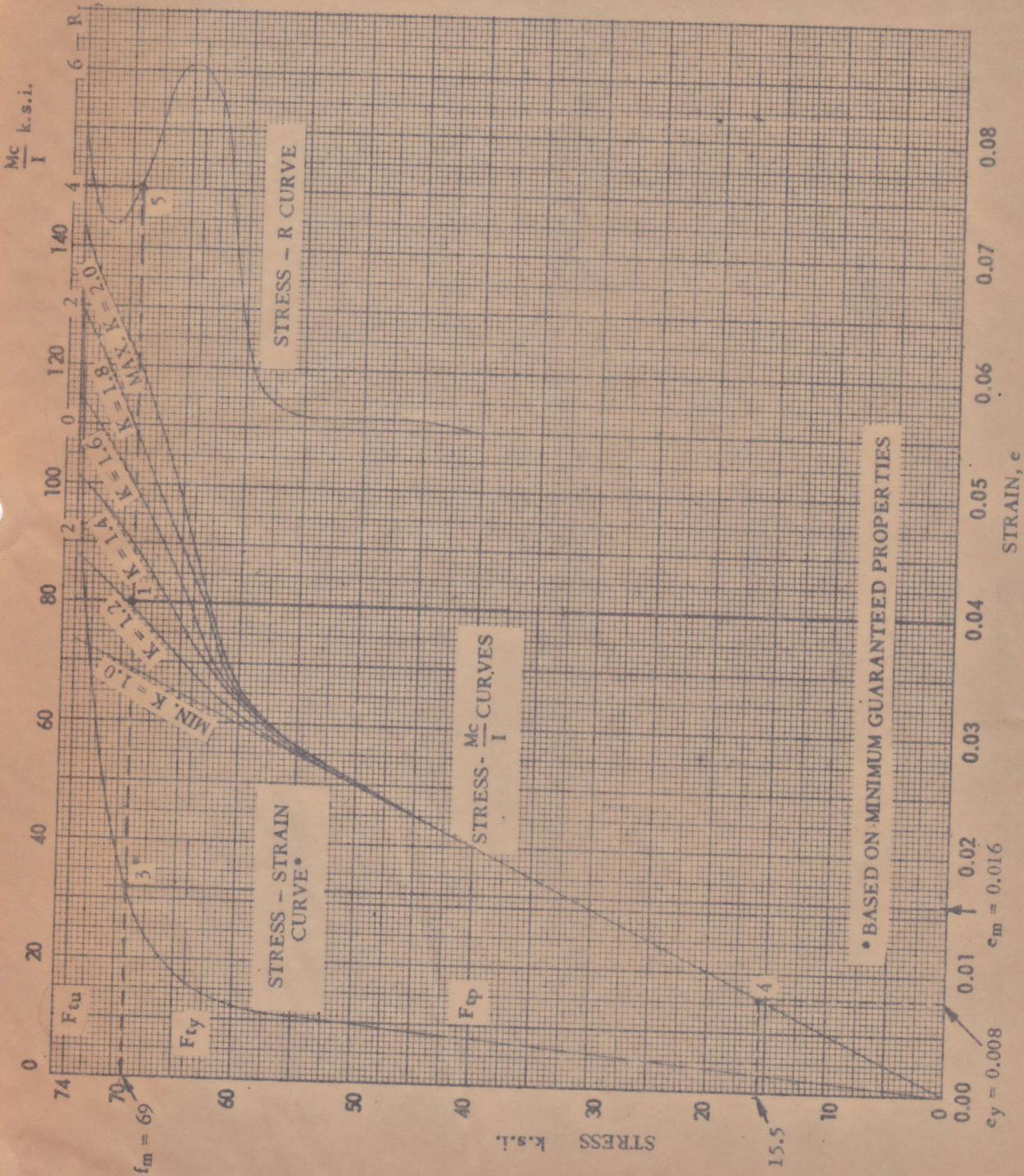


Fig. 3.8-8 755-T Aluminum Bar The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

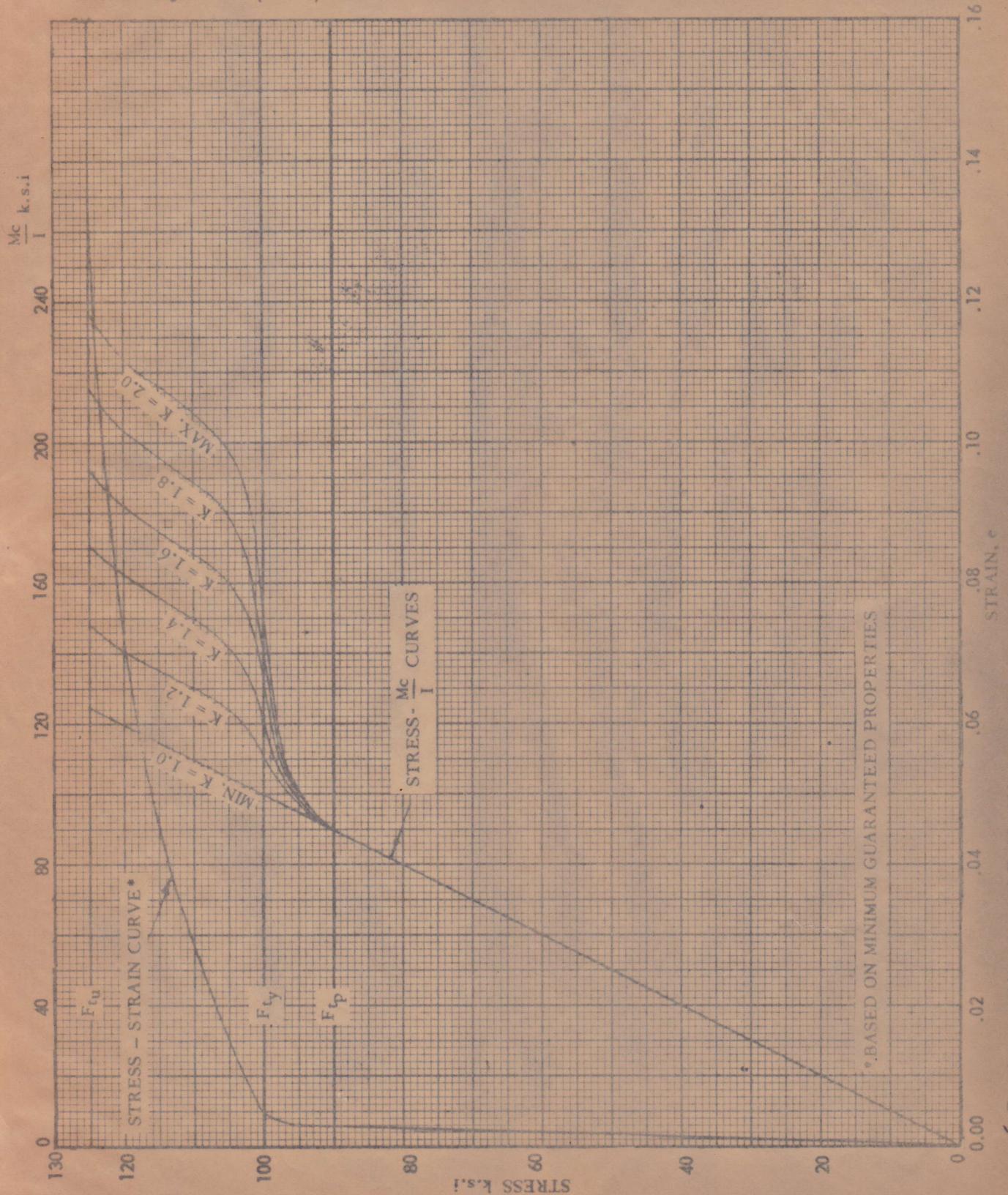


Fig. 3.8-9 Alloy Steel H.T. 125,000 p.s.i. The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

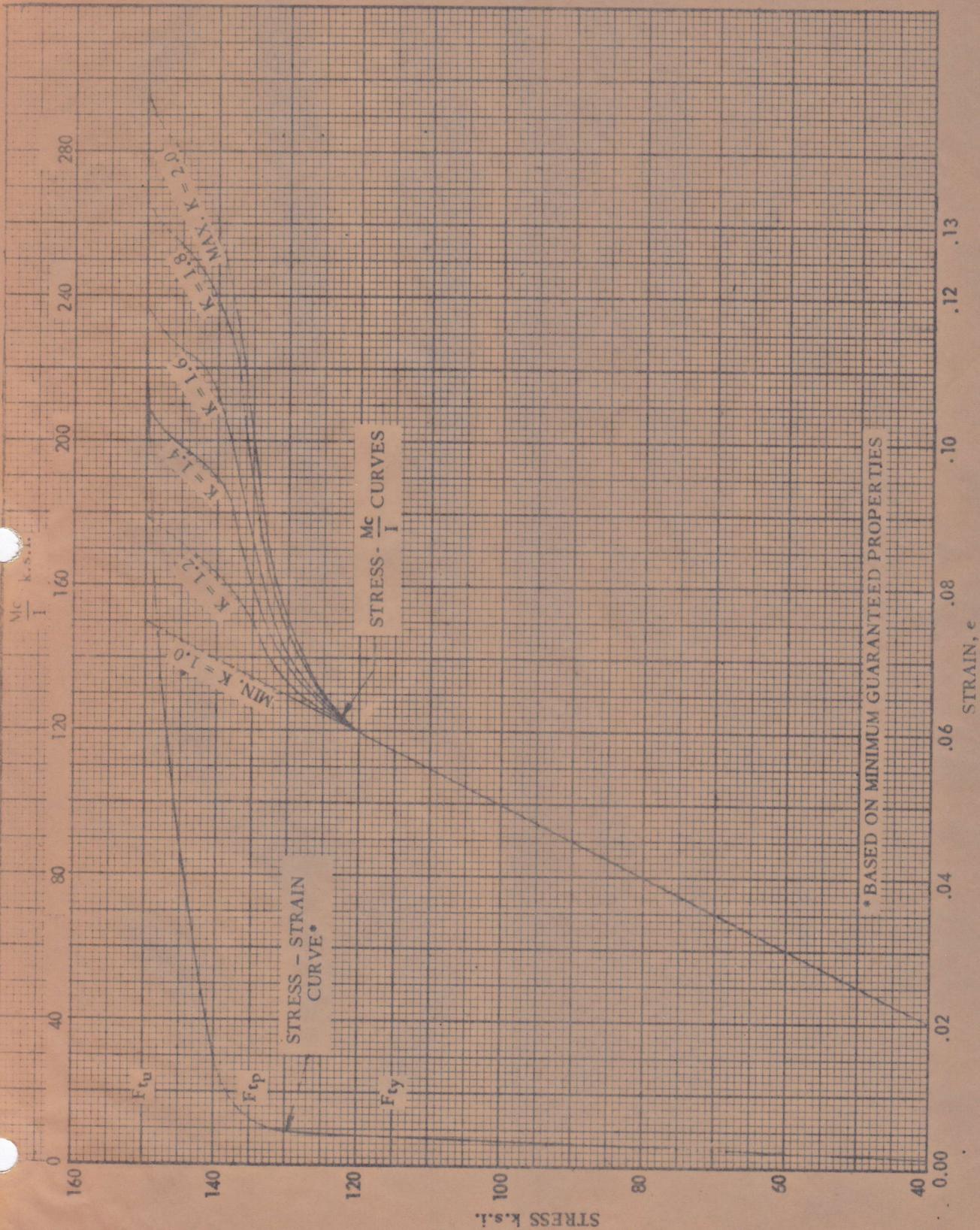


Fig. 3.8-10 Alloy Steel H.T. 150,000 p.s.i. The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

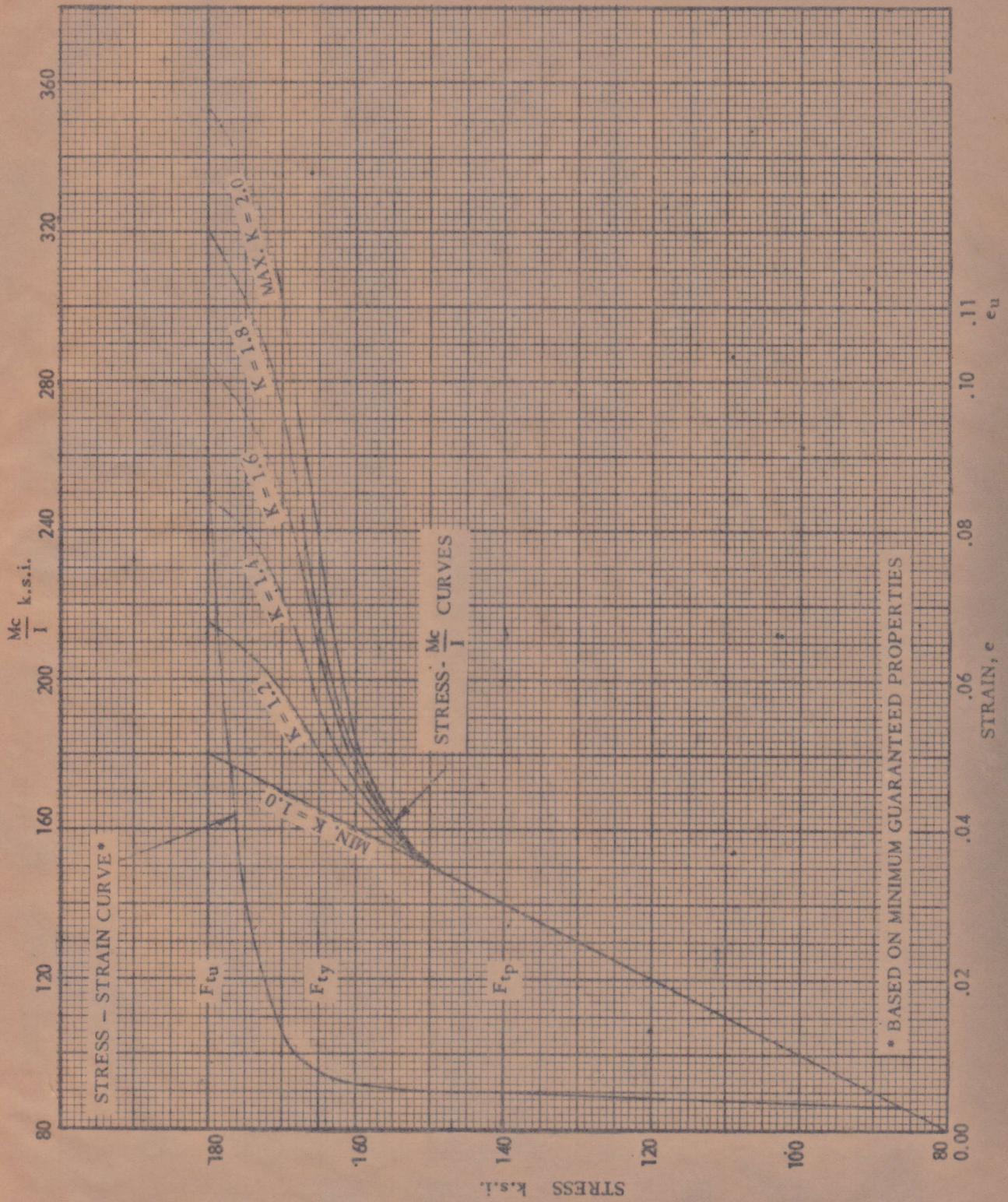


Fig. 3.8-11 Alloy Steel H.T. 180,000 p.s.i. The effects of shape and material properties on bending in the plastic range



3.8.3 Design Charts (cont'd)

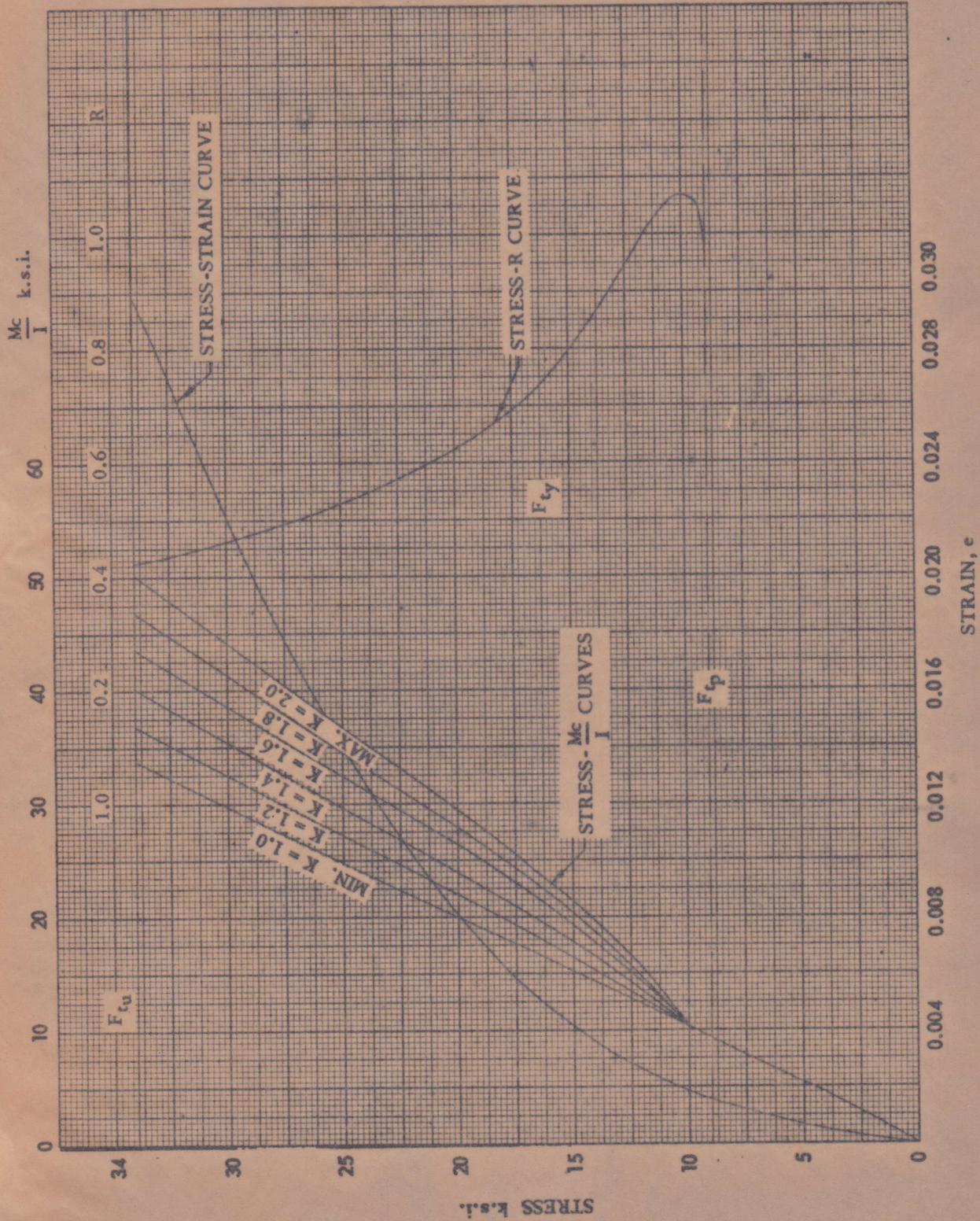


Fig. 3.8-12 HTA Magnesium Sand Castings The effects of shape and material properties on bending in the plastic range



3.8.4 Bending Stress at Extreme Fibre

(a) Find K from Table 3.8-1 or when this is not possible, use the formula $K = \frac{2Q_m}{I/c}$

(b) Find $\frac{mc}{I}$, the engineers' bending stress at the extreme fibre.

(c) Enter the appropriate Design Chart on the upper horizontal $\frac{mc}{I}$ scale and obtain the maximum applied stress f_m at the extreme fibre for the particular value of K found in (a).

NUMERICAL EXAMPLE

I section beam shown in Fig. 3.8-13 subjected to a bending moment of 31600 lb. in. about the neutral axis. Specification 75S-T Aluminum Alloy Bar.

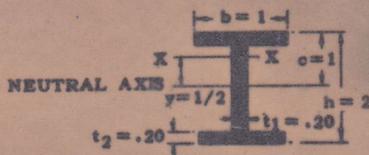


Fig. 3.8-13

(a) Enter Fig. 3.8-3 on the base line at $t_1/b = 0.20$. Read the value of $t_2/h = 0.10$ on the vertical scale. The intersection gives $K = 1.234$ by interpolation between the curves of $K = 1.2$ and $K = 1.3$.

As a check on this value, the general method, using the formula $K = \frac{2Q_m}{I/c}$, will be used.

$$Q_m = 1.0 \times .20 \times .90 + .80 \times .20 \times .40$$

$$= .18 + .064 = \underline{0.244 \text{ in}^3}$$

$$I = \frac{1}{12} [1 \times 2^3 - .80 \times 1.6^3]$$

$$= \frac{1}{12} [8.0 - 3.26] = \underline{0.395 \text{ in}^4}$$

$$I/c = .395 \text{ in}^3$$

$$K = \frac{2 \times .244}{.395} = \underline{1.236}$$

(b) Stress developed by engineers' formula

$$= \frac{mc}{I} = \frac{31600}{.395} = 80,000 \text{ p.s.i.}$$

(c) Enter Fig. 3.8-8 on the upper $\frac{Mc}{I}$ scale at 80,000 p.s.i. and proceed vertically downwards (as indicated by the dotted line) until point (1) on the $\frac{Mc}{I}$ curve representative of $K = 1.236$ (by interpolation between $K = 1.2$ and $K = 1.4$) is reached.

Corresponding stress value on the vertical scale is 69,000 p.s.i.

RESERVE FACTOR

Since stress is no longer proportional to bending moment, the reserve factor must be obtained on a bending moment basis.

Enter Fig. 3.8-8 with the allowable ultimate tensile stress F_{tu} to obtain an $\frac{Mc}{I}$ value for the proper section factor ($K = 1.236$ in this case) as indicated by point (2). $\frac{Mc}{I} = 91,000 \text{ p.s.i.}$ by interpolation.

Multiply this value by I/c to find the allowable ultimate bending moment viz $M = 91000 \times .395 = 36000 \text{ lb. in.}$

$$\text{Reserve Factor on an ultimate basis} = \frac{\text{allowable ultimate B.M.}}{\text{applied B.M.}}$$

$$= \frac{36000}{31600} = 1.13$$



3.8.5 Bending Stress at Intermediate Fibre

(a) On the appropriate Design Chart find the strain ϵ_m (on the lower horizontal scale) corresponding to the maximum applied stress f_m on the stress-strain curve.

(b) The strain at any point distant y from the neutral axis is:

$$\epsilon_y = \epsilon_m \times y/c$$

(c) Read off the vertical scale the stress corresponding to this strain on the stress-strain curve.

NUMERICAL EXAMPLE

To find the bending stress of the I section beam shown in Fig. 3.8-13 at Section XX, which is at a distance $y = 1/2$ in. from the neutral axis.

Applied bending moment = 31600 lb. in.

(a) $\epsilon_m = 0.016$ given by Point (3) on the stress-strain curve. $f_m = 69,000$ p.s.i.

(b) $\epsilon_y = 0.016 \times \frac{0.5}{1.0} = 0.008$. This gives Point (4) on the stress-strain curve.

(c) Corresponding stress = 15500 p.s.i.

3.8.6 Shear Flow

The shear stress distribution shows a marked change from that obtained by the conventional formula for shear flow viz, $q = \frac{VQ}{I}$ lb. per in.

It is shown by Cozzone that the maximum shear flow in the plastic range for a doubly-symmetrical section is given by:

$$q = \frac{VQ_m}{I} \left[\frac{1 + R \left(\frac{A \times c}{Q_m} - 1 \right)}{1 + R(K - 1)} \right]$$

Values of the coefficient R appear in Charts 3.8-5 to 12 in the form of a "Stress - R " curve.

Procedure

(a) Knowing f_m (from above) find the value of R on the stress - R curve.

(b) Substitute this value in the above expression for q .

NUMERICAL EXAMPLE

Consider the same I section as above.

(a) $f_m = 69,000$ p.s.i.

$R = 4$ for Point (5) on the Stress - R curve of Fig 3.8-8.

(b) $A = 1 \times 0.2 + 0.8 \times 0.2 = 0.36$ in².

$$\frac{A \times c}{Q_m} = \frac{0.36 \times 1.0}{0.244} = 1.472$$

$$\frac{A \times c}{Q_m} - 1 = 0.472$$

$$R \left(\frac{A \times c}{Q_m} - 1 \right) = 4 \times 0.472 = 1.888$$

$$\left[1 + R \left(\frac{A \times c}{Q_m} - 1 \right) \right] = 2.888$$

$$(K - 1) = 1.236 - 1 = 0.236$$

$$R(K - 1) = 0.944$$

$$\left[1 + R(K - 1) \right] = 1.944$$

$$q = \frac{VQ_m}{I} \times \frac{2.888}{1.944} = 1.485 \frac{VQ_m}{I}$$



SECTION PROPERTIES

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4.1 General Section Properties

4.1.1 Tabular Summary

The following tabular summary is intended to supplement Roark, pp. 68, 69 and 70, Table I.

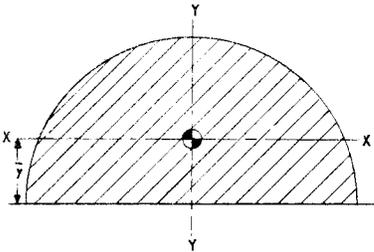
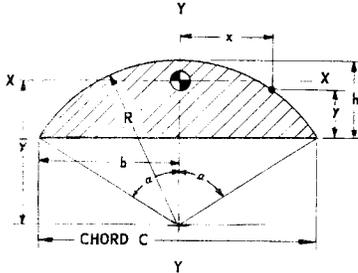
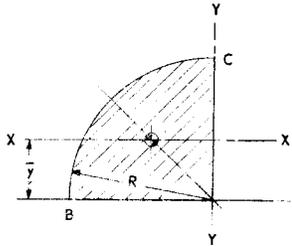
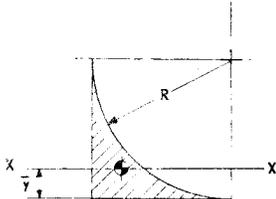
<p style="text-align: center;">CIRCULAR ARC (GENERAL)</p> <p style="text-align: center;">$\frac{t}{R}$ small</p>	<p style="text-align: center;">Area $A = 2 a R t$</p> <p style="text-align: center;">$\bar{y} = \frac{R}{a} \sin a \cos \theta$</p> <p style="text-align: center;">$I_B = R^3 t [a + \frac{1}{2} \sin 2 a \cos 2 \theta]$</p> <p style="text-align: center;">$I_X = I_B - A \bar{y}^2$</p>
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4.1.1 Tabular Summary (Cont'd)

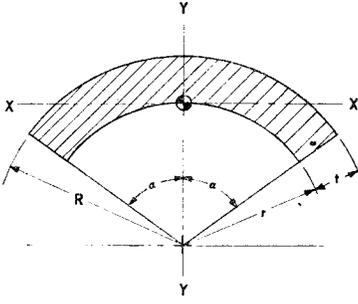
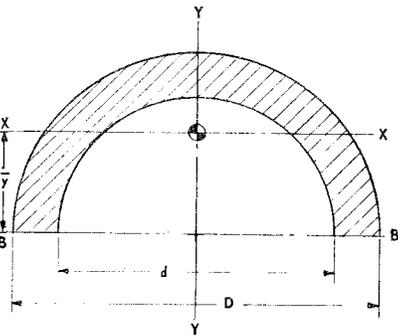
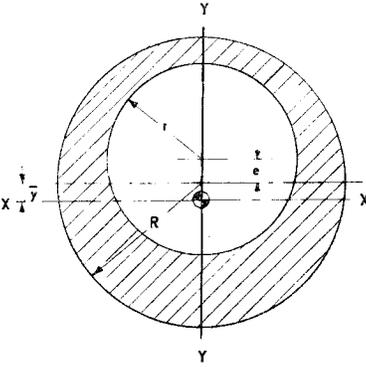
<p>CIRCULAR ARC (SYMMETRICAL) $\frac{t}{R}$ small</p>	<p>Area $A = 2 a R t$ <i>C-C CURVE LINE</i> $\bar{y} = R \frac{\sin a}{a}$ $I_B = R^3 t [a + \frac{1}{2} \sin 2 a]$ $I_X = R^3 t [a + \frac{1}{2} \sin 2 a - \frac{2 \sin^2 a}{a}]$ $I_Y = R^3 t [a - \frac{1}{2} \sin 2 a]$ <i>$I_C = R^3 t [2a - \frac{3}{2} \sin 2a + a \cos 2a]$</i></p>
<p>SEMI-CIRCULAR ARC $\frac{t}{R}$ small</p>	<p>Area $A = \pi R t$ $\bar{y} = .637 R$ $I_B = 1.57 R^3 t$ $I_X = .298 R^3 t$ $I_Y = 1.57 R^3 t$</p>
<p>QUARTER-CIRCULAR ARC (90° Bend) $\frac{t}{R}$ small</p>	<p>Area $A = \frac{\pi}{2} R t$ $\bar{x} = .637 R$ $\bar{y} = .637 R$ $I_B = .785 R^3 t$ $I_X = .149 R^3 t$ $I_Y = .785 R^3 t$</p>
<p>CIRCULAR SEGMENT $a > 90^\circ$</p>	<p>Area $A = \frac{R^2}{2} (2 a - \sin 2 a)$ $\bar{y} = \frac{2 R^3 \sin^3 a}{3 A}$ $I_B = \frac{R^4}{4} [a - \frac{1}{2} \sin 2 a + \sin^2 a \sin 2 a]$ $I_Y = \frac{R^4}{4} [a - \frac{1}{2} (1 + \frac{2}{3} \sin^2 a) \sin 2 a]$ $I_X = I_B - A \bar{y}^2$</p>

4.1.1 Tabular Summary (Cont'd)

<p style="text-align: center;">SEMI-CIRCULAR SEGMENT</p> 	$\text{Area } A = \frac{\pi R^2}{2}$ $\bar{y} = .4244 R$ $I_X = .1098 R^4$ $I_Y = \frac{\pi R^4}{8}$
<p style="text-align: center;">CIRCULAR SEGMENT <i>a</i> less than 90°</p> 	$R = \frac{b^2 + h^2}{2h}$ $\text{Ordinate } y = \sqrt{R^2 - x^2} - \sqrt{R^2 - b^2}$ $\text{Area } A = \frac{R^2}{2} (2a - \sin 2a)$ $\bar{y} = \frac{2}{3} \frac{b^3}{A} \quad \text{See also Fig. 4.1.2}$ $I_B = \frac{R^4}{4} [a - \frac{1}{2} \sin 2a + \sin^2 a \sin 2a]$ $I_Y = \frac{R^4}{4} [a - \frac{1}{2} (1 + \frac{2}{3} \sin^2 a) \sin 2a]$ $I_X = I_B - A \bar{y}^2$
<p style="text-align: center;">CIRCULAR QUADRANT</p> 	$\text{Area } A = .785 R^2$ $\bar{y} = .4244 R$ $I_X = .0549 R^4$
<p style="text-align: center;">CIRCULAR FILLET</p> 	$\text{Area } A = .2146 R^2$ $\bar{y} = .2234 R$ $I_X = .00754 R^4$

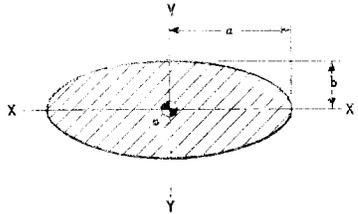
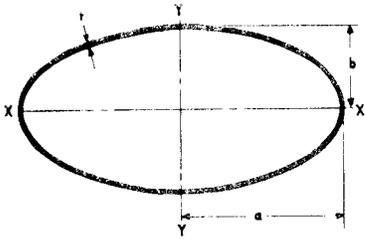
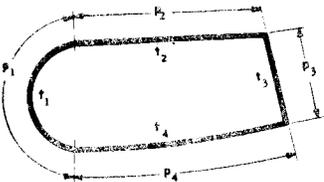
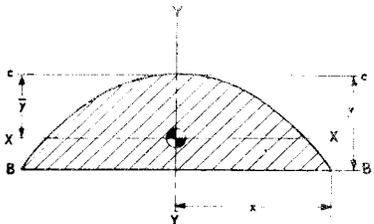


4.1.1 Tabular Summary (Cont'd)

<p>HOLLOW CIRCULAR SECTOR (Not Thin-Walled)</p> 	<p><i>X-X = CENTROIDAL AXIS ~ NOT NEEDED AS Y-AXIS THROUGH C.A.</i></p> $\text{Area } A = a (R^2 - r^2)$ $\bar{y} = .667 \frac{\sin a}{A} (R^3 - r^3)$ $I_X = .25 A (R^2 + r^2) \left[1 + \frac{\sin 2a}{2a} \right] - A \bar{y}^2$ $I_Y = .25 A (R^2 + r^2) \left[1 - \frac{\sin 2a}{2a} \right]$
<p>HOLLOW SEMI-CIRCLE (Not Thin-Walled)</p> 	$\text{Area } A = \frac{\pi}{2} (R^2 - r^2)$ $\bar{y} = \frac{4}{3\pi} \frac{R^3 - r^3}{(R^2 - r^2)}$ $I_Y = A/4 (R^2 + r^2)$ $I_B = A/4 (R^2 + r^2)$ $I_X = I_B - A \bar{y}^2$
<p>HOLLOW CIRCLE ECCENTRIC</p> 	<p>Eccentricity = e</p> $\text{Area } A = \pi (R^2 - r^2)$ $\bar{y} = \frac{r^2 e}{(R^2 - r^2)}$ $I_X = .25 A (R^2 + r^2) - 9.87 \frac{R^2 r^2 e^2}{A}$ $I_Y = .7854 (R^4 - r^4)$

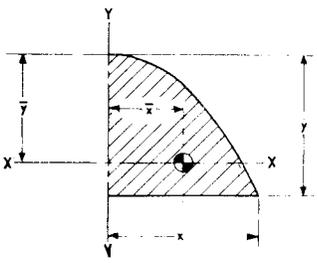
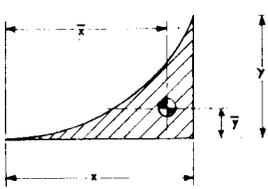
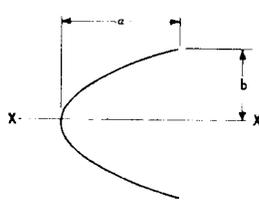


4.1.1 Tabular Summary (Cont'd)

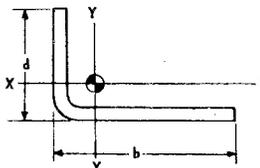
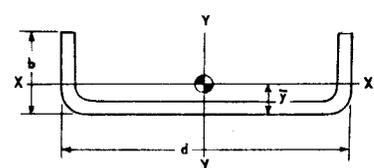
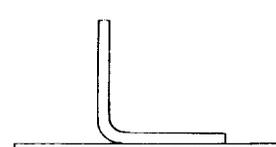
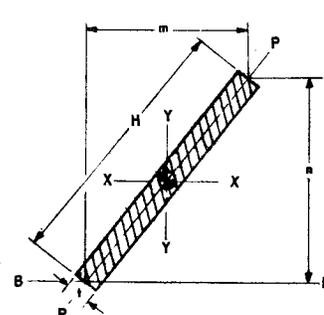
<p style="text-align: center;">SOLID ELLIPSE</p> 	$\text{Perimeter} = \pi \sqrt{2(a^2 + b^2) - \frac{(a - b)^2}{2.2}} \quad (\text{Approx.})$ $\text{Area } A = \pi a b$ $I_X = \frac{\pi}{4} a b^3$ $I_Y = \frac{\pi}{4} a^3 b$
<p style="text-align: center;">THIN-WALLED ELLIPSE</p> 	$I_X = \frac{\pi}{4} b^2 t [3a + b] \quad (\text{Approx.})$ $I_Y = \frac{\pi}{4} a^2 t [3b + a] \quad (\text{Approx.})$ $\text{Perimeter} = \pi \sqrt{2(a^2 + b^2) - \frac{(a - b)^2}{2.2}} \quad (\text{Approx.})$ <p style="text-align: center;">See Fig. 4.1.3-1</p>
<p style="text-align: center;">ANY THIN-WALLED ENCLOSED SECTION</p> 	$\text{Perimeter } p = p_1 + p_2 + p_3 + p_4$ $\text{Area } A = p_1 t_1 + p_2 t_2 + p_3 t_3 + p_4 t_4$ <p style="text-align: center;">Torsional stiffness constant</p> $J = \frac{4A_1^2}{\sum P/t} = \frac{4A_1^2 t}{P} \quad (\text{for constant } t)$ <p>where $\sum P/t = \frac{p_1}{t_1} + \frac{p_2}{t_2} + \frac{p_3}{t_3} + \frac{p_4}{t_4}$</p> <p>and $A_1 = \text{area enclosed by mean perimeter}$</p>
<p style="text-align: center;">PARABOLIC SEGMENT</p> 	$\text{Area} = \frac{4}{3} xy$ $\bar{y} = .60 y$ $I_C = .5714 xy^3$ $I_X = .0914 xy^3$ $I_B = .3048 xy^3$ $I_Y = .2667 x^3 y$

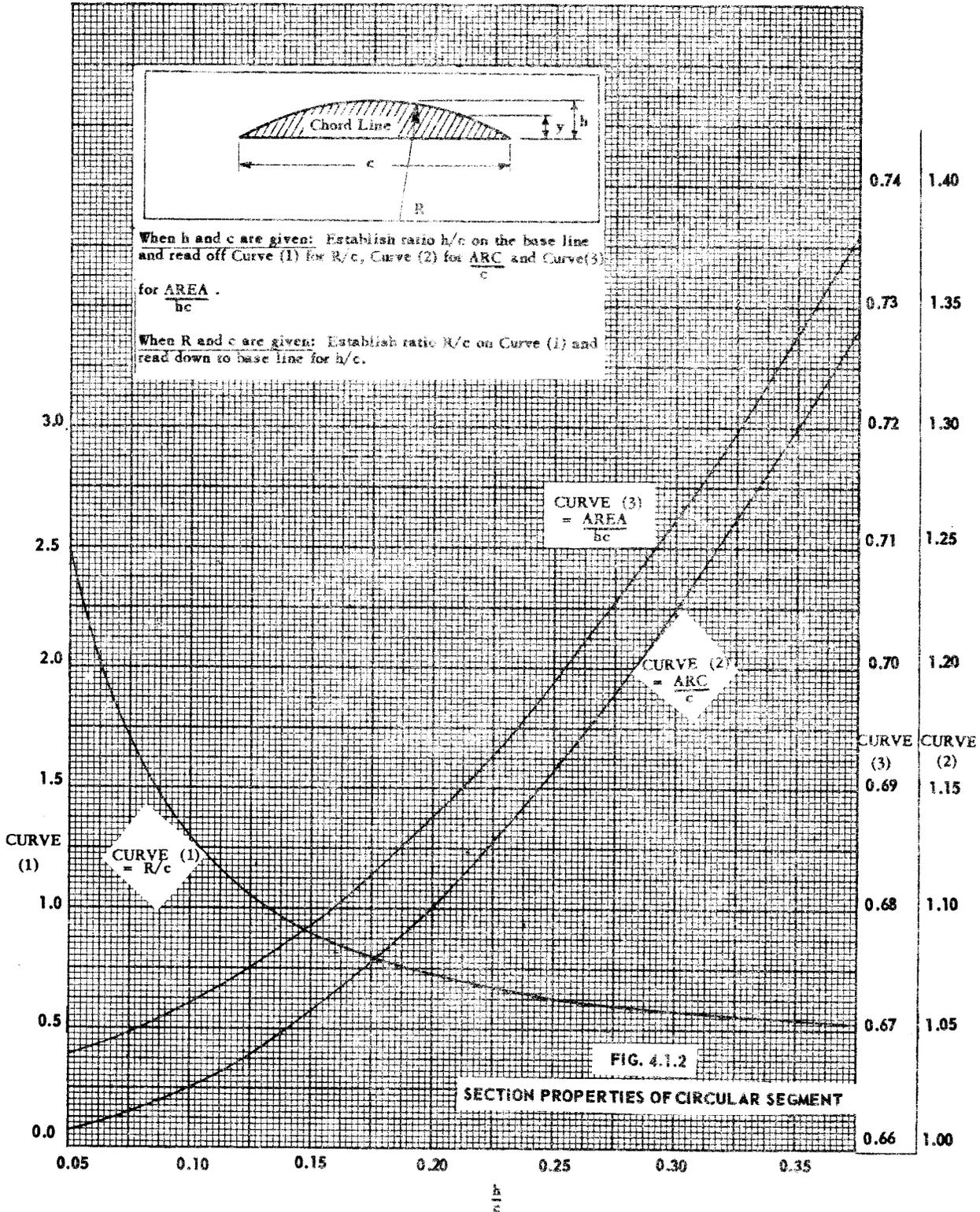


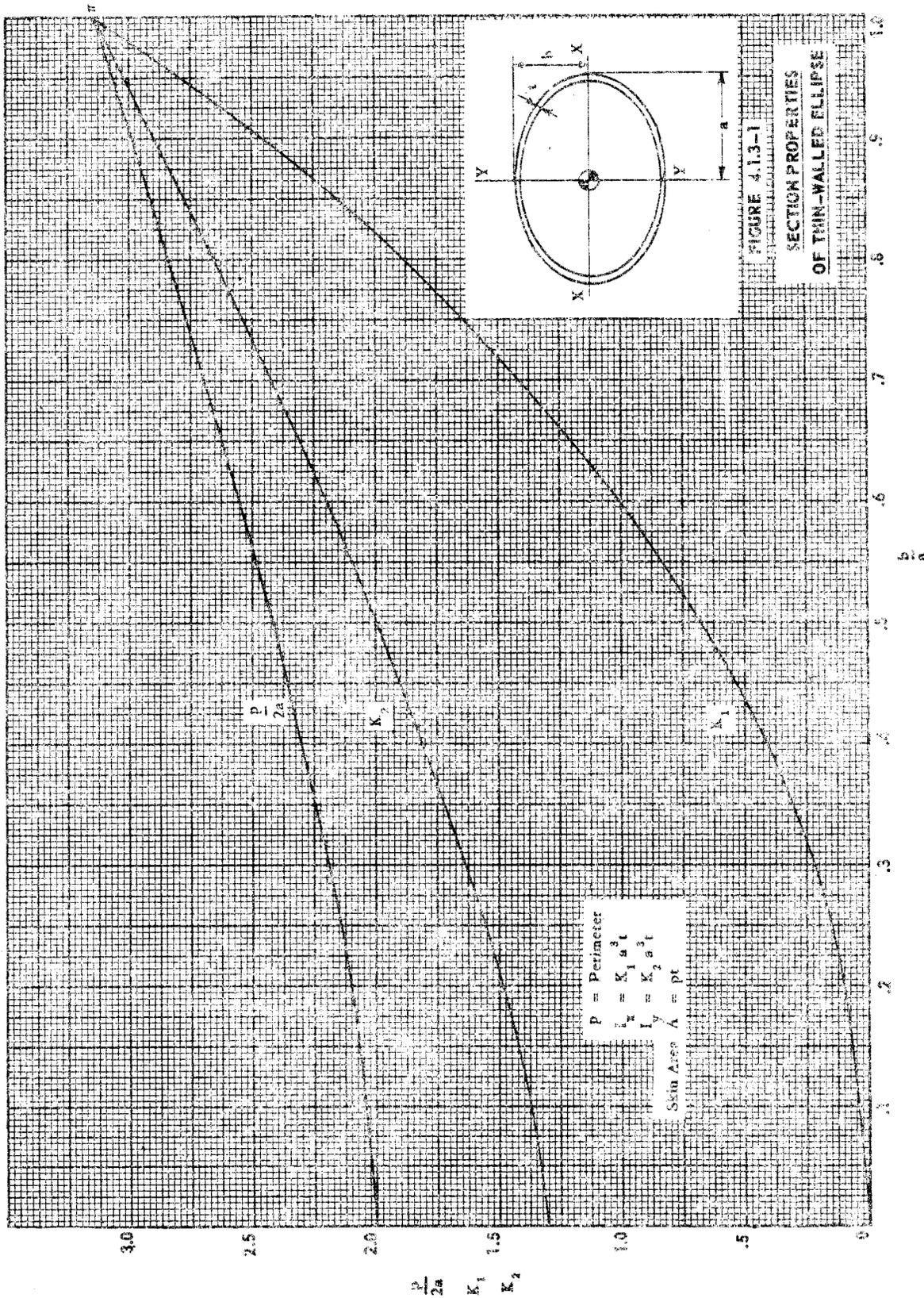
4.1.1 Tabular Summary (Cont'd)

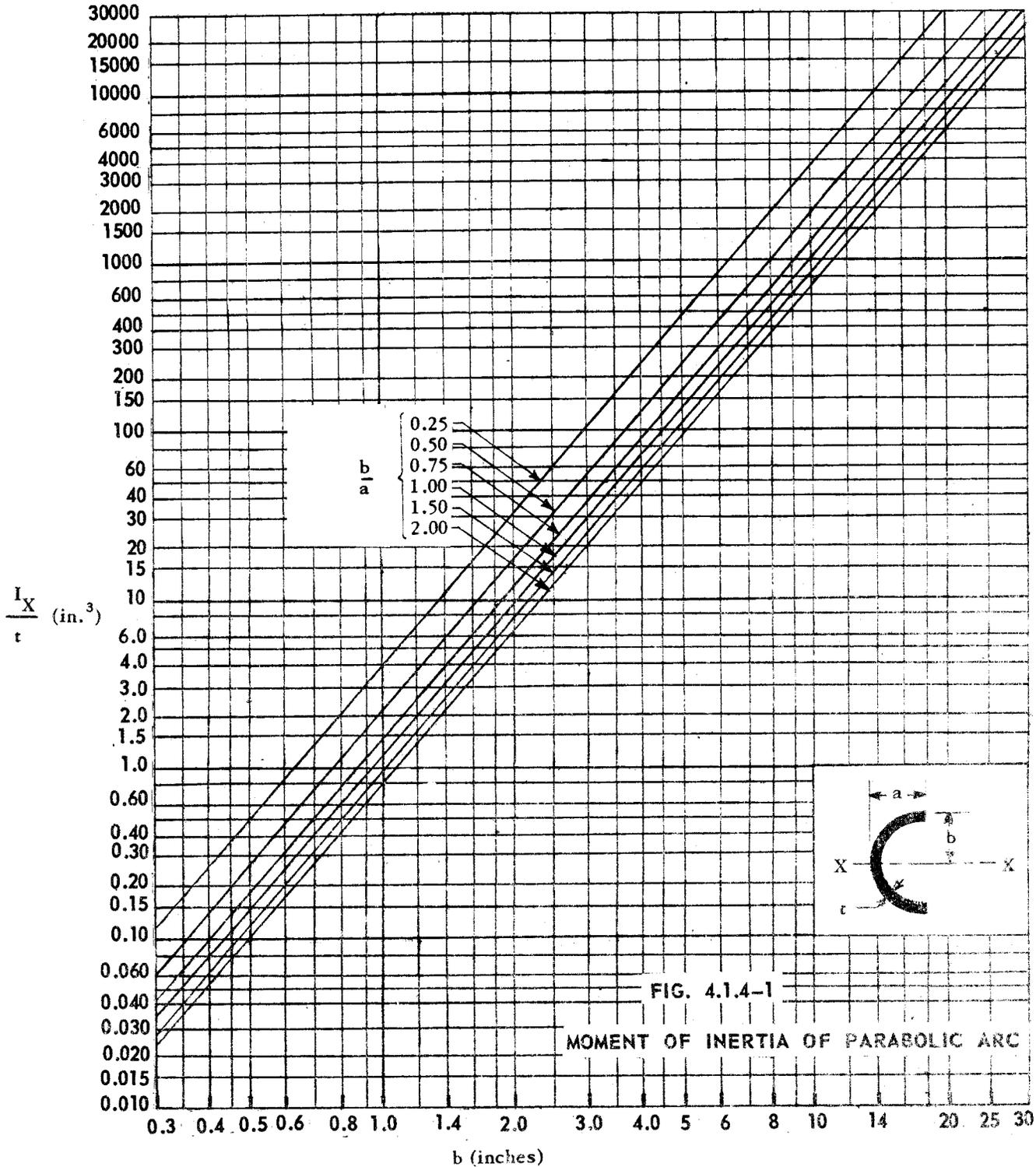
<p>PARABOLIC QUADRANT</p> 	$\text{Area} = \frac{2}{3} xy$ $\bar{x} = .375 x$ $\bar{y} = .60 y$
<p>PARABOLIC FILLET</p> 	$\text{Area} = \frac{1}{3} xy$ $\bar{x} = .75 x$ $\bar{y} = .30 y$
<p>PARABOLIC ARC</p> 	<p>See Fig. 4.1.4-1</p>
<p>TRAPEZOID</p> 	<p>See Fig. 4.1.5-1 and Fig. 4.1.5-2</p>

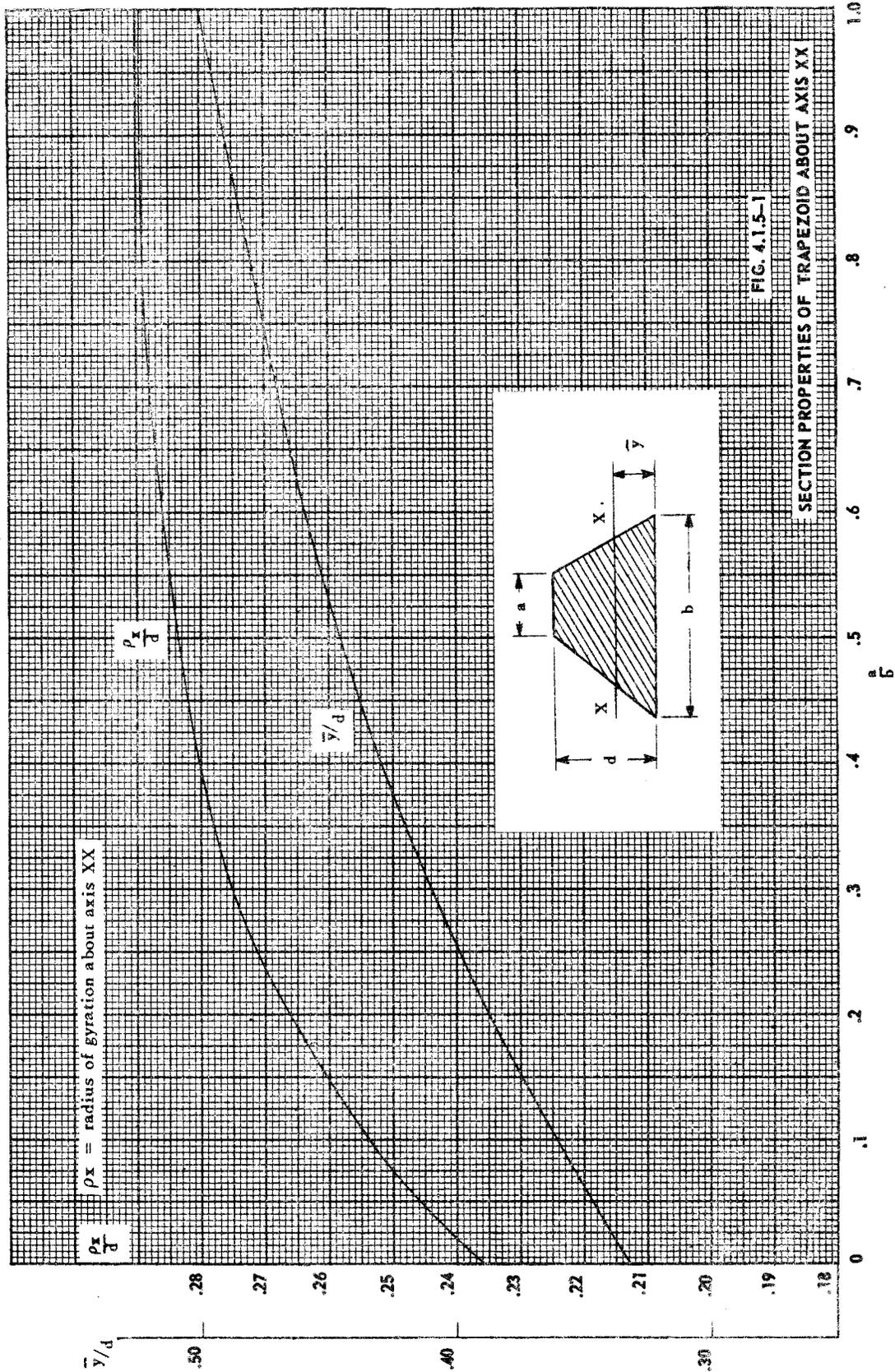
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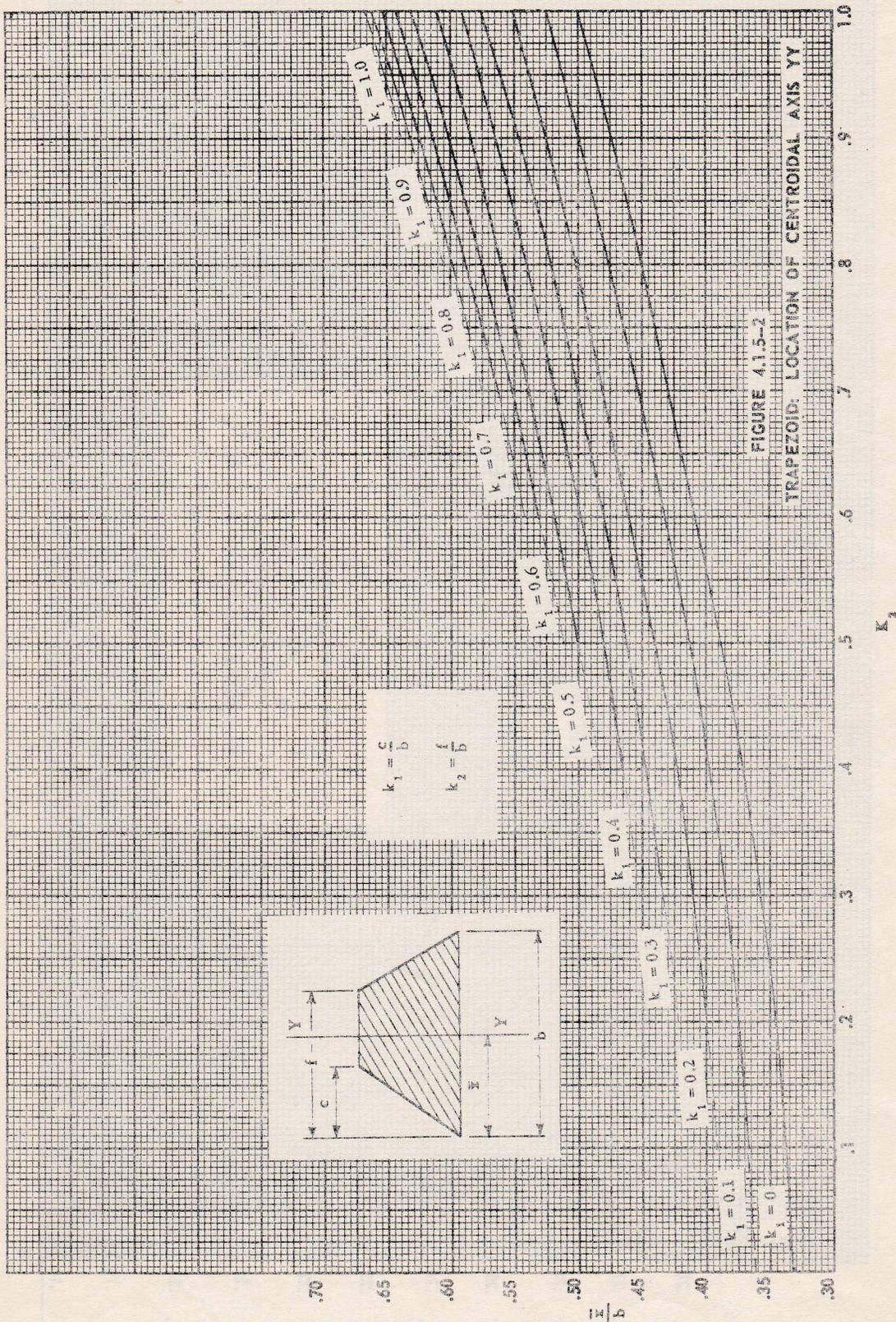
<p style="text-align: center;">90° ANGLE SECTION</p> 	<p>For approximate values of \bar{y} and ρ_x see Figs. 4.1.6-1 and 4.1.6.-2. Accurate values for certain cases appear in C.S. Drawings and Gen/TD/33 and 34.</p> <p>See R.Ae.S. Data Sheet 01.00.04 for Lipped Angle.</p>
<p style="text-align: center;">CHANNEL SECTION</p> 	<p>For approximate values of \bar{y} and ρ_x see Figs. 4.1.7.-1, 2 and 3. Accurate values for certain cases appear in C.S. Drawings and Gen/TD/34.</p> <p>See R.Ae.S. Data Sheet 01.00.02 for Lipped Channel.</p>
<p>ANY STRINGER - SKIN COMBINATION SUCH AS:</p> 	<p>See Section 4.1.8.</p>
<p style="text-align: center;">INCLINED RECTANGLE</p> 	<div style="display: flex; align-items: center;"> <div style="flex: 1;"> $\left. \begin{aligned} \text{Area } A &= Ht \\ I_X &= \frac{1}{12} An^2 \\ I_Y &= \frac{1}{12} Am^2 \\ I_B &= \frac{1}{3} An^2 \\ I_{XY} &= \frac{1}{12} Amn \end{aligned} \right\}$ </div> <div style="flex: 1; padding-left: 20px;"> <p>Formulas are approximate and assume that ratio t is small.</p> <p style="text-align: center;">\bar{H}</p> </div> </div>











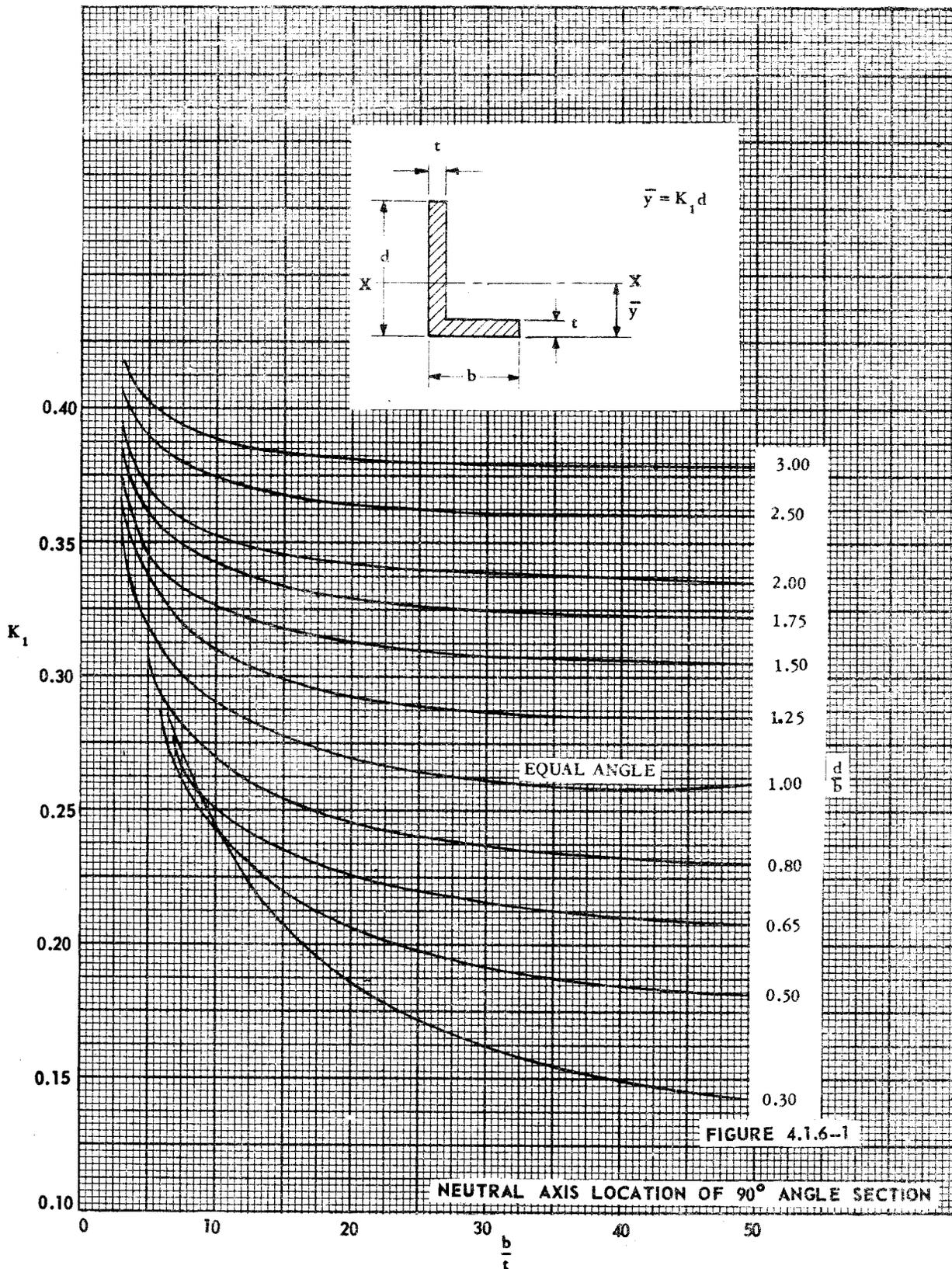
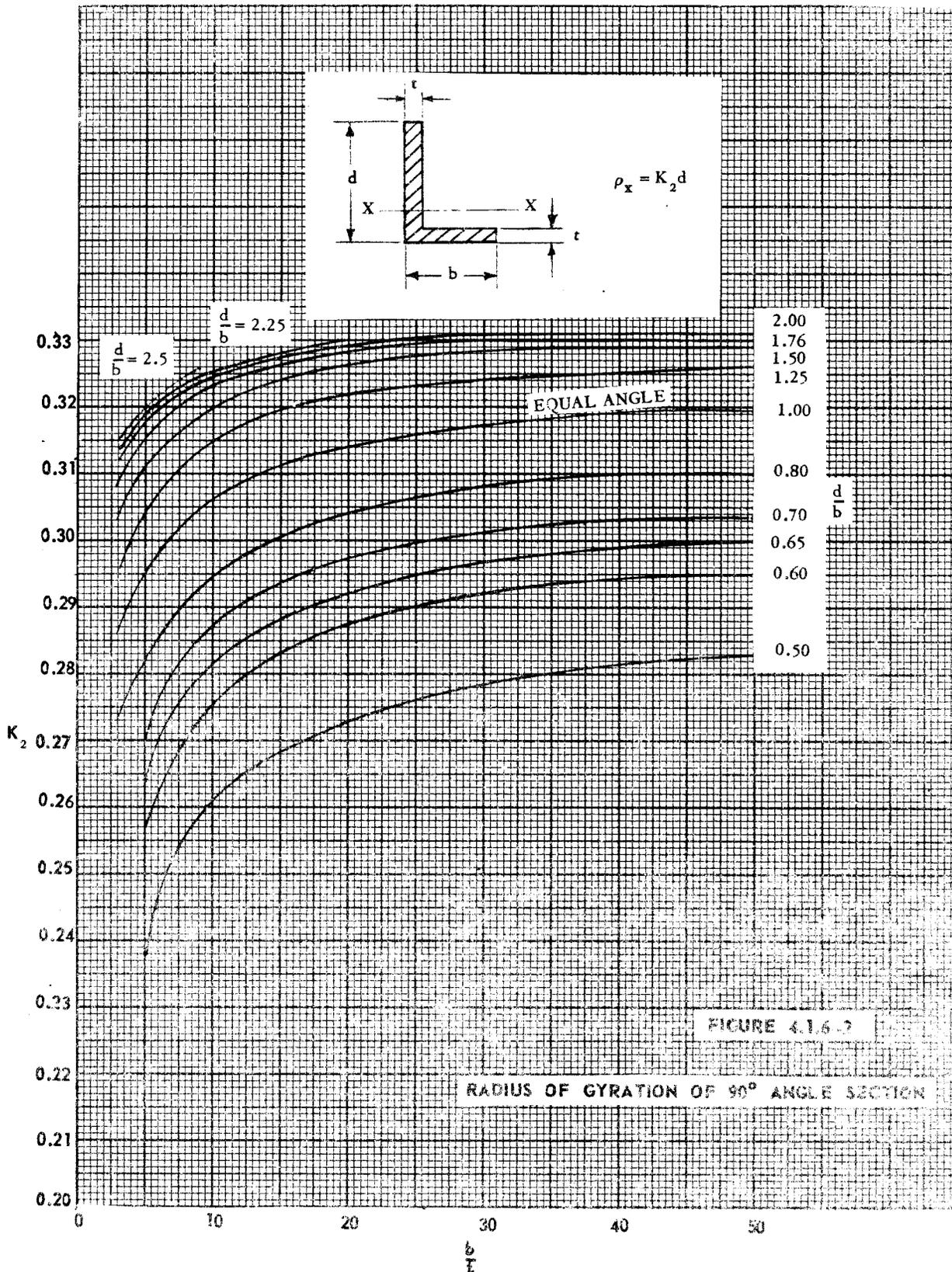
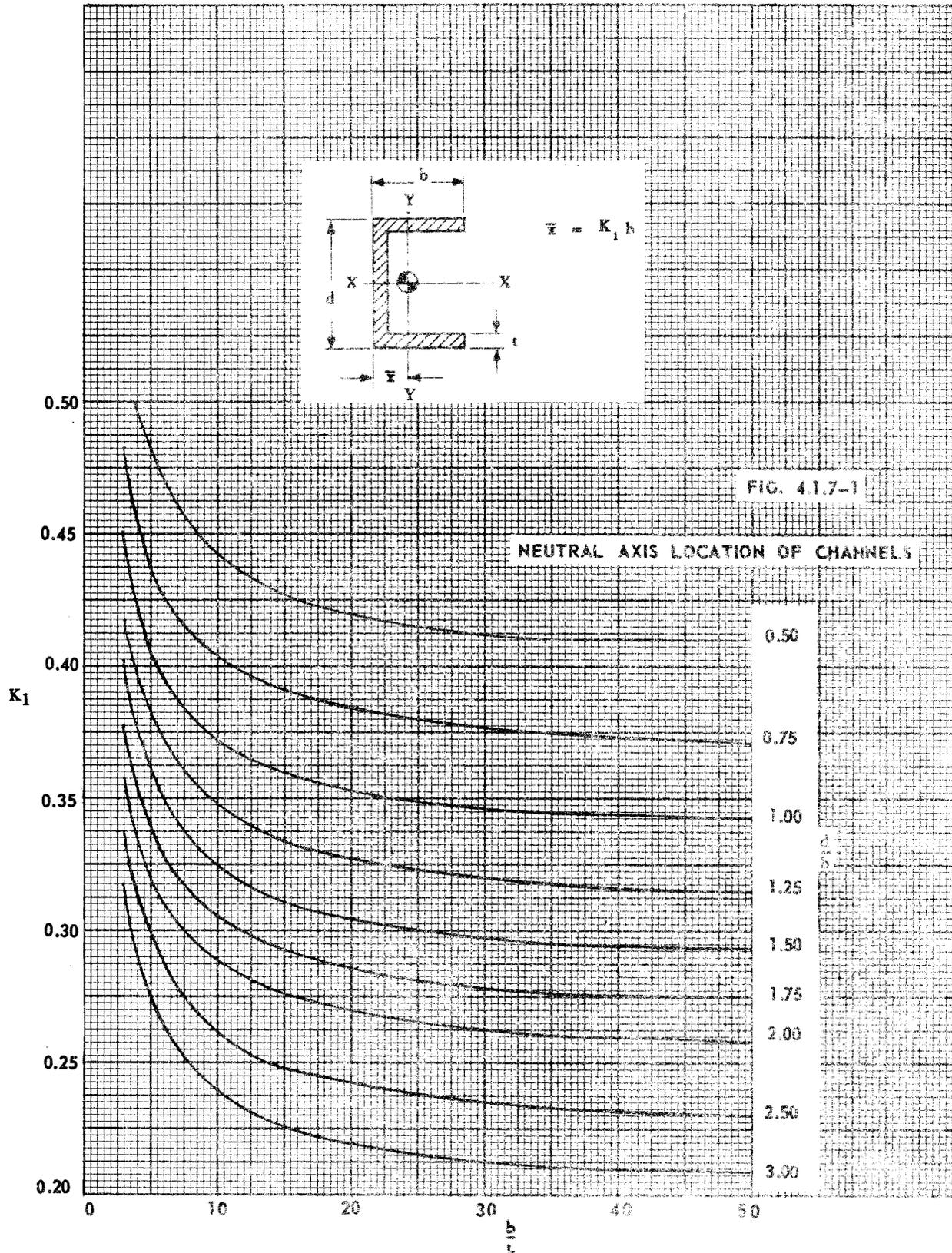
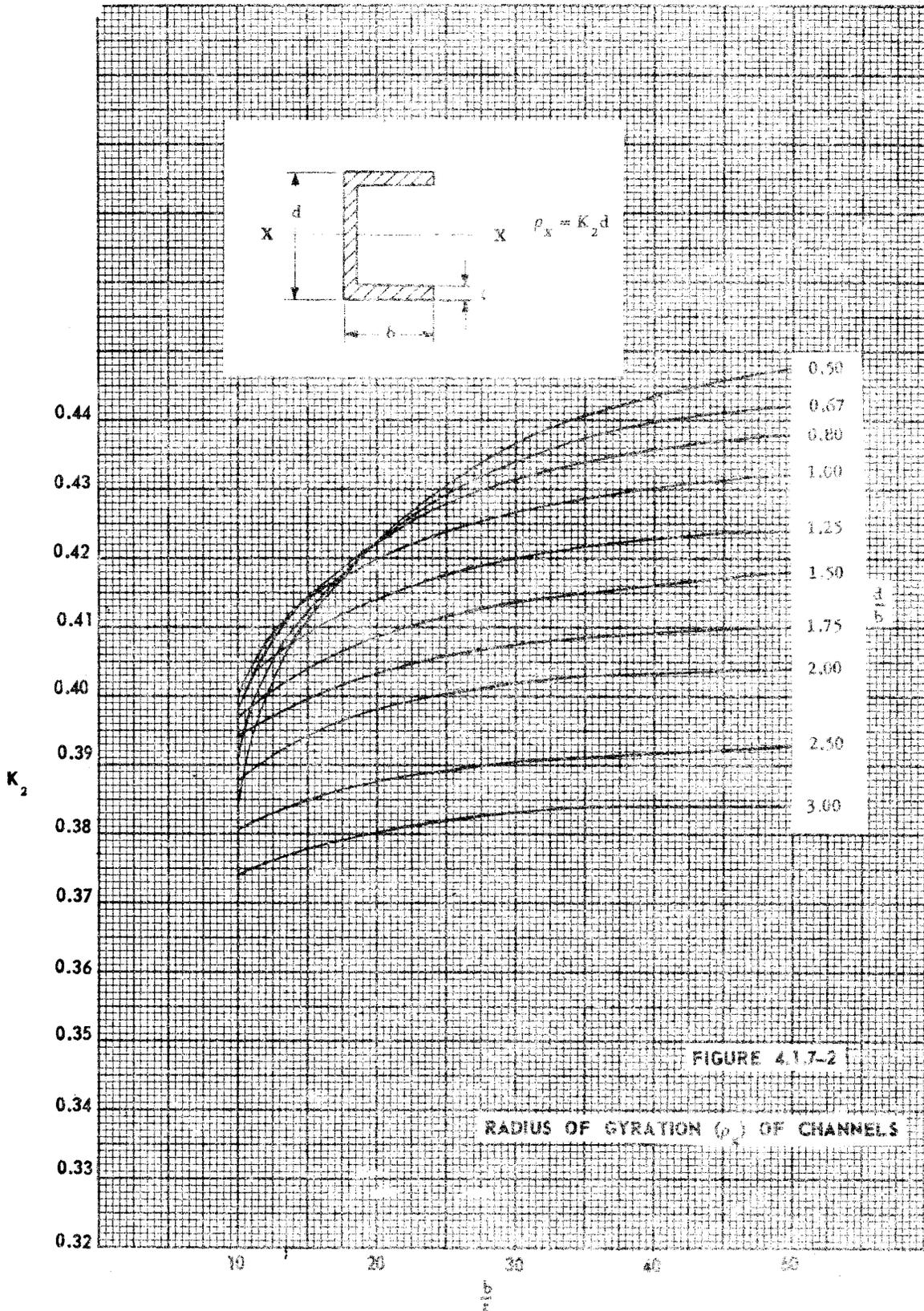


FIGURE 4.1.6-1

NEUTRAL AXIS LOCATION OF 90° ANGLE SECTION







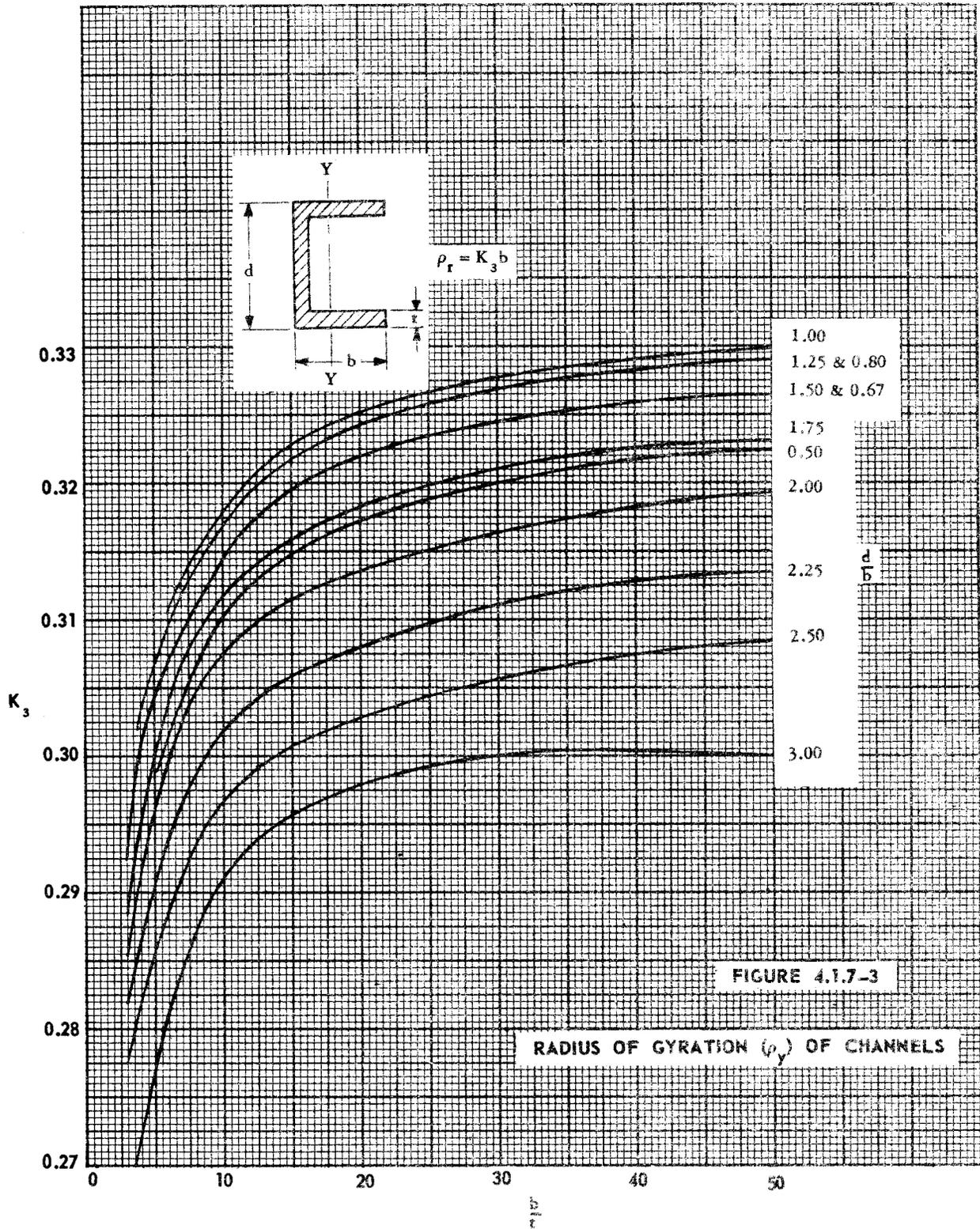
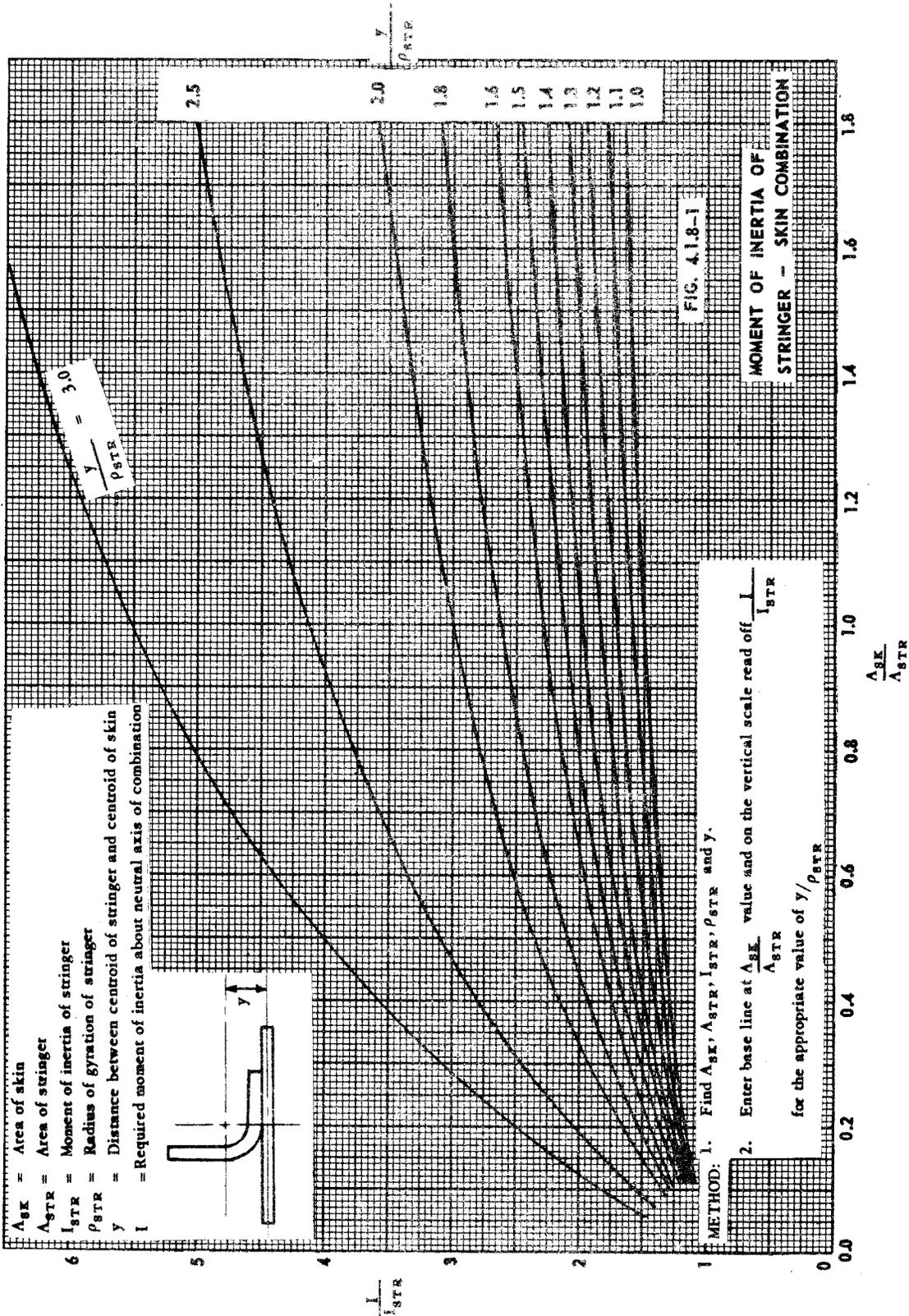


FIGURE 4.1.7-3

RADIUS OF GYRATION (ρ_y) OF CHANNELS



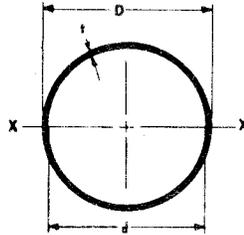


AIRCRAFT ENGINEERING MANUAL

VOL. I (DESIGN)

Sect V
Sub-Sect 4.2.1
Round Tubes

4.2.1 Round Tubes



$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$I_x = \frac{\pi}{64} (D^4 - d^4)$$

$$\rho_x = \frac{1}{4} \sqrt{D^2 + d^2}$$

$$Z_x = \frac{\pi}{32} \frac{(D^4 - d^4)}{D}$$

D (in.)	t (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	D/t	WEIGHT lb/100 in.		D (in.)	t (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	D/t	WEIGHT lb/100 in.	
							Steel	Dural								Steel	Dural
3/16	.028	.01403	.05726	.000046	.000491	6.70	.40	.14	5/8	.028	.05252	.2113	.002345	.007503	22.32	1.49	.53
	.035	.01677	.05531	.000051	.000547	5.36	.48	.17		.035	.06487	.2090	.002833	.009065	17.85	1.84	.66
1/4	.022	.01576	.08100	.000103	.000825	11.38	.45	.16	3/4	.049	.08867	.2044	.003704	.01185	12.77	2.51	.90
	.028	.01953	.07911	.000122	.000978	8.93	.55	.20		.058	.1033	.2016	.004195	.01342	10.79	2.93	1.05
	.035	.02364	.07701	.000140	.001122	7.14	.67	.24		.065	.1144	.1993	.004543	.01454	9.62	3.24	1.16
	.049	.03094	.07314	.000166	.001324	5.10	.88	.31		.083	.1413	.1939	.005311	.01700	7.53	4.00	1.43
	.058	.03498	.07091	.000176	.001407	4.31	.99	.35		.095	.1582	.1904	.005732	.01834	6.58	4.48	1.60
	.065	.03778	.06933	.000182	.001453	3.85	1.07	.38		.120	.1904	.1835	.006412	.02052	5.21	5.30	1.92
5/16	.022	.02008	.1030	.000213	.001363	14.20	.57	.20	11/16	.028	.05801	.2334	.003160	.009192	24.55	1.64	.59
	.028	.02503	.1011	.000256	.001636	11.16	.71	.25		.035	.07175	.2310	.003829	.01114	19.64	2.03	.73
	.035	.03051	.09889	.000298	.001910	8.93	.86	.31		.049	.09829	.2264	.005038	.01466	14.03	2.78	.99
	.049	.04056	.09476	.000364	.002331	6.38	1.15	.41		.058	.1147	.2235	.005730	.01667	11.85	3.25	1.16
	.056	.04637	.09228	.000395	.002527	5.39	1.31	.47	3/4	.028	.06351	.2555	.004145	.01105	26.80	1.80	.65
	.065	.05054	.09048	.000414	.002648	4.81	1.43	.51		.035	.07862	.2531	.005036	.01343	21.42	2.23	.80
3/8	.028	.03053	.1231	.000462	.002466	13.39	.86	.31	.049	.1079	.2485	.006661	.01776	15.30	3.06	1.09	
	.035	.03738	.1208	.000546	.002912	10.72	1.06	.38	.058	.1261	.2455	.007601	.02027	12.94	3.57	1.28	
	.049	.05018	.1166	.000682	.003636	7.65	1.43	.51	.065	.1399	.2433	.008278	.02208	11.53	3.96	1.42	
	.058	.05776	.1139	.000750	.003999	6.47	1.64	.58	.083	.1739	.2376	.009820	.02619	9.04	4.93	1.76	
	.065	.06330	.1120	.000794	.004234	5.77	1.79	.64	.095	.1955	.2340	.01070	.02854	7.89	5.54	1.98	
	.083	.07614	.1073	.000877	.004678	4.52	2.16	.77	.120	.2375	.2267	.01221	.03256	6.25	6.73	2.40	
7/16	.028	.03602	.1451	.000759	.003468	15.63	1.02	.36	7/8	.156	.2911	.2171	.01372	.03660	4.81	8.25	2.94
	.035	.04426	.1428	.000903	.004128	12.50	1.25	.45		.028	.07451	.2996	.006689	.01529	31.23	2.11	.76
	.049	.05981	.1384	.001146	.005240	8.93	1.69	.60		.035	.09236	.2973	.008161	.01865	25.00	2.62	.94
	.058	.06915	.1357	.001274	.005824	7.54	1.96	.70		.049	.1272	.2925	.01088	.02487	17.85	3.60	1.29
	.065	.07607	.1337	.001360	.006215	6.73	2.16	.77		.058	.1489	.2896	.01248	.02853	15.10	4.22	1.51
	.083	.09244	.1287	.001532	.007002	5.27	2.62	.93		.065	.1654	.2865	.01365	.03121	13.47	4.66	1.68
1/2	.028	.03304	.1692	.000946	.003782	22.73	.94	.33	1	.095	.2328	.2778	.01797	.04107	9.21	6.59	2.35
	.035	.04152	.1672	.001160	.004641	17.86	1.18	.42		.120	.2846	.2703	.02079	.04753	7.29	8.06	2.88
	.049	.05693	.1604	.001390	.005559	14.28	1.45	.52		.156	.3524	.2601	.02384	.05450	5.61	9.98	3.56
	.058	.08054	.1576	.002001	.008002	8.62	2.28	.81		.028	.08550	.3438	.01011	.02021	35.71	2.42	.86
	.065	.08883	.1555	.002148	.008592	7.69	2.52	.90		.035	.1061	.3414	.01237	.02474	28.56	3.01	1.07
	.083	.1087	.1503	.002457	.009828	6.02	3.08	1.10		.049	.1464	.3367	.01659	.03319	20.40	4.15	1.48
9/16	.028	.04702	.1892	.001684	.005986	20.09	1.33	.48	1-1/8	.058	.1716	.3337	.01911	.03822	17.25	4.86	1.74
	.035	.05800	.1869	.002026	.007205	16.07	1.64	.59		.065	.1909	.3314	.02097	.04193	15.38	5.41	1.93
	.049	.07905	.1824	.002629	.009348	11.48	2.24	.80		.083	.2391	.3255	.02534	.05068	12.05	6.77	2.42
	.058	.09193	.1795	.002963	.01054	9.70	2.60	.93		.095	.2701	.3217	.02796	.05591	10.53	7.65	2.73
	.065	.10913	.1775	.003296	.01176	8.31	2.96	1.03		.120	.3318	.3140	.03271	.06542	8.33	9.40	3.35
	.120	.1433	.1409	.002844	.01137	4.17	4.06	1.45		.156	.4142	.3034	.03809	.07618	6.41	11.72	4.18
3/4	.028	.04702	.1892	.001684	.005986	20.09	1.33	.48	1-1/8	.187	.4776	.2949	.04155	.08310	5.35	13.53	4.83
	.035	.05800	.1869	.002026	.007205	16.07	1.64	.59		.028	.09640	.3880	.01453	.02582	40.18	2.73	.98
	.049	.07905	.1824	.002629	.009348	11.48	2.24	.80		.035	.1198	.3856	.01782	.03168	32.10	3.40	1.21
	.058	.09193	.1795	.002963	.01054	9.70	2.60	.93		.049	.1656	.3808	.02402	.04270	22.95	4.68	1.68
	.065	.10913	.1775	.003296	.01176	8.31	2.96	1.03		.058	.1944	.3780	.02775	.04933	19.40	5.51	1.97
	.120	.1468	.1621	.004383	.01558	4.69	4.73	1.69		.065	.2165	.3755	.03052	.05425	17.30	6.14	2.20
5/8	.028	.04702	.1892	.001684	.005986	20.09	1.33	.48	1-1/8	.083	.2717	.3696	.03711	.06597	13.55	7.70	2.75
	.035	.05800	.1869	.002026	.007205	16.07	1.64	.59		.095	.3074	.3657	.04111	.07309	11.84	8.71	3.11
	.049	.07905	.1824	.002629	.009348	11.48	2.24	.80		.120	.3789	.3578	.04852	.08625	9.38	10.73	3.83
	.058	.09193	.1795	.002963	.01054	9.70	2.60	.93									
	.065	.10913	.1775	.003296	.01176	8.31	2.96	1.03									
	.120	.1468	.1621	.004383	.01558	4.69	4.73	1.69									



4.2.1 Round Tubes (Cont'd)

D (in.)	t (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	D/t	WEIGHT lb/100 in.		D (in.)	t (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	D/t	WEIGHT lb/100 in.		
							Steel	Dural								Steel	Dural	
1-1/4	.028	.1075	.4321	.02007	.03212	44.64	3.05	1.09	2	.120	.7087	.6660	.3144	.3144	16.67	20.08	7.17	
	.035	.1336	.4297	.02467	.03948	35.70	3.78	1.35		.156	.9037	.6543	.3869	.3869	12.82	25.60	9.14	
	.049	.1849	.4250	.03339	.05342	25.50	5.23	1.87		.188	1.070	.6441	.4440	.4440	10.64	30.32	10.82	
	.058	.2172	.4219	.03867	.06187	21.55	6.15	2.20		.250	1.374	.6250	.5259	.5369	8.00	38.94	13.90	
	.065	.2420	.4196	.04260	.06816	19.22	6.86	2.45		2-1/8	.035	.2298	.7390	.1255	.1181	60.71	6.51	2.32
	.083	.3043	.4136	.05206	.08330	15.06	8.62	3.08			.049	.3196	.7342	.1723	.1621	43.37	9.05	3.23
	.095	.3447	.4097	.05787	.09259	13.16	9.77	3.49			.058	.3766	.7311	.2013	.1895	36.64	10.67	3.81
	.120	.4260	.4018	.06876	.1100	10.42	12.07	4.31			.065	.4207	.7287	.2234	.2102	32.69	11.92	4.25
	.156	.5362	.3907	.08184	.1310	8.01	15.19	5.42			.083	.5325	.7226	.2780	.2616	25.60	15.08	5.38
	1-3/8	.028	.1185	.4764	.02689	.03911	49.11	3.36			1.20	.095	.6059	.7185	.3128	.2944	22.37	17.16
.035		.1473	.4739	.03309	.04814	39.25	4.17	1.49	.120	.7559	.7101	.3812	.3588	17.71	21.41	7.64		
.049		.2041	.4691	.04492	.06534	28.05	5.78	2.07	.156	.9650	.6983	.4706	.4429	13.62	27.34	9.76		
.058		.2400	.4661	.05213	.07583	23.70	6.80	2.43	2-1/4	.035	.2436	.7832	.1494	.1328	64.29	6.90	2.46	
.065		.2675	.4636	.05753	.08367	21.15	7.58	2.70		.049	.3388	.7783	.2052	.1824	45.90	9.59	3.43	
.083		.3369	.4577	.07059	.1027	16.57	9.54	3.41		.058	.3994	.7753	.2401	.2134	38.80	11.30	4.05	
.095		.3820	.4538	.07867	.1144	14.47	10.82	3.86		.065	.4462	.7728	.2665	.2369	34.60	12.64	4.52	
.120		.4731	.4457	.09400	.1367	11.46	13.40	4.78		.083	.5651	.7667	.3322	.2953	27.15	16.01	5.72	
.156		.5974	.4345	.1129	.1642	8.81	16.92	6.04		.095	.6432	.7626	.3741	.3325	23.70	18.22	6.51	
.187		.6979	.4252	.1262	.1835	7.35	19.77	7.07	.120	.8030	.7543	.4568	.4061	18.75	22.75	8.12		
1-1/2	.028	.1295	.5205	.03508	.04678	53.57	3.67	1.31	.156	1.026	.7424	.5656	.5028	14.42	29.07	10.38		
	.035	.1611	.5181	.04324	.05765	42.80	4.56	1.63	.188	1.218	.7320	.6526	.5801	11.97	34.50	12.31		
	.049	.2234	.5132	.05885	.07847	30.60	6.32	2.26	.250	1.571	.7216	.7977	.7090	9.00	44.50	15.88		
	.058	.2628	.5102	.06841	.09121	25.85	7.45	2.66	2-5/16	.250	1.620	.7345	.8740	.7559	9.25	45.89	16.38	
	.065	.2930	.5079	.07558	.1008	23.05	8.30	2.97		2-3/8	.049	.3581	.8225	.2423	.2040	48.47	10.14	3.62
	.083	.3695	.5018	.09305	.1241	18.08	10.47	3.74			.058	.4222	.8194	.2835	.2387	40.95	11.96	4.27
	.095	.4193	.4979	.1039	.1386	15.79	11.98	4.24			.065	.4717	.8170	.3149	.2651	36.54	13.36	4.77
	.120	.5202	.4897	.1248	.1664	12.50	14.74	5.26			.083	.5976	.8109	.3930	.3309	28.61	16.93	6.04
	.156	.6587	.4784	.1507	.2010	9.62	18.66	6.66			.095	.6805	.8068	.4429	.3730	25.00	19.28	6.88
	.187	.7714	.4689	.1696	.2261	8.02	21.85	7.80	.120		.8501	.7984	.5419	.4563	19.79	24.08	8.59	
1-5/8	.028	.1405	.5647	.04480	.05514	58.04	3.98	1.42	.156	1.088	.7865	.6727	.5664	15.22	30.81	10.99		
	.035	.1748	.5622	.05528	.06803	46.40	4.95	1.77	.188	1.292	.7761	.7780	.6551	12.63	36.59	13.06		
	.049	.2426	.5575	.07540	.09279	33.15	6.87	2.46	.250	1.669	.7565	.9551	.8043	9.50	47.28	16.87		
	.058	.2855	.5544	.08776	.1080	28.00	8.09	2.89	.375	2.356	.7194	1.220	1.027	6.33	66.75	23.82		
	.065	.3186	.5520	.09707	.1195	25.00	9.05	3.23	2-1/2	.049	.3773	.8667	.2834	.2267	51.00	10.68	3.82	
	.083	.4021	.5459	.1198	.1475	19.58	11.40	4.06		.058	.4450	.8635	.3318	.2655	43.10	12.60	4.50	
	.095	.4566	.5420	.1341	.1651	17.11	12.94	4.62		.065	.4972	.8613	.3688	.2950	38.45	14.09	5.03	
	.120	.5674	.5338	.1617	.1990	13.54	16.07	5.74		.083	.6302	.8550	.4607	.3686	30.10	17.85	6.38	
	1-3/4	.028	.1515	.6089	.05616	.06418	62.50	4.29		1.53	.095	.7178	.8509	.5197	.4158	26.30	20.34	7.27
		.035	.1885	.6065	.06936	.07927	50.00	5.32		1.91	.120	.8972	.8425	.6369	.5095	20.83	25.42	9.07
.049		.2618	.6017	.09478	.1083	35.70	7.42	2.65	.156	1.149	.8306	.7925	.6340	16.03	32.54	11.61		
.058		.3083	.5986	.1105	.1262	30.20	8.73	3.12	.188	1.366	.8201	.9184	.7347	13.30	38.68	13.81		
.065		.3441	.5962	.1223	.1398	26.90	9.75	3.48	.250	1.767	.8004	1.132	.9057	10.00	50.06	17.87		
.083		.4347	.5901	.1514	.1730	21.10	12.32	4.40	2-3/4	.049	.4158	.9551	.3793	.2759	56.10	11.78	4.20	
.095		.4939	.5861	.1697	.1939	18.42	13.99	4.99		.058	.4905	.9521	.4446	.3233	47.40	13.90	4.96	
.120		.6145	.5778	.2052	.2345	14.58	17.41	6.21		.065	.5483	.9496	.4944	.3596	42.30	15.50	5.55	
1-7/8		.028	.1625	.6531	.06930	.07392	66.96	4.60		1.64	.083	.6954	.9434	.6189	.4501	33.15	19.70	7.04
		.035	.2023	.6507	.08565	.09136	53.60	5.73		2.04	.095	.7924	.9393	.6991	.5084	28.95	22.48	8.03
	.049	.2811	.6458	.1172	.1250	38.25	7.95	2.84		.120	.9915	.9308	.8590	.6247	22.92	28.09	10.02	
	.058	.3311	.6427	.1368	.1459	32.30	9.38	3.35	.156	1.271	.9188	1.073	.7805	17.63	36.02	12.85		
	.065	.3696	.6404	.1516	.1617	28.80	10.47	3.74	.188	1.513	.9082	1.248	.9078	14.63	42.87	15.30		
	.083	.4673	.6342	.1880	.2005	22.60	13.25	4.73	.250	1.964	.8883	1.549	1.127	11.00	55.63	19.85		
	.095	.5312	.6302	.2110	.2251	19.74	15.05	5.37	3	.049	.4543	1.044	.4946	.3298	61.22	12.87	4.59	
	.120	.6616	.6219	.2559	.2730	15.63	18.74	6.69		.058	.5361	1.040	.5802	.3868	51.70	15.18	5.42	
	2	.028	.1735	.6973	.08434	.08434	71.43	4.91		1.75	.065	.5993	1.038	.6457	.4305	46.20	16.95	6.06
		.035	.2161	.6948	.1043	.1043	57.14	6.12		2.18	.083	.7606	1.032	.8097	.5398	36.15	21.55	7.70
.049		.3003	.6900	.1430	.1430	40.80	8.50	3.04		.095	.8670	1.028	.9156	.6104	31.58	24.56	8.78	
.058		.3539	.6869	.1670	.1670	34.45	10.03	3.58		.120	1.086	1.019	1.128	.7518	25.00	30.76	11.00	
.065		.3951	.6845	.1851	.1851	30.75	11.19	4.00	.156	1.394	1.007	1.413	.9423	19.23	39.49	14.09		
.083		.4999	.6783	.2300	.2301	24.10	14.16	5.06	.188	1.661	.9964	1.649	1.099	15.96	47.05	16.79		
.095		.5685	.6744	.2586	.2586	21.05	16.11	5.76	.250	2.160	.9763	2.059	1.372	12.00	61.19	21.84		
.109		.6476	.6697	.2904	.2904	18.35	18.35	6.55	.375	3.093	.9375	2.718	1.812	8.00	87.61	31.27		

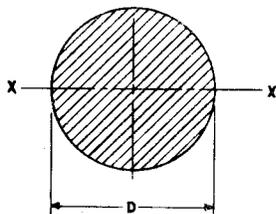


4.2.1 Round Tubes (Cont'd)

D (in.)	t (in.)	A (in. ²)	ρ _x (in.)	I _x (in. ⁴)	Z _x (in. ³)	D/t	WEIGHT lb/100 in.		D (in.)	t (in.)	A (in. ²)	ρ _x (in.)	I _x (in. ⁴)	Z _x (in. ³)	D/t	WEIGHT lb/100 in.		
							Steel	Dural								Steel	Dural	
3-1/4	.049	.4928	1.132	.6313	.3885	66.33	13.96	4.98	4-1/4	.095	1.240	1.469	2.677	1.260	44.74	35.13	12.54	
	.053	.5816	1.129	.7410	.4560	55.10	16.47	5.89		.120	1.557	1.461	3.322	1.564	35.42	41.11	15.74	
	.065	.6804	1.126	.8251	.5077	50.00	13.40	6.58		.134	1.733	1.456	3.673	1.741	31.78	49.10	17.55	
	.083	.8258	1.120	1.036	.6376	39.15	23.38	8.35		.155	2.006	1.448	4.210	1.981	27.24	56.84	20.29	
	.095	.9416	1.116	1.173	.7217	34.20	26.66	9.52		.188	2.399	1.433	4.859	2.334	22.60	67.97	24.25	
	.120	1.180	1.107	1.447	.8906	27.10	33.43	11.95		.219	2.773	1.427	5.630	2.659	19.46	78.57	28.04	
	.156	1.516	1.095	1.819	1.119	20.83	42.96	15.33		.250	3.142	1.417	6.303	2.968	17.06	89.00	31.76	
	.188	1.808	1.085	2.127	1.309	17.29	51.23	18.28		.313	3.871	1.395	7.348	3.352	13.53	109.67	39.14	
	.250	2.355	1.064	2.669	1.643	13.00	66.75	23.82										
	3-1/2	.065	.7014	1.215	1.035	.5914	53.80	19.85		7.09	4-1/2	.065	.9056	1.558	2.227	.9898	69.23	25.66
.083		.8910	1.208	1.301	.7435	42.20	25.20	9.01	.083	1.152		1.562	2.810	1.249	54.22	32.65	11.64	
.095		1.016	1.204	1.474	.8422	36.85	28.70	10.25	.095	1.315		1.558	3.190	1.418	47.37	37.24	13.29	
.120		1.274	1.196	1.822	1.041	29.15	36.00	12.89	.120	1.651		1.549	3.963	1.761	37.50	46.78	16.69	
.156		1.639	1.184	2.296	1.312	22.44	46.43	16.57	.156	2.129		1.537	5.028	2.235	28.80	60.40	21.55	
.188		1.956	1.173	2.691	1.538	18.62	55.42	19.78	.188	2.547		1.526	5.930	2.636	23.94	72.15	25.75	
.250		2.553	1.152	3.390	1.937	14.00	72.31	25.81	.219	2.945		1.516	6.755	3.007	20.55	83.44	29.78	
									.250	3.338		1.505	7.562	3.361	18.00	94.56	33.75	
3-3/4	.065	.7525	1.303	1.278	.6814	57.60	21.30	7.60	4-3/4	.065	.9567	1.657	2.025	1.195	73.08	27.10	9.67	
	.083	.9562	1.297	1.608	.8576	45.20	27.06	9.67		.083	1.217	1.650	3.314	1.396	57.23	34.48	12.30	
	.095	1.091	1.293	1.823	.9722	39.50	30.84	11.04		.095	1.389	1.646	3.765	1.585	50.00	39.36	14.05	
	.120	1.368	1.284	2.256	1.204	31.25	38.70	13.82		.120	1.745	1.638	4.680	1.971	39.58	49.45	17.65	
	.156	1.761	1.272	2.849	1.520	24.04	49.90	17.81		.156	2.251	1.625	5.946	2.504	30.45	63.78	22.76	
	.188	2.104	1.261	3.346	1.784	19.95	59.60	21.27		.188	2.694	1.614	7.021	2.926	25.25	76.25	27.20	
	.250	2.749	1.241	4.231	2.256	15.00	77.88	27.79		.219	3.117	1.604	8.019	3.376	21.69	88.32	31.52	
4	.083	.8035	1.392	1.556	.7779	61.50	22.75	8.12	5	.083	1.282	1.739	3.876	1.550	60.24	36.32	12.96	
	.095	1.021	1.385	1.960	.9799	48.20	28.95	10.32		.095	1.464	1.734	4.404	1.762	52.63	41.47	14.80	
	.120	1.463	1.372	2.755	1.378	33.33	41.40	14.80		.120	1.840	1.726	5.480	2.192	41.67	52.12	18.60	
	.156	1.884	1.360	3.485	1.743	25.64	53.37	19.05		.156	2.374	1.714	6.970	2.788	32.05	67.26	24.00	
	.188	2.251	1.349	4.099	2.050	21.28	63.78	22.76		.188	2.842	1.703	8.239	3.295	26.50	80.52	28.73	
	.219	2.601	1.339	4.664	2.322	18.26	73.70	26.30		.219	3.289	1.692	9.418	3.767	22.83	93.19	33.26	
	.250	2.945	1.329	5.200	2.600	16.00	83.44	29.78		.250	3.731	1.682	10.55	4.220	20.00	105.69	37.72	
4-1/4	.065	.8546	1.480	1.871	.8805	55.38	24.21	8.64		.313	4.609	1.661	12.7	5.085	15.97	130.07	46.60	
	.083	1.087	1.474	2.359	1.110	51.20	30.78	10.99		.375	5.449	1.640	14.66	5.856	13.33	154.36	55.09	



4.2.2 Round Bar



$$A = \frac{\pi}{4} D^2$$

$$\rho_x = D/4$$

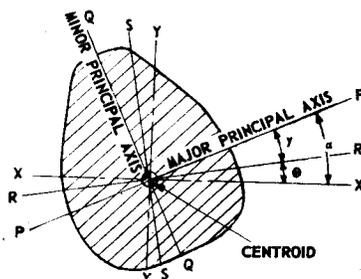
$$I_x = \frac{\pi}{64} D^4$$

$$Z_x = \frac{\pi}{32} D^3$$

D (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	WEIGHT lb/100 in.		D (in.)	A (in. ²)	ρ_x (in.)	I_x (in. ⁴)	Z_x (in. ³)	WEIGHT lb/100 in.	
					Steel	Dural						Steel	Dural
1/8	.01227	.03125	.000012	.000192	.35	.12	1-3/4	2.405	.4375	.4604	.5262	68.14	23.24
1/4	.04909	.06250	.000192	.001534	1.39	.47	2	3.142	.5000	.7854	.7854	89.00	30.35
3/8	.1104	.09375	.000971	.005177	3.13	1.07	2-1/4	3.976	.5625	1.258	1.118	112.64	38.41
1/2	.1963	.1250	.003068	.01227	5.56	1.90	2-1/2	4.909	.6250	1.917	1.534	139.06	47.42
5/8	.3068	.1562	.007490	.02397	8.69	2.96	2-3/4	5.940	.6875	2.807	2.042	168.27	57.38
3/4	.4418	.1875	.01553	.04142	12.52	4.27	3	7.069	.7500	3.976	2.651	200.25	68.29
7/8	.6013	.2188	.02877	.06577	17.04	5.81	3-1/2	9.621	.8750	7.366	4.209	272.57	92.95
1	.7854	.2500	.04909	.09817	22.25	7.59	4	12.57	1.000	12.57	6.283	356.00	121.40
1-1/4	1.227	.3125	.1198	.1917	34.77	11.86	4-1/2	15.90	1.125	20.13	8.946	450.57	153.64
1-1/2	1.767	.3750	.2485	.3313	50.06	17.07	5	19.63	1.250	30.68	12.27	556.26	189.68

4.3 Principal Axes

4.3.1 Tabular Summary of Formulas



Notation:

- XX and YY = convenient rectangular axes through the centroid
- PP = axis of greatest moment of inertia (*Major principal axis*)
- QQ = axis of least moment of inertia (*Minor principal axis*)
- RR and SS = any rectangular axes through the centroid
- α = angle between major principal axis PP and axis XX



4.3.1 Tabular Summary of Formulas (Cont'd)

- γ = angle between major principal axis PP and axis RR
- θ = angle between axis RR and axis XX
- I_X = moment of inertia about axis XX
- I_Y = moment of inertia about axis YY
- I_P = moment of inertia about axis PP
- I_Q = moment of inertia about axis QQ
- I_R = moment of inertia about axis RR
- I_S = moment of inertia about axis SS
- I_{RS} = product of inertia about axes RR and SS
- I_{XY} = product of inertia about axes XX and YY

KNOWING	TO FIND	FORMULA
I_X and I_Y	I_P and I_Q	$I_P = \frac{I_X \cos^2 a - I_Y \sin^2 a}{\cos 2a} = \frac{I_X + I_Y}{2} + \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + I_{XY}^2}$ $I_Q = I_X + I_Y - I_P$ $\text{TAN } 2a = \frac{-2 I_{XY}}{I_X - I_Y}$ <p><i>Note that I_{XY} must be determined with due regard to sign, as shown in the numerical example of 4.3.2.</i></p>
I_X and I_Y	I_R and I_S	$I_R = I_X \cos^2 \theta + I_Y \sin^2 \theta - I_{XY} \sin 2\theta$ $I_S = I_X + I_Y - I_R$ $I_{RS} = \frac{1}{2}(I_X - I_Y) \sin 2\theta + I_{XY} \sin 2\theta$
I_P and I_Q	I_R and I_S	$I_R = I_P \cos^2 \gamma + I_Q \sin^2 \gamma$ $I_S = I_P + I_Q - I_R$
I_P and I_Q	I_X and I_Y	$I_X = I_P \cos^2 a + I_Q \sin^2 a$ $I_Y = I_P + I_Q - I_X$



4.3.2 Numerical Example

To find I_P and I_Q for the unequal angle shown in Fig. 4.3.2-1 from the given data:-

$$\begin{aligned}
 A &= .2336 \\
 \bar{x} &= .197 \\
 \bar{y} &= .698 \\
 I_X &= .0995 \\
 I_Y &= .0178
 \end{aligned}$$

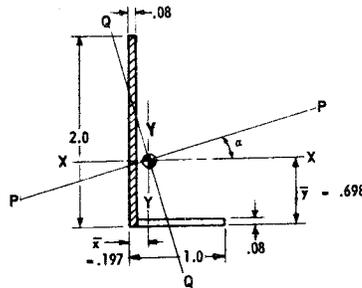


Fig. 4.3.2-1

Find I_{XY} as follows, with due regard to sign:-

ITEM	SHADED RECTANGLE	PLAIN RECTANGLE
Self-product of inertia	NIL	NIL
Area A	$2 \times .08 = .16$	$.92 \times .08 = .0736$
x	-.157	.343
y	.302	-.658
xy	-.0474	-.2255
Axy	-.0076	-.0166
	$I_{XY} = -.0242$	

$$\tan 2a = \frac{-2 I_{XY}}{I_X - I_Y} = \frac{-(-.0484)}{.0817} = +0.592$$

$$2a = 30^\circ 38'$$

$$a = 15^\circ 19'$$

$$\sin a = .2642 : \cos a = .9645$$

$$I_P = \frac{I_X \cos^2 a - I_Y \sin^2 a}{\cos 2a} \quad \sin^2 a = .0697 : \cos^2 a = .931$$

$$= \frac{.0995 \times .931 - .0178 \times .0697}{.8604}$$

$$= 0.1062$$

$$I_Q = I_X + I_Y - I_P$$

$$= .0995 + .0178 - .1062$$

$$= 0.0111$$

4.3.2 Numerical Example (Cont'd)

Alternatively,

$$I_P = \frac{I_X + I_Y}{2} + \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + I_{XY}^2}$$

$$= .0587 + \sqrt{.0411^2 + .0242^2} = \underline{.1063}$$

4.4 Moment of Inertia of a Series of Masses

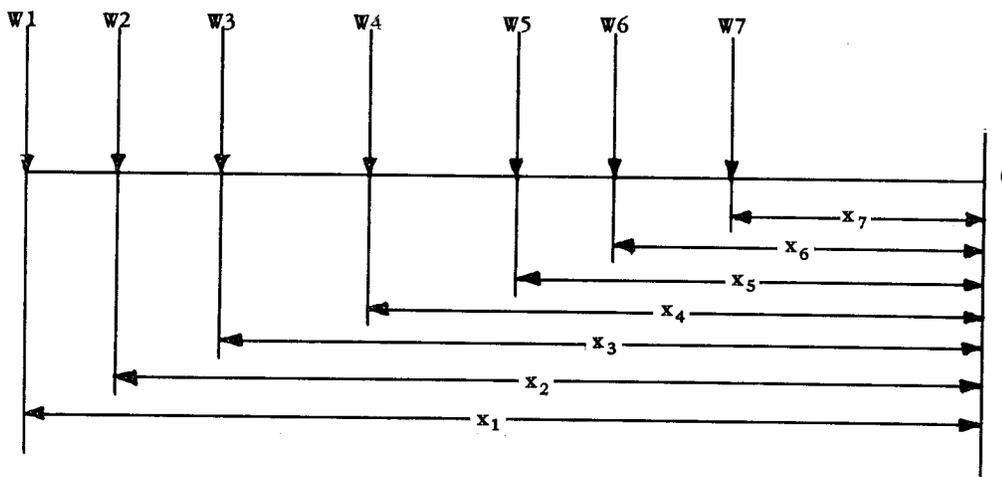


Fig. 4.4-1

Consider the system of masses shown in Fig. 4.4-1. The moment of inertia (second moment) about 0 of W_1 is $W_1 x_1^2$ and of W_2 it is $W_2 x_2^2$ and so on.

The total moment of inertia of all the masses about 0 is:-

$$I_O = W_1 x_1^2 + W_2 x_2^2 + W_3 x_3^2 + \dots = \sum W x^2$$

In practice, the amount of calculation is reduced by finding Wx for each item, then $M_O = \sum Wx$, that is, $W_1 x_1 + W_2 x_2 + W_3 x_3 + \dots$ and finally $M_O \Delta x$, where Δx is the increment of length between any two items, and $I_O = \sum M_O \Delta x$.

This is because $M_O = \sum Wx$ is the first moment of the masses about 0. The area under the curve of M_O against $x = \sum M \Delta x$, the second moment.

EXAMPLE: Masses disposed as shown in Fig. 4.4-2.



4.4 Moment of Inertia of a Series of Masses (Cont'd)

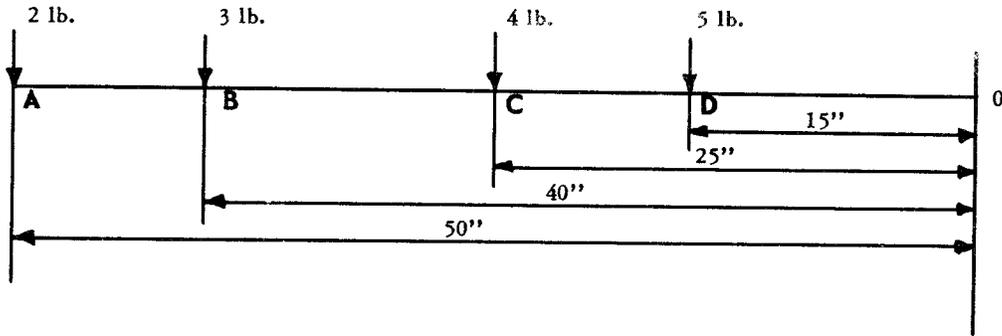


Fig. 4.4-2

ITEM	W	x	Wx	$M_o = \sum Wx$	Δx	$M_o \Delta x$
A	2	50	100	100	10	1000
B	3	40	120	220	15	3300
C	4	25	100	320	10	3200
D	5	15	75	395	15	5925

$I_o = 13425$



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INTRODUCTION

Solid sections are dealt with in 5.1 and in 5.1.1 there is a summary of the results in relation to the few cross-sections for which a mathematical solution can be obtained.

A method based on the membrane analogy is given in 5.1.2 for obtaining the shear stresses in any solid section.

In 5.2 formulas for hollow sections, thin-walled and otherwise, are summarised, whilst 5.3 deals with thin-walled open sections. After giving formulas for the shear stress and torsion constant of many common sections, a method of calculating the warp of an open section is shown.

It is necessary to find the warp in order that the warping (or torsion bending) constant, and hence the torsional instability, can be calculated, as explained in 5.3.

5.4 deals with thin multi-walled sections having constant spanwise dimensions throughout and shows how to determine the shear stresses (or shear fluxes) due to a concentrated torque.



5.1 Solid Sections

Rigorous mathematical analysis can be employed for only a few solid sections such as those having circular, elliptical, rectangular, square or equilateral triangular form.

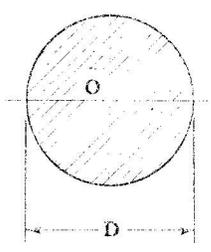
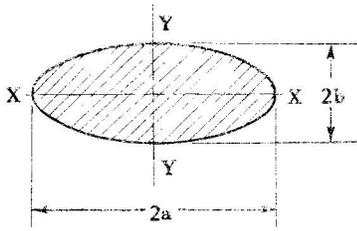
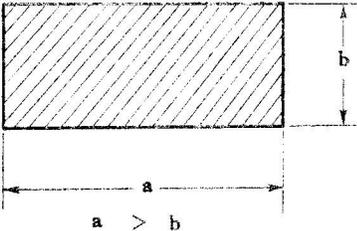
The resulting formulas (for small angles of twist) are given in the table of 5.1.1 (Cases 1 to 5 inclusive); Cases 3 and 4 are, however, simplified approximations of the exact results.

The formulas of Cases 6 to 10 inclusive are approximations based on the membrane analogy. For greater accuracy in these cases, and for the general case of tapering flanges, see 5.1.2, which employs Griffith's method based on soap-film technique and the membrane analogy.

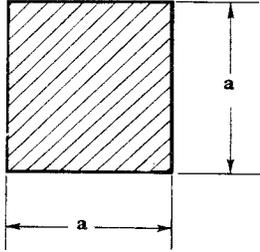
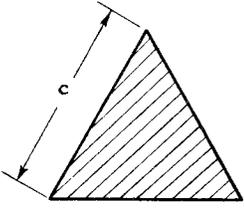
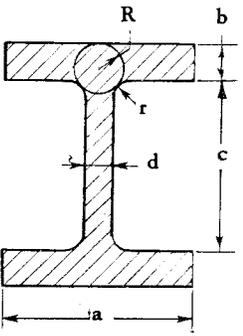
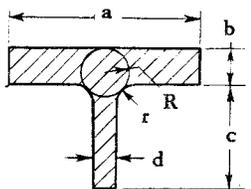
5.1.1 Summary of Common Cases

- T = Torque (lb.in.)
J = Torsion Constant (in.⁴)
θ = Angle of Twist (Radians)
- G = Shear Modulus (p.s.i.)
τ = Shear Stress (p.s.i.)
L = Length of Member (in.)

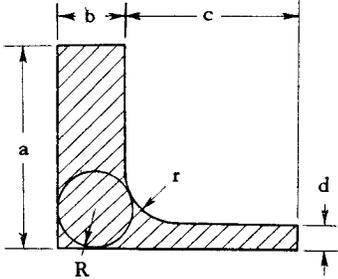
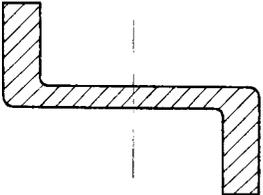
$$\frac{\theta G}{L} = \frac{T}{J}$$

SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>1</p> <p>SOLID CIRCULAR</p> 	<p>Max. $\tau = \frac{16T}{\pi D^3}$ at boundary</p>	<p>$J = \frac{\pi D^4}{32}$ In this particular case, J is equal to the polar moment of inertia of the section about the centre O.</p>
<p>2</p> <p>SOLID ELLIPTICAL</p> 	<p>Max. $\tau = \frac{2T}{\pi ab^2}$ at ends of minor axis YY</p> <p>$\tau = \frac{2T}{\pi a^2 b}$ at ends of major axis XX</p>	<p>$J = \frac{\pi a^3 b^3}{(a^2 + b^2)}$</p>
<p>3</p> <p>SOLID RECTANGULAR</p> 	<p>Max. $\tau = \frac{T}{ab^2} (3 + 1.8 \frac{b}{a})$ This is the approximate value of τ at the mid. point of each longer side.</p>	<p>$J = ab^3 [\frac{1}{3} - 0.21 \frac{b}{a} (1 - \frac{1}{12} \frac{b^4}{a^4})]$ (approx.)</p>



SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>4</p> <p style="text-align: center;">SOLID SQUARE</p> 	<p>Max. $\tau = 4.8 \frac{T}{a^3}$ (approx.)</p> <p>at mid. point of each side</p>	<p>$J = 0.1408 a^4$ (approx.)</p>
<p>5</p> <p style="text-align: center;">SOLID EQUILATERAL TRIANGLE</p> 	<p>Max. $\tau = \frac{20T}{c^3}$</p> <p>at mid. point of each side</p>	<p>$J = \sqrt{3} \frac{c^4}{80}$</p>
<p>6</p> <p style="text-align: center;">I SECTION Flange Thickness Constant</p> 	<p>$R =$ radius of largest inscribed circle</p> <p>$r =$ fillet radius</p> <p>See note below for maximum shear stress</p>	<p>$J = 2J_F + J_W + 32aR^4$ (approx.)</p> <p>Where</p> <p>$J_F = ab^3 \left[\frac{1}{3} - 0.21 \frac{b}{a} \left(1 - \frac{b^4}{12a^4} \right) \right]$</p> <p>$J_W = \frac{1}{3} cd^3$</p> <p>$a = K \left(0.15 + 0.1 \frac{r}{b} \right)$</p> <p>$K =$ the smaller of d/b or b/d</p>
<p>7</p> <p style="text-align: center;">T SECTION Flange Thickness Constant</p> 	<p>$R =$ radius of largest inscribed circle</p> <p>$r =$ fillet radius</p> <p>See note below for maximum shear stress</p>	<p>$J = J_F + J_W + 16 a R^4$ (approx.)</p> <p>Where</p> <p>$J_F = ab^3 \left[\frac{1}{3} - 0.21 \frac{b}{a} \left(1 - \frac{b^4}{12a^4} \right) \right]$</p> <p>$J_W = cd^3 \left[\frac{1}{3} - 0.105 \frac{d}{c} \left(1 - \frac{d^4}{192c^4} \right) \right]$</p> <p>$a = K \left(0.15 + 0.10 \frac{r}{b} \right)$</p> <p>$K =$ the smaller of d/b or b/d</p>



SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>8</p> <p>ANGLE SECTION b equal to, or greater than, d</p> 	<p>R = radius of largest inscribed circle</p> <p>r = fillet radius</p> <p>See note below for maximum shear stress.</p>	<p>$J = J_F + J_W + 16\alpha R^4$ (approx.)</p> <p>Where</p> <p>$J_F = ab^3 \left[\frac{1}{3} - 0.21 \frac{b}{a} \left(1 - \frac{b^4}{12a^4} \right) \right]$</p> <p>$J_W = cd^3 \left[\frac{1}{3} - 0.105 \frac{d}{c} \left(1 - \frac{d^4}{192c^4} \right) \right]$</p> <p>$\alpha = \frac{b}{d} \left(0.07 + 0.076 \frac{r}{b} \right)$</p>
<p>9</p> <p>CHANNEL SECTION</p> 	<p>See note below for maximum shear stress.</p>	<p>J = the sum of the J values for constituent angle sections, computed as for Case 8.</p>
<p>10</p> <p>Z SECTION</p> 		

NOTE REGARDING SHEAR STRESS (Cases 6 - 10 inclusive):-

The maximum shear stress occurs, in general, at or near one of the points of contact of the largest inscribed circle at the point where the radius of curvature is least.

Empirical formulae are:-

Equation (a): When the boundary is convex or straight,

$$\text{Max. } \tau = \frac{2R}{1+m} \left[1 + 0.15 \left(m^2 - \frac{R}{\rho} \right) \right] \frac{T}{J}$$

$$\text{Where } m = \frac{\pi R^2}{A}$$

ρ = radius of curvature of the boundary (positive when the boundary is convex)

A = area of section

R = radius of largest inscribed circle



Equation (b): When the boundary is concave,

Max. $\tau =$

$$\frac{2R}{1+m^2} \left[1 + \tanh \frac{2\beta}{\pi} \left\{ 0.118 \log_e \left(1 - \frac{R}{\rho} \right) - 0.238 \frac{R}{\rho} \right\} \right] \frac{T}{J}$$

Where $\beta =$ angle turned through by the tangent to the boundary of the re-entrant portion.

The application of these formulas is given in Section 5.1.2.

5.1.2 Approximate Method for any Solid Section

It can be shown that the mean stress around the boundary of any solid section of area A and perimeter p is equal to the boundary stress in a circular bar of radius $\frac{2A}{p}$.

Since the torsion constant of a solid circular section of cross sectional-area A is $J = \frac{\pi D^4}{32} = \frac{\pi r^4}{2} = \frac{A}{2} r^2$ this value of $\frac{2A}{p}$ may be looked upon as an equivalent torsional radius. Denoting it by C gives

$$J = \frac{A}{2} C^2$$

A preliminary estimate of the torsion constant is consequently given by this expression.

In most practical cases it will be necessary to divide the section into suitable compartments and find the contribution to J of each part, the total J value then being the sum of the component J 's.

The general rules for division into compartments are as follows:—

Imagine a circle of varying radius, always touching the profile at two points at least, to move inside the section. Then there will be some positions when the circle has three or more points of contact and between each such pair of positions there will be a position of the circle where its radius is a minimum.

The division lines are drawn through the points of contact (with the profile) of this minimum circle (Fig. 5.1.2-1).

Where the figure contains a thick portion, from which project one or more thinner parts whose boundaries in the neighbourhood of the junctions are parallel or nearly parallel lines, the division should be such as to include with the thick component a length of each projection equal to half its thickness at the junction.

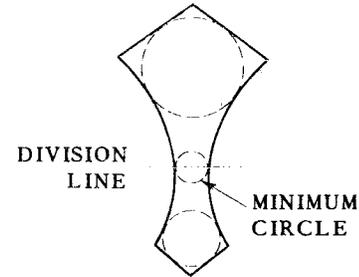


Figure 5.1.2-1

5.1.2.1 Procedure for First Approximation to J : Numerical example.

Consider the channel section shown in Fig. 5.1.2-2. Divide the section into five compartments numbered 1, 1¹, 2, 2¹ and 3 in accordance with the rule just given.

The dividing line between 1 and 2 (and hence between 1¹ and 2¹) is taken at a distance from P , where the fillet starts, equal to half the depth at P , viz $\frac{0.77}{2} = 0.385$ in., as shown dotted at BC (and KL).

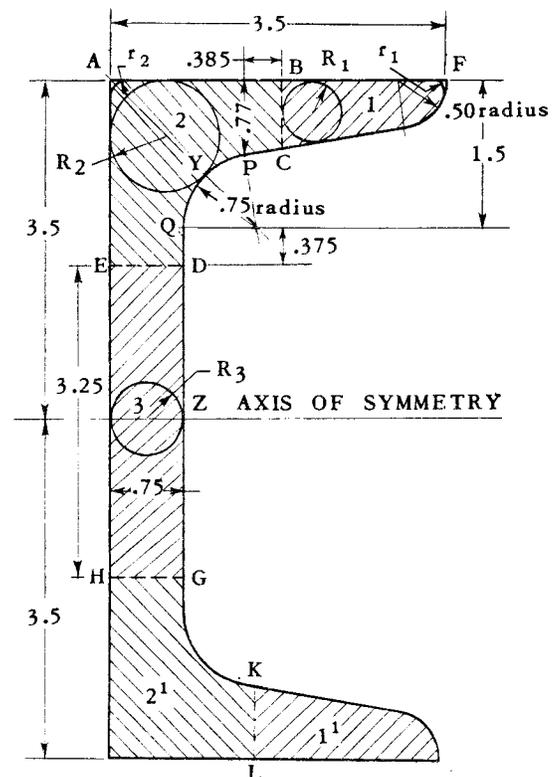


Figure 5.1.2-2



Similarly, the dividing line between 2 and 3 (and likewise between 2¹ and 3) is at half the web thickness viz $\frac{0.75}{2} = 0.375$ in. from Q, where the fillet ends, as shown dotted by ED (and HG).

For each compartment in turn find the area A, perimeter p and the ratio $C = \frac{2A}{p}$. Note, however, that

in determining p no division line is included.

These values are given in Table 5.1.2-1 .

5.1.2.2 More Accurate Value of Torsion Constant

In order to find a more refined value of J, proceed as follows:-

(a) Draw the maximum inscribed circle of radius R in each compartment. The circles drawn in Compartments 1, 1¹, 2 and 2¹ touch the boundary at three points, as shown. The radii of the circles are:-

COMPARTMENT	R (IN.)
1	.32
2	.56
3	.375
2 ¹	.56
1 ¹	.32

(b) Round off the outwardly projecting corners (on the main boundary only) with a radius "r" which is a certain fraction of the radius R of the circle inscribed in any particular compartment.

This fraction depends on the angle β turned through by the tangent to the boundary at the corner in question: values of r/R are given on the accompanying graph (Fig. 5.1.2-3).

For a right-angled corner, $r/R = .50$. These values are:-

STATION	R (IN.)	β (deg.)	r/R	r (IN.)
F	.32	90	.50	.16
A	.56	90	.50	.28

(c) Draw in the corner radii. This gives a new figure having a component area and perimeter A₁ and p₁ respectively. Calculate A₁, p₁ and the ratio $C_1 = \frac{2A_1}{p_1}$ for each compartment.

(d) The torsion constant is then given by:-

$$J = \frac{A}{2} \lambda C_1^2 \text{ (see Table 5.1.2-2)}$$

λ in this expression is a coefficient, values of which are given in terms of $\frac{R}{C}$ in Fig. 5.1.2-4.

TABLE 5.1.2-1

COMPARTMENT	A	2A	p	$C = \frac{2A}{p}$	C^2	$\frac{A}{2}$	$\Delta J = \frac{A}{2} C^2$
1	.91	1.82	3.59	.506	.255	.455	.116
1 ¹	.91	1.82	3.59	.506	.255	.455	.116
2	2.29	4.58	5.67	.81	.655	1.145	.750
2 ¹	2.29	4.58	5.67	.81	.655	1.145	.750
3	2.43	4.86	6.50	.746	.558	1.215	.678

First approximation to torsion constant $J = 2.410 \text{ in.}^4$

TABLE 5.1.2-2

COMPARTMENT	R	C	R/C	λ	A ₁	2A ₁	p ₁	$\frac{2A_1}{p_1}$	C_1^2	λC_1^2	ΔJ
1	.320	.506	.63	.827	.87	1.74	3.55	.501	.25	.207	.090
1 ¹	.320	.506	.63	.827	.87	1.74	3.55	.501	.25	.207	.090
2	.560	.81	.69	.887	2.26	4.52	5.51	.820	.671	.595	.672
2 ¹	.560	.81	.69	.887	2.26	4.52	5.51	.820	.671	.595	.672
3	.375	.75	.50	.667	2.43	4.86	6.50	.746	.556	.371	.451

NOTE: This example is taken from "Strength of Materials" by Case, page 486. It should be noted that the above value of $\lambda = .827$ does not agree with his value of 0.845. The final J value is consequently at variance too.

$$J = 1.975 \text{ in.}^4$$

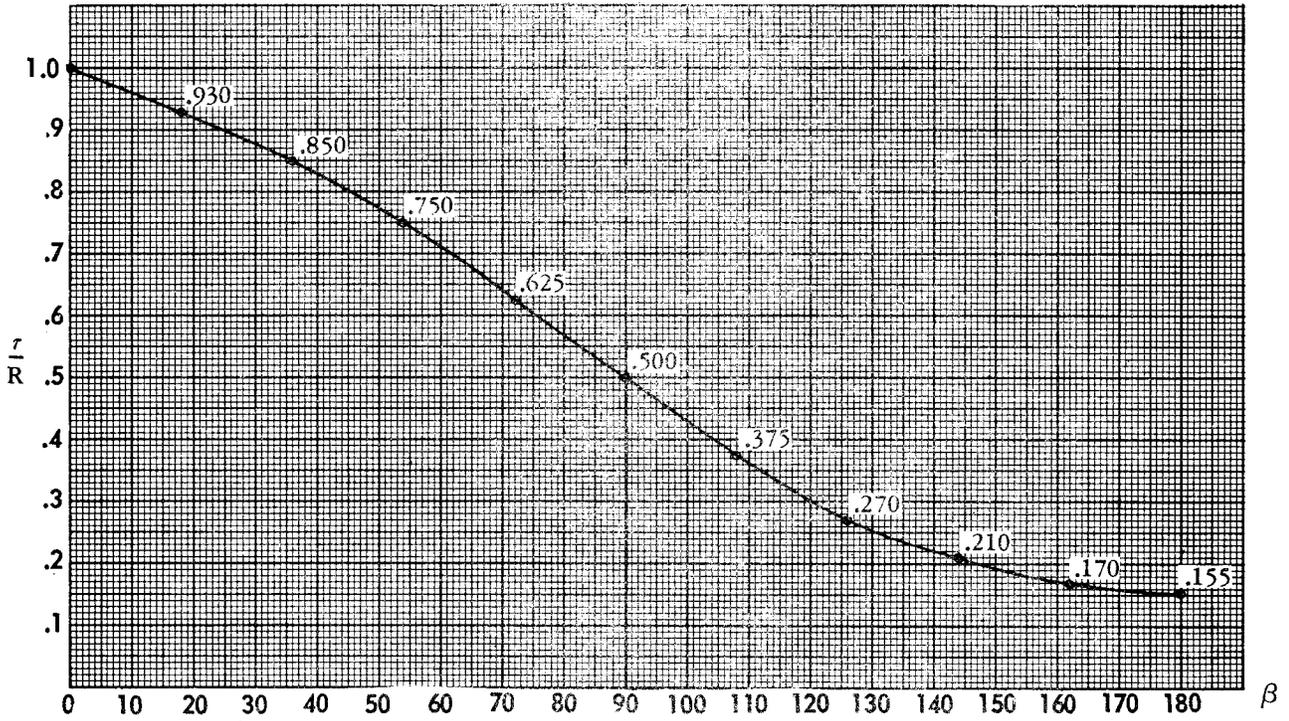


Figure 5.1.2-3 Plot of $\frac{r}{R}$ versus β

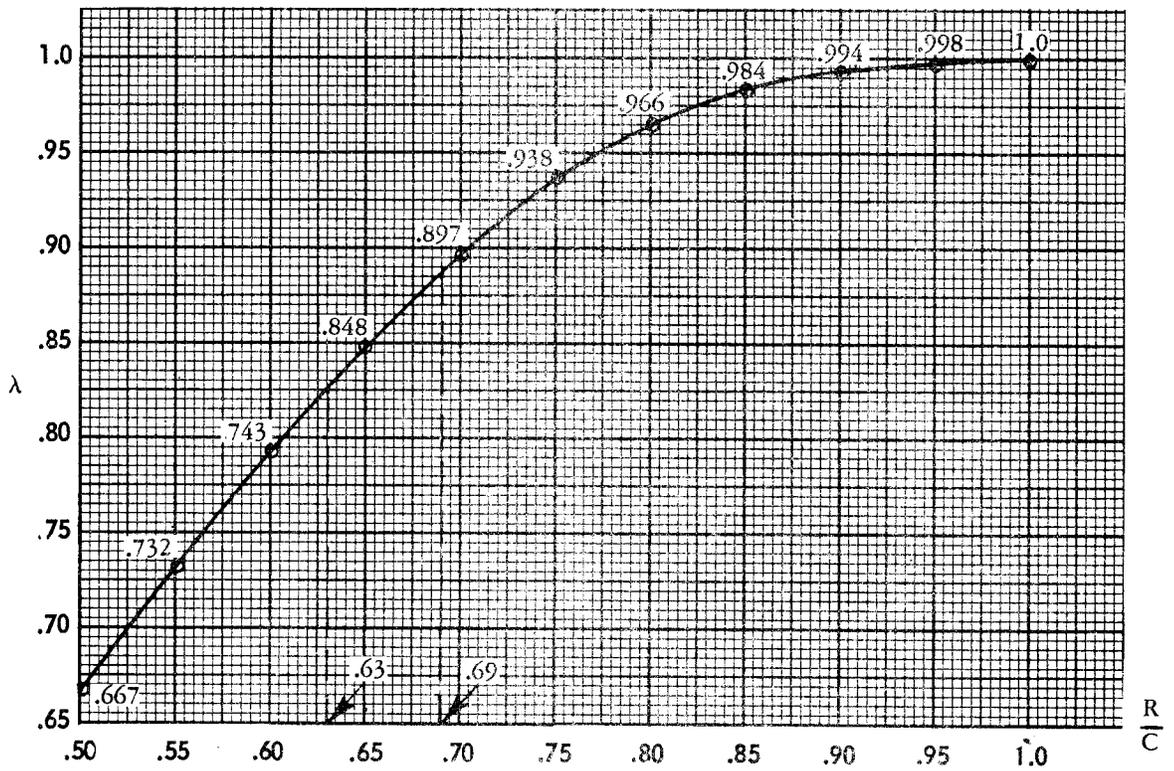


Figure 5.1.2-4 Plot of λ versus $\frac{R}{C}$



5.1.2.3 Shear Stresses

The maximum stress in any compartment occurs, in general, at or near one of the points of contact of the largest inscribed circle at the point where the radius of curvature is least.

Empirical formulas were given in 5.1.1 for the determination of the maximum shear stress.

COMPARTMENT 1

The shear stress at the corner F is zero. The mean shear stress on the section is

$$\frac{2A}{p} \frac{T}{J} = .506 \frac{T}{J} = \frac{.506}{1.975} T = \underline{.2565T}$$

The maximum shear stress for this compartment is given by Equation (a) of 5.1.1, since it has no re-entrant angle and ρ is infinite.

$$\text{Max. } \tau = \frac{2R}{1+m^2} \left[1 + 0.15 \left(m^2 - \frac{R}{\rho} \right) \right] \frac{T}{J}$$

$$m = \frac{\pi R^2}{A} = \frac{\pi \times .32^2}{.91} = .353$$

$$m^2 = .124 \quad .15m^2 = .0186$$

$$1 + m^2 = 1.124 \quad 2R = .64$$

$$\frac{2R}{1+m^2} = .568$$

ρ is infinite since the boundary is uncurved.

$$\text{Max. } \tau = .568 [1 + .0186] \frac{T}{J} = .579 \frac{T}{J} = \underline{.292T}$$

This occurs at or near point X in Fig. 5.1.2-2, the point of contact of the maximum inscribed circle.

COMPARTMENT 2

Since the boundary is concave, use Equation (b). ρ is negative.

Max. $\tau =$

$$\frac{2R}{1+m^2} \left[1 + \tanh \frac{2\beta}{\pi} \left\{ 0.118 \log_e \left(1 - \frac{R}{\rho} \right) - 0.238 \frac{R}{\rho} \right\} \right] \frac{T}{J}$$

$$\rho = -.75 \quad R = .56 \quad R/\rho = -.747$$

$$(1 - R/\rho) = 1.747 \quad .238 R/\rho = -.178$$

$$\log_e 1.747 = .5579 \quad 0.118 \log_e 1.747 = 0.066$$

β , the angle turned through by the tangent to the boundary in passing from P to Q, is equal to $81^\circ = 1.412$ radians.

$$2\beta/\pi = \frac{2.824}{\pi} = .898$$

$$\text{Tanh } 2\beta/\pi = .716 \text{ (from Tables)}$$

$$\left\{ 0.118 \log_e \left(1 - \frac{R}{\rho} \right) - 0.238 \frac{R}{\rho} \right\} = 0.066 + 0.178 = 0.244$$

$$m = \frac{\pi R^2}{A} = \frac{\pi \times 0.313}{2.29} = 0.429$$

$$m^2 = 0.184 \quad 1 + m^2 = 1.184$$

$$\frac{2R}{1+m^2} = \frac{1.12}{1.184} = 0.944$$

$$\text{Max. } \tau = 0.944 [1 + 0.716 \times 0.244] \frac{T}{J} = 1.11 \frac{T}{J} = \underline{.559T}$$

This occurs at Y in Fig. 5.1.2-2: it is the maximum stress on the whole section.

COMPARTMENT 3

Use Equation (a)

$$R = .375 \quad 2R = .75$$

$$m = \frac{\pi R^2}{A} = \frac{\pi \times .14}{2.43} = .181$$

$$m^2 = .0327 \quad 1 + m^2 = 1.0327$$

$$\frac{2R}{1+m^2} = \frac{.75}{1.0327} = .724$$

$$\rho = \infty \quad .15m^2 = .00491$$

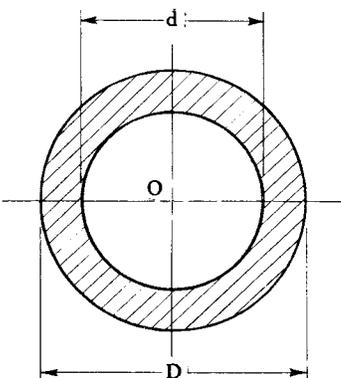
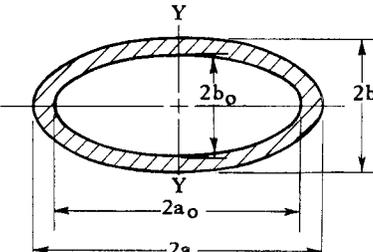
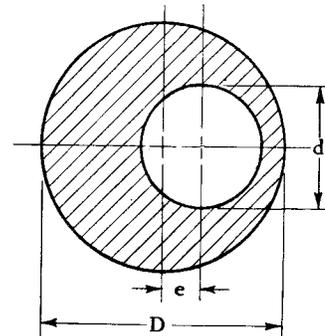
$$\text{Max. } \tau = .724 \times 1.00491 \frac{T}{J} = .728 \frac{T}{J} = \underline{.367T}$$

This occurs at Z on the axis of symmetry.

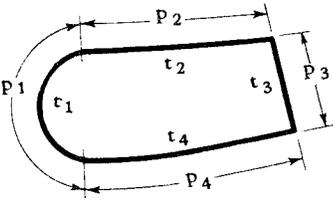
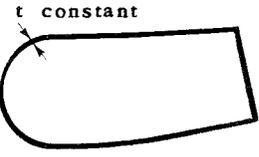
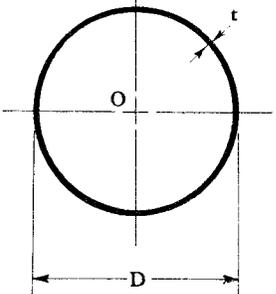
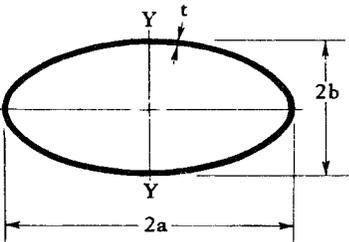
5.2 Hollow Sections: Thin walled and otherwise.

T = Torque (lb. in.) G = Shear Modulus (p.s.i.)
 J = Torsion Constant (in.⁴) τ = Shear Stress (p.s.i.)
 θ = Angle of Twist (Radians) L = Length of Member (in.)

$$\frac{\theta G}{L} = \frac{T}{J}$$

SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>1</p> <p style="text-align: center;">HOLLOW CONCENTRIC CIRCULAR TUBE</p> 	$\text{Max. } \tau = \frac{16TD}{\pi(D^4 - d^4)}$ <p>at outer boundary.</p> <p>For allowables see ANC-5, pages 29, 81 and 121.</p>	$J = \frac{\pi}{32}(D^4 - d^4)$ <p>In this particular case, J is equal to the polar moment of inertia of the section about the centre O.</p>
<p>2</p> <p style="text-align: center;">HOLLOW ELLIPTICAL SECTION</p> <p>Outer and inner boundaries are similar ellipses: thickness not constant.</p> 	$\text{Max. } \tau = \frac{2T}{\pi a_o b_o^2 [(1+K)^4 - 1]}$ <p>on outer surface at ends of minor axis YY.</p> $K = \frac{a - a_o}{a_o} = \frac{b - b_o}{b_o}$	$J = \frac{\pi a_o^3 b_o^3}{a_o^2 + b_o^2} [(1+K)^4 - 1]$
<p>3</p> <p style="text-align: center;">ECCENTRIC HOLLOW SHAFT</p> 	<p>See Roark, page 172, Case 21</p>	



SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>4</p> <p>ANY THIN-WALLED ENCLOSED SECTION OF VARYING THICKNESS</p> 	$\tau_1 = q/t_1$ $\tau_2 = q/t_2$ $\tau_3 = q/t_3$ $\tau_4 = q/t_4$ <p>where $q = \frac{T}{2A}$ lb. per in.</p> <p>(Batho's formula)</p>	$J = \frac{4A^2}{\sum P/t}$ <p>where $\sum P/t = \frac{P_1}{t_1} + \frac{P_2}{t_2} + \text{etc.}$ and $A =$ area enclosed by the mean perimeter</p>
<p>5</p> <p>ANY THIN-WALLED ENCLOSED SECTION OF CONSTANT THICKNESS</p> 	<p>Average shear stress = q/t where $q = \frac{T}{2A}$ lb. per in.</p> <p>(Batho's formula)</p>	$J = \frac{4A^2t}{p}$ <p>where $p =$ the mean perimeter and $A =$ area enclosed by the mean perimeter</p>
<p>6</p> <p>THIN-WALLED CIRCULAR TUBE</p> <p>D/t large t constant</p> 	<p>Max. $\tau = \frac{2T}{\pi D^2t}$</p> <p>For allowables see ANC-5, pages 29, 81 and 121.</p>	$J = \frac{\pi}{4} D^3t$ <p>In this particular case, J is equal to the polar moment of inertia of the section about the centre O.</p>
<p>7</p> <p>THIN-WALLED ELLIPTICAL SECTION OF CONSTANT THICKNESS</p> 	<p>Average shear stress = q/t where $q = \frac{T}{2A}$ lb. per in.</p> <p>(Batho's formula)</p>	$J = \frac{4A^2t}{p}$ <p>where $p =$ mean perimeter (for which see 4.1.1 or Fig. 4.1.3-1) $A =$ area enclosed by mean perimeter = πab</p>



5.3 Thin-walled Open Sections

5.3.1 Introduction

When a thin-walled open section is subjected to pure (St. Venant) torsion as a result of the application of equal and opposite end torques T (Fig. 5.3-1), cross-sections of the tube warp in a longitudinal direction. In other words, points in the plane of the end sections do not remain in the same plane.

Shear stresses only are present in the walls when the tube is free to warp, that is, when there is no end constraint.

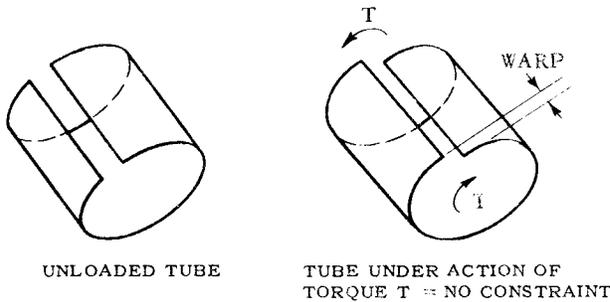


Fig. 5.3-1

It will be apparent that any member having a small thickness has practically the same torsional stiffness when bent into a channel, angle or other

section as it has when flat. Thin open sections are consequently primarily unsuited as torque carrying members; in many cases, however, they have to take a certain amount of torque in addition to carrying out their primary function.

For a rectangular section (Case 1, 5.3.2) the distribution of shear stress is as shown in Fig. 5.3-2, the shear stresses acting in a direction parallel to the boundary, for small t/b ratios.

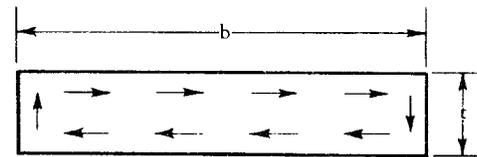


Fig. 5.3-2

Formulas for Cases 2 - 5 inclusive assume that the torsion constant J for the whole section is equal to the sum of the J values for the component rectangles of which the section is composed.

Similarly the J value for the remaining cases is based on the developed rectangle.

A comparison between a split and a solid tube appears in GEN 1090/809.

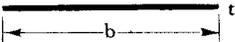
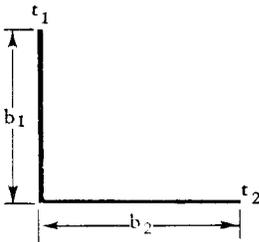
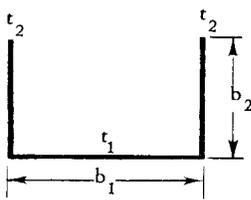
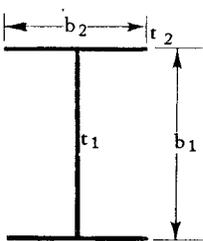


5.3.2 Formulas for Shear Stress and Torsion Constant

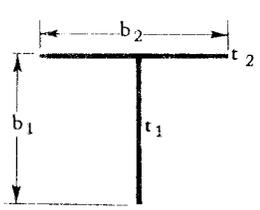
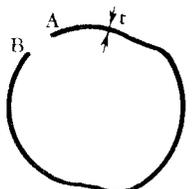
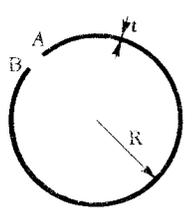
T = Torque (lb. in.)
 J = Torsion Constant (in.⁴)
 θ = Angle of Twist (Radians)

G = Shear Modulus (p.s.i.)
 τ = Shear Stress (p.s.i.)
 L = Length of Member (in.)

$$\frac{\theta G}{L} = \frac{T}{J}$$

SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>1</p> <p>NARROW RECTANGLE OF CONSTANT THICKNESS $b/t > 20$</p> 	$\tau = \frac{3T}{bt^2}$	$J = \frac{1}{3} bt^3$
<p>2</p> <p>ANGLE t_1 and t_2 small compared with b_1 and b_2</p> 	$\tau = \frac{3T}{b_1 t_1^2 + b_2 t_2^2}$ <p>For Bulb Angle see R.Ae.S. Data Sheet 00.07.01</p>	$J = \frac{1}{3} \Sigma bt^3$ $= \frac{1}{3} [b_1 t_1^3 + b_2 t_2^3]$
<p>3</p> <p>CHANNEL t_1 and t_2 small compared with b_1 and b_2</p> 	$\tau = \frac{3T}{(b_1 t_1^2 + 2b_2 t_2^2)}$	$J = \frac{1}{3} \Sigma bt^3$ $= \frac{1}{3} [b_1 t_1^3 + 2b_2 t_2^3]$
<p>4</p> <p>I SECTION t_1 and t_2 small compared with b_1 and b_2</p> 	$\tau = \frac{3T}{(b_1 t_1^2 + 2b_2 t_2^2)}$	$J = \frac{1}{3} \Sigma bt^3$ $= \frac{1}{3} [b_1 t_1^3 + 2b_2 t_2^3]$



SECTION	SHEAR STRESS	TORSION CONSTANT J
<p>5</p> <p style="text-align: center;">T SECTION</p> <p>t_1 and t_2 small compared with b_1 and b_2</p> 	$\tau = \frac{3T}{(b_1 t_1^2 + b_2 t_2^2)}$	$J = \frac{1}{3} \sum b t^3$ $= \frac{1}{3} [b_1 t_1^3 + b_2 t_2^3]$
<p>6</p> <p style="text-align: center;">ANY THIN-WALLED OPEN TUBE</p> <p style="text-align: center;">t constant and small</p> 	$\tau = \frac{3T}{s t^2}$	$J = \frac{1}{3} \sum_A^B t^3 ds$ $= \frac{1}{3} s t^3$ <p style="text-align: center;">$s =$ length of periphery of cross-section from A to B</p>
<p>7</p> <p style="text-align: center;">THIN-WALLED OPEN CIRCULAR TUBE</p> <p style="text-align: center;">t constant and small</p> 	$\tau = \frac{3T}{2\pi R t^2}$	$J = \frac{1}{3} s t^3$ $= \frac{2}{3} \pi R t^3$ <p style="text-align: center;">$s =$ length of periphery of cross-section from A to B</p>



5.3.3 Warp

For any open tube (Fig. 5.3-3) let

θ = angle of twist (radians),

L = length of tube (in.)

$\phi = \frac{\theta}{L}$ = angle of twist per unit length (radians per inch),

w = warp (in.)

$W = \frac{w}{\phi}$ = unit warp (in./radians per inch),

p_t = perpendicular distance from shear centre O to tangent at P , and

ds = element on periphery at P (Fig. 5.3-4).

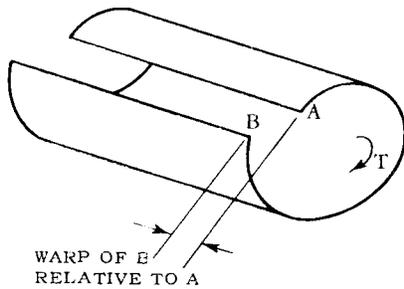


Fig. 5.3-3

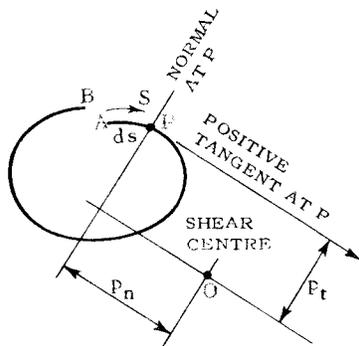


Fig. 5.3-4

Then from R.Ae.Soc. Data Sheet 00.07.00, but using the above notation,

$$W = - \int_A^B p_t ds$$

The minus sign signifies that the warp of B relative to A is into the paper for clockwise T.

SIGN OF p_t

When s is measured from A (Fig. 5.3-4), the tangent at any point P is considered positive in the direction of increasing s and p_t is positive if the positive tangent acts clockwise relative to the shear centre O . Consequently, in Fig. 5.3-4, p_t at P is positive.

Similarly, measuring s from A in the channel section of Fig. 5.3-5, all the p_t values are positive except in CD, whereas in the Z section of Fig. 5.3-6, p_t is positive for AB but negative for CD.

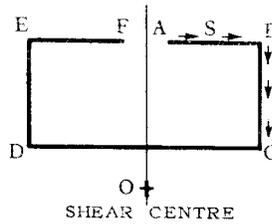


Fig. 5.3-5



Fig. 5.3-6

5.3.3.1 Numerical Example - To find the unit warp W for the section 10CS-R-525 shown in Fig. 5.3-7 under the action of a contra-clockwise torque T (a contra-clockwise torque has been chosen in order to avoid the necessity for a minus sign in the expression for unit warp).

$$W = + \int_0^s p_t ds$$

The section is divided into convenient lengths ds and the p_t value for each section obtained.

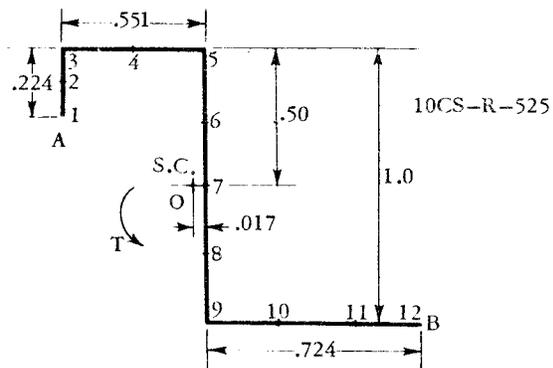


Fig. 5.3-7



STN	ds	p _t	p _t ds × 10 ²	W × 10 ²	s
1				0	
2	.10	.534	5.34	5.34	.10
3	.124	.534	6.62	11.96	.224
4	.25	.50	12.50	24.46	.474
5	.301	.50	15.05	39.51	.775
6	.25	.017	.43	39.94	1.025
7	.25	.017	.42	40.36	1.275
8	.25	.017	.43	40.79	1.525
9	.25	.017	.42	41.21	1.775
10	.25	-.50	-12.50	28.71	2.025
11	.25	-.50	-12.50	16.21	2.275
12	.224	-.50	-11.20	5.01	2.499

Plotted in Fig. 5.3-8

MEAN UNIT WARP

By plotting W against s in Fig. 5.3-8 the area under the curve is obtained. This is, $\sum_A^B W ds = .7244$.

$$\text{Mean unit warp } \bar{W} = \frac{\sum_A^B W ds}{\text{Length A to B}} = \frac{.7244}{2.499} = .29$$

This quantity is required for the calculation of the warping constant Γ (gamma).

5.3.4 Torsion Bending

When the cross-section is not free to warp, due to the constraint caused by building-in at one end, pure torsional stresses will be accompanied by tension or compression in the longitudinal direction of the beam.

These longitudinal stresses are proportional to the torsional warping displacement, now prevented.

The rate of change of angle of twist $\phi = \frac{d\theta}{dz}$ along the axis of the beam is not constant, a state of affairs which is consequently referred to as NON-UNIFORM TORSION.

The problem is normally tackled by splitting the torque into two parts, namely:—

T_1 = torque taken in shear
and T_2 = torque taken in torsion-bending, which for an I beam consists of differential bending of the flanges in opposite directions.

The relative proportion of T_1 and T_2 will depend on the dimensions of the cross-section and the length of the beam.

It is shown in GEN/1090/809.5.2 that the total torque T is given by the following expression:—

$$T = T_1 + T_2 = GJ\phi - E\Gamma \frac{d^2\phi}{dz^2}$$

Where $\phi = \frac{d\theta}{dz}$

Γ = warping (torsion-bending) constant of the section, values of which for common sections are given in 5.3.5.

This is the fundamental equation for the torsion of a thin-walled open section with end constraint.

5.3.5 Warping Constants

The warping (torsion bending) constant Γ (gamma) given in the fundamental equation above is equal to the sum of Γ_1 and Γ_2 , where Γ_1 is the primary warping constant and Γ_2 is the secondary warping constant.

Γ_2 is normally very small compared with Γ_1 and can therefore be neglected except, for example, in the case of rectangular, angle and tee sections, for which $\Gamma_1 = 0$.

Values of Γ_1 (and Γ_2 where applicable) for common sections are given in the following tabular summary (5.3.5.1), and a method of calculation for Γ_1 in 5.3.5.2 for an unsymmetrical Zee section.

$$\Gamma_2 = \frac{1}{12} \int_0^s p_n^2 t^3 ds$$

where p_n is the perpendicular from the shear centre O to the normal to the median line at any point P in Fig. 5.3-4.

For a rectangular section:—



$$\begin{aligned} \Gamma_2 &= \frac{1}{12} \int_0^s p_n^2 t^3 ds \\ &= \frac{1}{12} t^2 \int_0^s p_n^2 t ds \end{aligned}$$

But $\int_0^s p_n^2 t ds = I = \frac{tb^3}{12}$

$$\Gamma_2 = \frac{1}{12} t^2 \times \frac{tb^3}{12} = \frac{1}{144} b^3 t^3$$

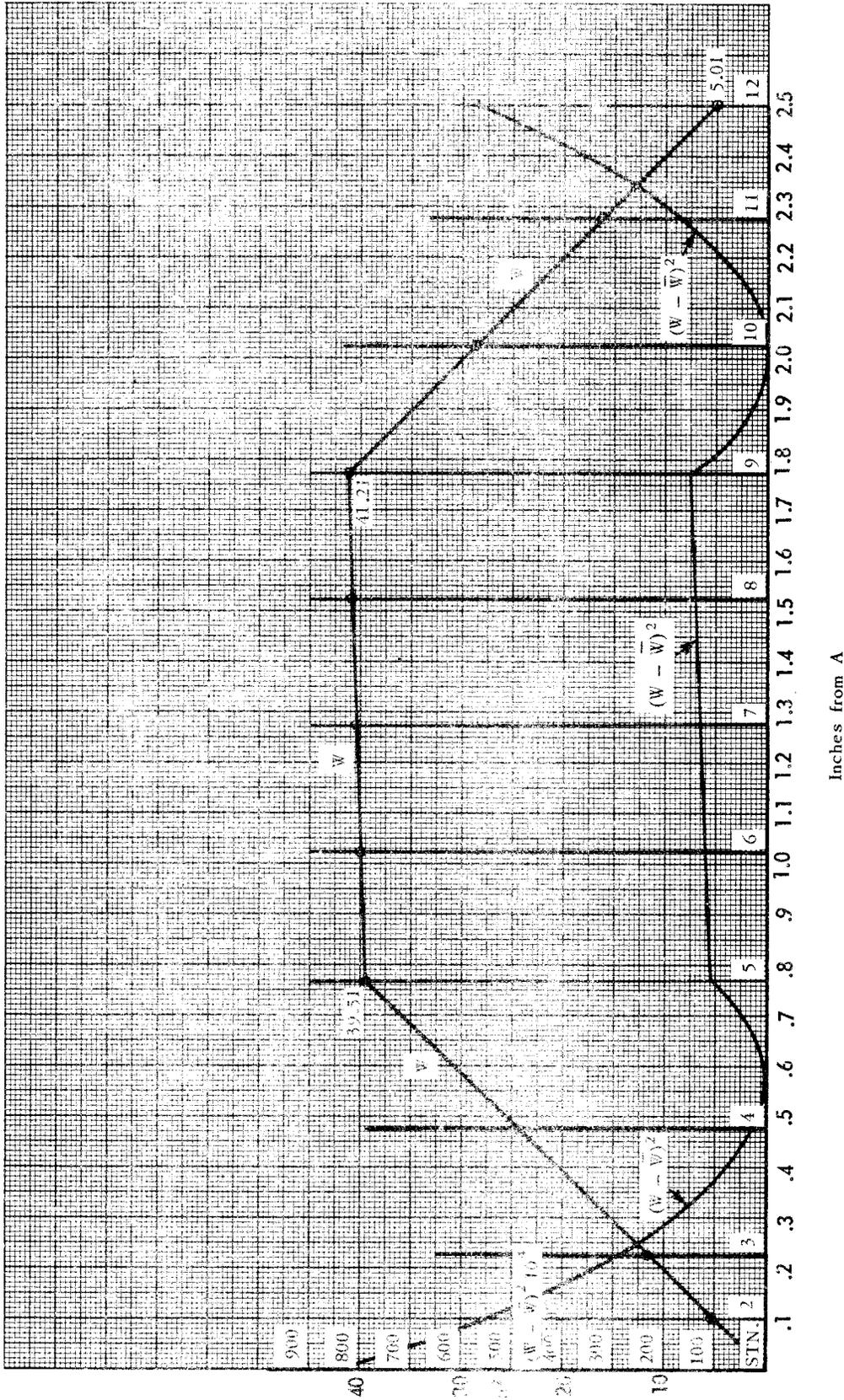
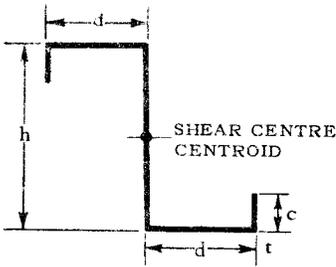
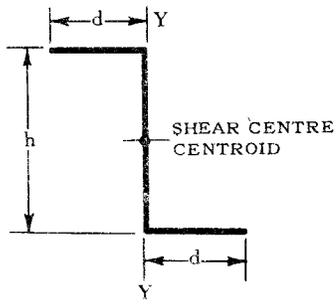
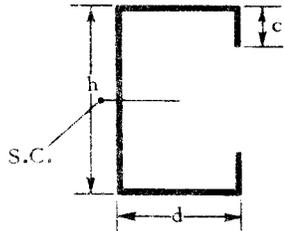
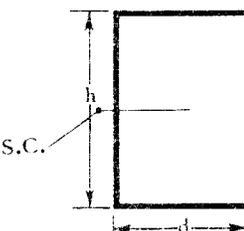


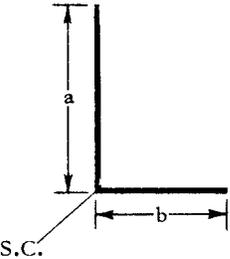
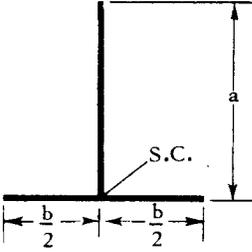
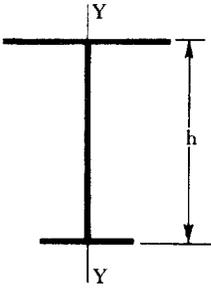
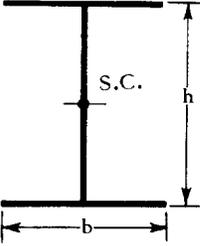
Fig. 5.3-8 Plot of W and $(W - \bar{W})^2$



5.3.5.1 Summary of Warping Constants

SECTION	
<p>1</p> <p style="text-align: center;">LIPPED Z-SECTION t constant</p>  <p style="text-align: center;">SHEAR CENTRE CENTROID</p>	<p style="text-align: center;">See Fig. 5.3-9 for Primary Warping Constant Γ_1</p>
<p>2</p> <p style="text-align: center;">Z-SECTION t constant</p>  <p style="text-align: center;">SHEAR CENTRE CENTROID</p>	<p style="text-align: center;">See Fig. 5.3-9 for Primary Warping Constant Γ_1 Take $c = 0$</p>
<p>3</p> <p style="text-align: center;">LIPPED CHANNEL t constant</p>  <p style="text-align: center;">S.C.</p>	<p style="text-align: center;">See Fig. 5.3-10</p>
<p>4</p> <p style="text-align: center;">CHANNEL t constant</p>  <p style="text-align: center;">S.C.</p>	<p style="text-align: center;">See Fig. 5.3-10</p>



SECTION	
<p>5</p> <p>ANGLE t constant</p> 	$\Gamma_1 = 0$ $\Gamma_2 = \frac{t^3}{36} (a^3 + b^3)$
<p>6</p> <p>TEE t constant</p> 	$\Gamma_1 = 0$ $\Gamma_2 = \frac{t^3}{36} \left[a^3 + \frac{1}{4} b^3 \right]$
<p>7</p> <p>I SECTION t constant unequal flanges</p> 	$\Gamma_1 = \frac{h^2 I_1 I_2}{I_1 + I_2}$ <p>I_1 = moment of inertia of upper flange about axis YY</p> <p>I_2 = moment of inertia of lower flange about axis YY</p>
<p>8</p> <p>I SECTION t constant equal flanges</p> 	$\Gamma_1 = \frac{1}{24} t h^2 b^3$

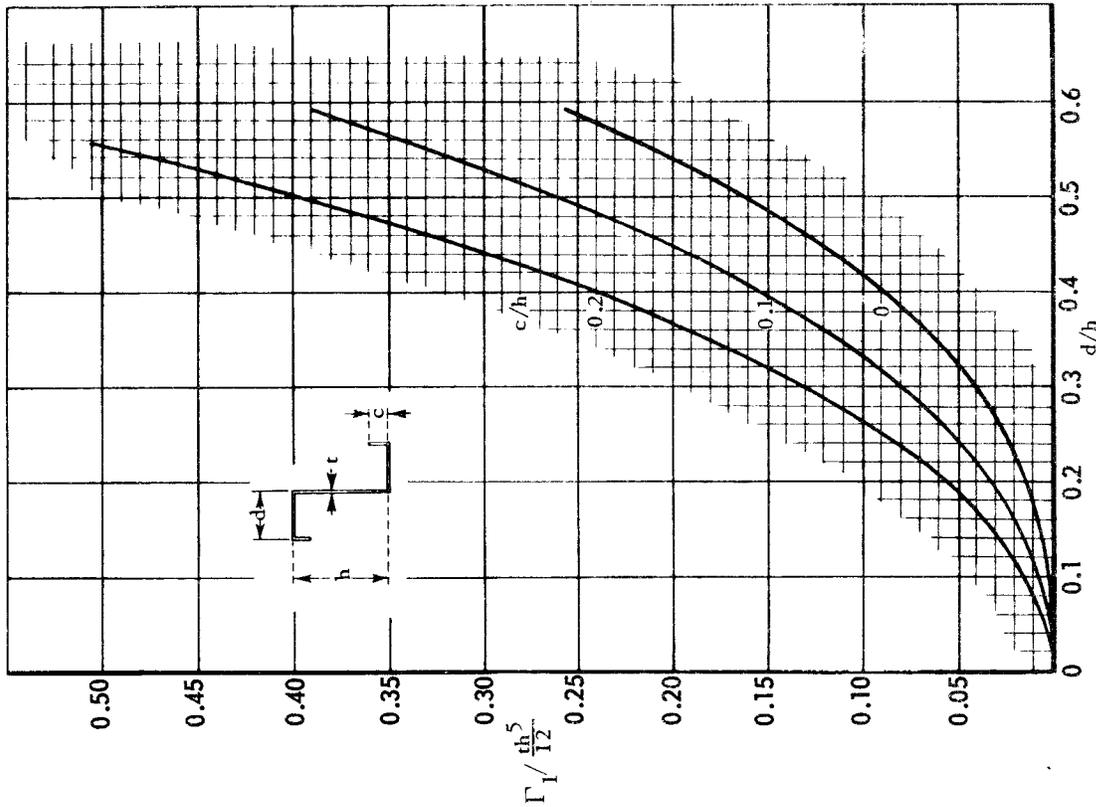


Fig. 5.3-9 Primary Warping Constant Γ_1 for Lipped Z-Sections

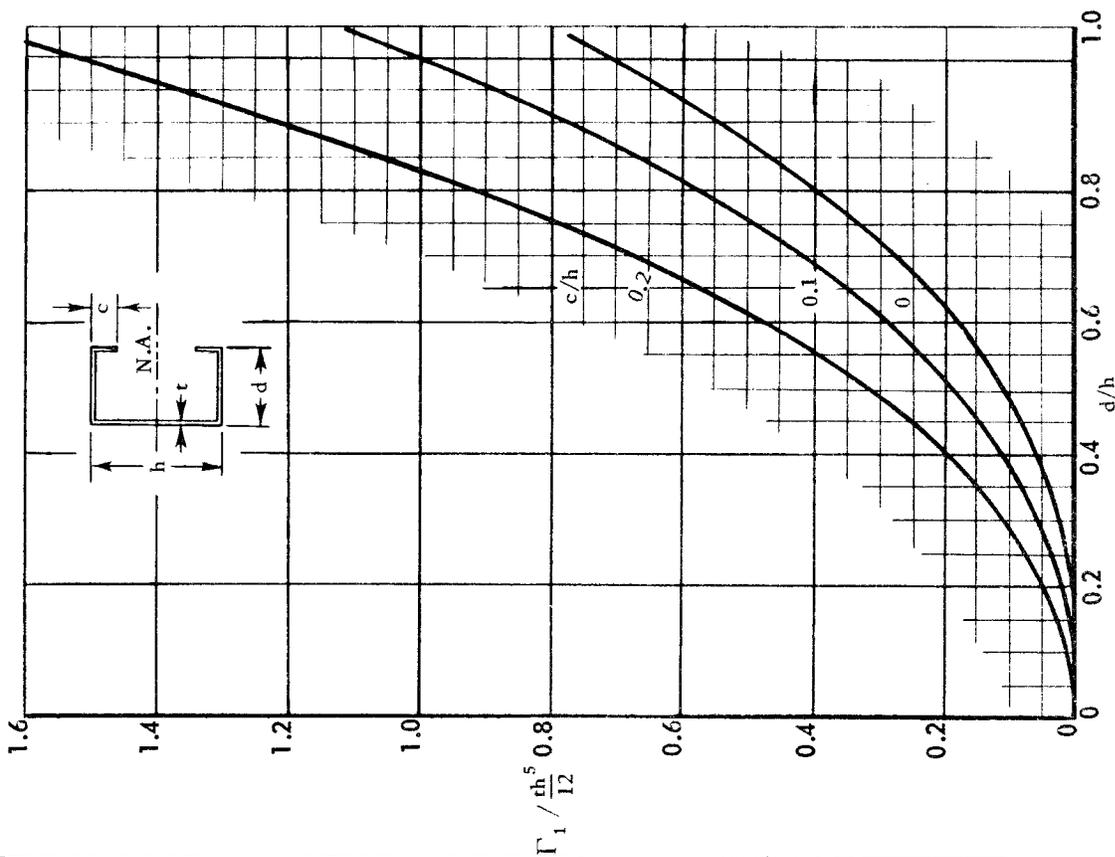


Fig. 5.3-10 Primary Warping Constant for a Lipped Channel Section
(Ref. S.M. Tech. Memo 30)



5.3.5.2 Method of Calculating Warping Constant in the General Case - From "Buckling Strength of Metal Structures", Bleich, p.119,

$$\begin{aligned} \text{Primary Warping Constant } \Gamma_1 &= \int_0^s (W - \bar{W})^2 t ds \\ &= t \int_0^s (W - \bar{W})^2 ds \\ &\text{for constant } t. \\ W &= \text{unit warp} = \int_0^s p_t ds \\ \bar{W} &= \text{mean unit warp} \\ &= \frac{1}{s} \int_0^s W ds \end{aligned}$$

Having found W and \bar{W} by the method of 5.3.3.1 tabulate values of $(W - \bar{W})$ and $(W - \bar{W})^2$ at each station. Plot $(W - \bar{W})^2$ against s and determine the area under this curve viz $\int_0^s (W - \bar{W})^2 ds$, as in the numerical example that follows.

NUMERICAL EXAMPLE
Z-Section as before

$\bar{W} = 29 \times 10^{-2}$ from 5.3.3.1 as are values of W

STN	$W \times 10^2$	$(W - \bar{W}) \times 10^2$	$(W - \bar{W})^2 \times 10^4$
1	0	- 29	841
2	5.34	- 23.66	560
3	11.96	- 17.04	290.4
4	24.46	- 4.54	20.6
5	39.51	+ 10.51	110
6	39.94	+ 10.94	119.5
7	40.36	+ 11.36	129
8	40.79	+ 11.79	139
9	41.21	+ 12.21	149
10	28.71	- .29	.09
11	16.21	- 12.79	163
12	5.01	- 23.99	575.5

(Plotted in Fig. 5.3-8)

$$\frac{\Gamma_1}{t} = \sum_A^B (W - \bar{W}) \Delta s = 394.38 \times 10^4 \text{ by graphical integration}$$

$$\frac{\Gamma_1}{t} = .0394$$

$$\Gamma_1 = .0394 \times .051 = 0.00201$$

Check from R.Ae.S. 00.07.02 assuming that the section is as in Fig. 5.3-11.

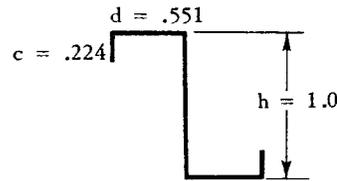


Fig. 5.3-11

$$\frac{d}{h} = \frac{.551}{1.0} = .551$$

$$c = .224 \quad \frac{c}{h} = .224$$

$$\frac{\Gamma_1}{t} = .50 \frac{h^5}{12} \text{ from curve} \quad h = 1.0 \quad h^5 = 1.0$$

$$= .50 \times .0832 \quad \frac{h^5}{12} = \frac{1}{12} = .0832$$

$$= .0416$$

as compared with .0394 above.

5.3.6 Torsional Instability

5.3.6.1 Stringer Section Alone

The critical torsional instability stress of an open section stringer is given below.

This assumes that plane cross-sections of the stringer warp but do not change their geometric shape, known as Primary failure, as distinct from local instability (for which see 6.2), characterised by distortion of the section.

$$\sigma_{T1} = \frac{1}{I_P} \left[\text{TWISTING } GJ + \frac{(\pi)^2}{L} \text{ BENDING } E_T \Gamma \right]$$

See GEN/1090/809.8.1

The first term in the bracket represents the resistance of the stringer to pure twisting and the second term its resistance to bending.

I_P = Polar moment of inertia of the section referred to the shear centre = $I_{P0} + Ar_0^2$

I_{P0} = Polar moment of inertia about the centroid

r_0 = Polar distance from centroid to shear centre

A = area of section

G = shear modulus

J = St. Venant torsion constant

t = thickness of stringer

L = length of stringer

E_T = tangent modulus of elasticity

Γ = warping constant relative to the shear centre



NUMERICAL EXAMPLE

$$\begin{aligned} A &= .1274 & r_o &= .017 \text{ (See GEN/1090/809/3.3)} \\ I_x &= .0225 & L &= 20 \\ I_y &= .01273 \end{aligned}$$

Polar moment of inertia about centroid

$$\begin{aligned} I_{P_O} &= I_x + I_y \\ &= .0225 + .0127 \\ &= .0352 \end{aligned}$$

Polar moment of inertia about shear centre

$$\begin{aligned} I_p &= I_{P_O} + Ar_o^2 \\ &= .0352 + .1274 \times .017^2 \\ &= .0352 \end{aligned}$$

$$\begin{aligned} J &= 1/3 \times \text{developed length} \times t^3 \\ &= 1/3 \times 2.499 \times (.051)^3 & .051^3 &= 132 \times 10^{-6} \\ &= 110 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} GJ &= .385 E J \\ &= 3.85 \times 10^6 \times 110 \times 10^{-6} = 423.5 \end{aligned}$$

$$E_T = E_{\text{say}} = 10 \times 10^6$$

$$\Gamma = .00206 = 20.6 \times 10^{-4} \quad (\text{from 5.3.5.2})$$

$$\left(\frac{\pi}{L}\right)^2 = \left(\frac{\pi}{20}\right)^2 = (.157)^2 = 2.47 \times 10^{-2}$$

$$\begin{aligned} \left(\frac{\pi}{L}\right)^2 E_T \Gamma &= 2.47 \times 10^{-2} \times 10 \times 10^6 \times 20.6 \times 10^{-4} \\ &= 509 \end{aligned}$$

$$\begin{aligned} \sigma_{T1} &= \frac{1}{I_p} \left[GJ + \left(\frac{\pi}{L}\right)^2 E_T \Gamma \right] \\ &= \frac{1}{.0352} [423.5 + 509] \\ &= \frac{932.5}{.0352} = \underline{26,500 \text{ p.s.i.}} \end{aligned}$$

It is customary to take the enforced axis of rotation at the intersection of the inside skin line and the principal axis of the stringer.

The torsional instability stress is then

$$\sigma_{T1} = \frac{1}{I_p} \left[\begin{array}{ccc} \text{TWISTING} & \text{BENDING} & \text{SKIN BENDING} \\ GJ^1 & + \left(\frac{\pi}{L}\right)^2 E_T \Gamma^1 & + \left(\frac{L}{\pi}\right)^2 K \end{array} \right]$$

See GEN/1090/809.8.1

Many of these items have the same significance as in 5.3.6.1 except that they are now found with reference to the enforced axis of rotation, as signified by the dash. Items not specified earlier are:-

K = skin support stiffness. It represents the resistance of the skin to bending due to twist of the stiffener = $\frac{1}{K_1 + K_2}$

$$\begin{aligned} K_1 &= \frac{3d}{Et_{SK}^3} \text{ for non-buckled skin} \\ &= \frac{d}{Et_{SK}^3} \text{ for buckled skin} \end{aligned}$$

d = stringer spacing = 4 in.

t_{SK} = skin thickness

$$K_2 = \frac{4h + 12r}{Et^3}$$

r = offset of rivet line from centre of rotation

h = depth of stringer

t = thickness of stringer

5.3.6.2 Stringer Attached to Skin

The stiffness of the skin in its own plane and the anchorage of the skin at the sides of the panel are controlling factors in the location of the enforced axis of rotation.

If the stiffness of the skin in its own plane is assumed to be infinite, the axis of rotation is forced to lie in the plane of the skin, because rotation of the stringer-skin about any axis not in the plane of the skin would require a movement of the skin in its own plane.

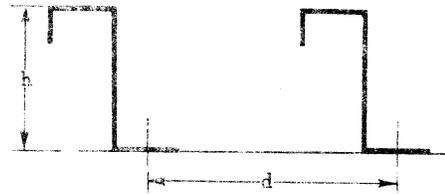


Fig. 5.3-12



NUMERICAL EXAMPLE

Same Z-section, attached to .036 in. skin. Take effskin width = $30t = 1.0$ in. say.

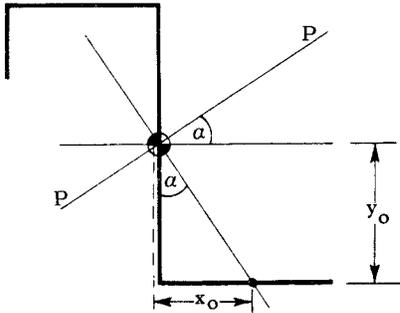


Fig. 5.3-13

(a) Warping Constant Γ^1

$$\Gamma^1 = \Gamma_{\text{STRINGER}}^1 + \Gamma_{\text{SKIN}}^1$$

$$\Gamma_{\text{STRINGER}}^1 = \Gamma + x_o^2 I_X + y_o^2 I_Y - 2x_o y_o I_{XY}$$

Where x_o and y_o are the co-ordinates of enforced axis relative to S.C. of stringer alone

$$y_o = -.50 \quad \text{See GEN/1090/809.1}$$

$$\begin{aligned} x_o &= .017 + .50 \tan \alpha & \alpha &= 34^\circ 40' \\ &= .017 + .3458 & \tan \alpha &= .6915 \\ &= .3628 \end{aligned}$$

$$x_o^2 = .132 \quad y_o^2 = .25$$

$$\begin{aligned} x_o^2 I_X &= .132 \times .0225 & y_o^2 I_Y &= .25 \times .01273 \\ &= .00297 & &= .00318 \end{aligned}$$

$$\begin{aligned} 2x_o y_o I_{XY} &= 2 \times .3628 \times -.50 \times -.012995 \\ &= +.00472 \end{aligned}$$

$$\begin{aligned} \Gamma_{\text{STRINGER}}^1 &= .00206 + .00297 + .00318 - .00472 \\ &= .00349 \end{aligned}$$

Γ_{SKIN}^1 = secondary warping constant of skin (primary warping constant is zero)

$$= \frac{1}{144} (\text{effskin})^3 t_{\text{SKIN}}^3 \quad \text{from 5.3.5}$$

$$= \frac{1}{144} \times 1.0^3 \times .036^3$$

= NEGLIGIBLE

$$\Gamma_{\text{GROSS}}^1 = .00349 = 34.9 \times 10^{-4}$$

$$\begin{aligned} \text{(b) } J_{\text{SK}} &= \frac{1}{3} \text{effskin} \times t_{\text{SK}}^3 \\ &= \frac{1}{3} \times 1.0 \times 46.5 \times 10^{-6} \\ &= 15.5 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} J^1 &= J + J_{\text{SK}} \\ &= 110 \times 10^{-6} + 15.5 \times 10^{-6} \\ &= 125.5 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} GJ^1 &= 3.85 \times 10^6 \times 125.5 \times 10^{-6} \\ &= 483 \end{aligned}$$

$$\begin{aligned} \text{(c) } I_{\text{PSTR}}^1 &= I_P + A r_o^2 & r_o^2 &= x_o^2 + y_o^2 \\ &= .0352 + .0487 & &= .132 + .25 \\ &= .0839 & &= .382 \end{aligned}$$

$$A r_o^2 = .1274 \times .382 = .0487$$

$$\begin{aligned} I_{\text{PSKIN}}^1 &= \frac{1}{12} (\text{effskin})^3 t_{\text{SK}} \quad \text{approx.} \\ &= \frac{1}{12} \times 1.0 \times .036 = .003 \end{aligned}$$

$$I_{\text{PGROSS}}^1 = .0839 + .003 = .0869$$

(d) Assume well-buckled skin

$$K_1 = \frac{d}{Et_{\text{SK}}^3} = \frac{4}{10^7 \times 46.5 \times 10^{-6}} = .0086$$

$$\begin{aligned} K_2 &= \frac{4h + 12r}{Et_{\text{SK}}^3} = \frac{4 \times 1.0}{10^7 \times 46.5 \times 10^{-6}} \\ &\quad \text{since } r = 0 \end{aligned}$$

$$= .0086$$

$$K_1 + K_2 = .0172$$

$$K = \frac{1}{.0172} = 58.2$$

$$\text{(e) } \left(\frac{\pi}{L}\right)^2 = 2.47 \times 10^{-2}$$

$$\left(\frac{L}{\pi}\right)^2 = \frac{10^2}{2.47} = 40.4$$



$$\begin{aligned} \sigma_{T1} &= \frac{1}{I_p} [GJ^1 + \left(\frac{\pi}{L}\right)^2 E_T I^1 + \left(\frac{L}{\pi}\right)^2 K] \\ &= \frac{1}{.0869} [483 + 24.7 \times 10^4 \times 34.9 \times 10^{-4} + 40.4 \times 58.2] \\ &= \frac{1}{.0869} [483 + 861 + 2351] \\ &= \frac{3695}{.0869} = 42,600 \text{ p.s.i} \end{aligned}$$

5.4 Thin Multi-walled Sections
(concentrated T, constant J)

In order to explain a method of finding the shear stresses due to torque in thin multi-walled sections, consider the two cell wing-beam shown in Fig. 5.4-1 under the action of a torque T, assumed concentrated at the ends.

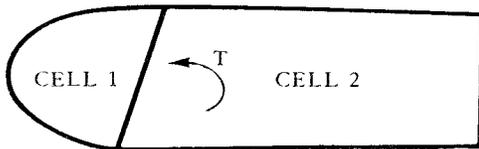


Fig. 5.4-1

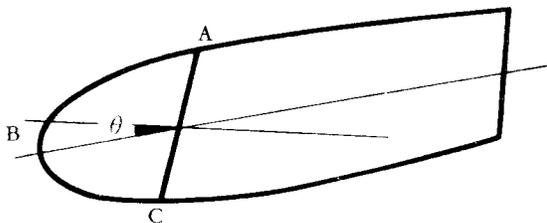


Fig. 5.4-2

Let θ be the angle of twist of the whole beam due to the action of the torque T. If there is no distortion of the section it will be clear that the boundary wall AC will rotate through some angle θ (Fig. 5.4-2) and consequently the angle of twist of Cell 1 will be the same as that of Cell 2.

It can be shown (see GEN/1090/810) that

$$\frac{\theta G}{L} = \frac{\Sigma \text{ shear flux} \times p/t}{2A}$$

Thus for the two cell beam under consideration it is necessary to determine the value of the right-hand side of this equation for both Cell 1 and for Cell 2 in terms of the unknown shear fluxes and then equate the two, since each is equal to $\frac{\theta G}{L}$.

A second relationship results from the fact that the total torque T on the section must equal the sum of the individual torques in each cell (see GEN/1090/810). That is,

$$\begin{aligned} T &= T_1 + T_2 \\ &= 2A_1 q_1 + 2A_2 q_2 \end{aligned}$$

where q_1 and q_2 are the shear fluxes in Cells 1 and 2 respectively.

The method will be demonstrated with reference to a 3 cell beam, the solution of which is exactly the same as the above in principle but of necessity a little more complicated.

NUMERICAL EXAMPLE

To determine the distribution of shear stress, due to a concentrated torque $T = 10,000$ lb. in., on the section shown in Fig. 5.4-3 given that:-

Enclosed areas are $A_1 = 89.3$ in.²

$A_2 = 250$ in.²

$A_3 = 125$ in.²

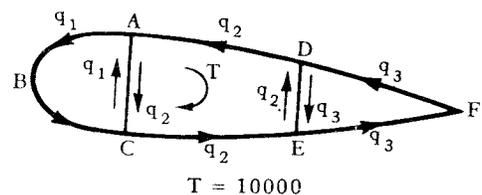


Fig. 5.4-3

Other given data is as follows:-

MEMBER	ABC	AD	CE	DFE	AC	DE
LENGTH p (in.)	25.72	25	25	51.93	10	10
THICKNESS t (in.)	.05	.04	.02	.01	.06	.03

Let the reactive shear fluxes in the members be as indicated in Fig. 5.4-3. The following information is tabulated:-



MEMBER	ABC	AD	CE	DFE	AC	DE
Shear flux	q_1	q_2	q_2	q_3	$q_1 - q_2$	$q_2 - q_3$
perimeter p	25.72	25	25	51.93	10	10
thickness t	.05	.04	.02	.01	.06	.03
p/t	514.4	625	1250	5193	166.7	333
Shear flux \times p/t	$514.4q_1$	$625q_2$	$1250q_2$	$5193q_3$	$166.7(q_1 - q_2)$	$333(q_2 - q_3)$

Cell 1 - Take contra-clockwise reactive shear flux as positive.

$$\frac{\theta G}{L} = \frac{\sum \text{shear flux} \times p/t}{2A_1} = \frac{514.4 q_1 + 166.7(q_1 - q_2)}{178.6}$$

$$= 3.83 q_1 - 0.932 q_2 \quad (1)$$

Cell 2

$$\frac{\theta G}{L} = \frac{625 q_2 + 1250 q_2 - 166.7(q_1 - q_2) + 333(q_2 - q_3)}{500}$$

$$= -0.333 q_1 + 4.75 q_2 - 0.66 q_3 \quad (2)$$

Cell 3

$$\frac{\theta G}{L} = \frac{5193 q_3 - 333(q_2 - q_3)}{250}$$

$$= 22.2 q_3 - 1.332 q_2 \quad (3)$$

By solving these three simultaneous equations the following results are obtained:

$$q_1 = 5.38 q_3$$

$$q_1 = 1.322 q_2$$

$$q_2 = 4.06 q_3$$

From the second relationship

$$T = 2A_1 q_1 + 2A_2 q_2 + 2A_3 q_3$$

$$10,000 = 178.6 q_1 + 500 q_2 + 250 q_3$$

Substituting in terms of q_3 ,

$$10,000 = 960 q_3 + 2030 q_3 + 250 q_3 = 3240 q_3$$

whence $q_3 = 3.10$ lb. per in.

$$q_2 = 12.52 \text{ lb. per in.}$$

$$q_1 = 16.60 \text{ lb. per in.}$$

$$(q_1 - q_2) = 4.08 \text{ lb. per in.}$$

$$(q_2 - q_3) = 9.42 \text{ lb. per in.}$$

As shown
in Fig. 5.4-4

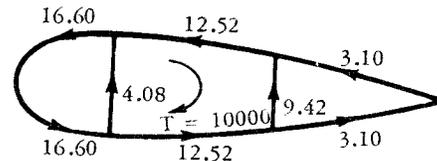


Fig. 5.4-4

These results are in agreement with those in J.Ae.Sc., January 1945, p.103, "A Simplified Analysis of Hollow Beams" by Eryk Kosko.

Check on $\frac{\theta G}{L}$

$$\text{Cell 1. } \frac{\theta G}{L} = 3.83 q_1 - 0.932 q_2 = 63.5 - 11.7 = \underline{51.8}$$

$$\text{Cell 2. } \frac{\theta G}{L} = -0.333 q_1 + 4.75 q_2 - 0.66 q_3$$

$$= -5.54 + 59.7 - 2.05 = \underline{52}$$

$$\text{Cell 3. } \frac{\theta G}{L} = 22.2 q_3 - 1.332 q_2 = 68.8 - 16.7 = \underline{52.1}$$

Check on T

$$T = 2A_1 q_1 + 2A_2 q_2 + 2A_3 q_3$$

$$= 178.6 \times 16.6 + 500 \times 12.52 + 250 \times 3.10$$

$$= 2970 + 6260 + 776 = \underline{10006} \text{ as against } 10000.$$



COLUMNS

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6.1.4	Miscellaneous Formulas	6.5.4	Numerical examples
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COLUMNS

Introduction and Reference Table

Column problems usually involve the determination of (a) the buckling load and/or (b) the distribution of bending moment.

The Column Reference Table below indicates the Section that deals with one or other of these subdivisions (a) and (b).

COLUMN REFERENCE TABLE

PROBLEM		(a) BUCKLING LOAD	(b) BENDING MOMENT
Single Span Column No Lateral Load	I Constant	See 6.1.1 for Euler formulas for pinned and fixed ends. See 6.1.3 for Column Stress curves for pinned and fixed ends.	
	I Varying	See 6.3.1 for standard cases and 6.3.2 for general method	
Continuous Column No Lateral Load	I Constant	See 6.4 for Charts	
Straight Beam-Column	I Constant	Buckling stress, corresponding to buckling load of 6.1, must not be exceeded by combined stress at any station.	See 6.5.2 for summary of standard cases, 6.5.3 for Perry's approximation and 6.5.5 for Polar diagrams
	I Varying		See 6.5.6 for general method
Column with small initial Bow	I Constant		See 6.6 for summary of standard cases.



6.1 Primary Instability in Compression

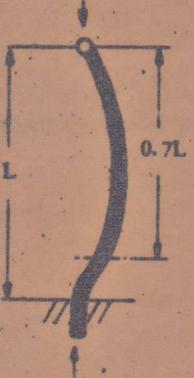
6.1.1 Long Columns

Euler's formula for the primary failure by lateral bending (buckling) of long columns, that is, columns loaded in the elastic range is:-

$$F_c = C \frac{\pi^2 E}{(L/\rho)^2} = \frac{\pi^2 E}{(L/\rho\sqrt{C})^2}$$

$$= \frac{\pi^2 E}{(L'/\rho)^2} \quad \text{ANC-5, Equation 1.381}$$

- Where F_c = allowable compressive stress
 E = Modulus of Elasticity
 L = length of column
 ρ = radius of gyration of cross section
 C = fixity coefficient, the value of which depends on the degree of end fixity or constraint. Values of C are given below: they are obtained in GEN/1090/811.1
 L/ρ = slenderness ratio
 L'/ρ = effective slenderness ratio
 L' = effective column length = $\frac{L}{\sqrt{C}}$

PINNED AT EACH END	PINNED AT ONE END FIXED AT OTHER END	FIXED AT BOTH ENDS	FREE AT ONE END FIXED AT OTHER END (LIKE A MAST)
 <p>$C = 1.0$</p>	 <p>$C = \frac{1}{(0.7)^2} = 2.047$</p>	 <p>$C = 4.0$</p>	 <p>$C = 1/4$</p>

ABC in Fig. 6.1.2-1 is a typical Euler curve of F_c plotted against the effective slenderness ratio L'/ρ for an aluminium alloy column having $E = 10 \times 10^6$ p.s.i. (Values used in plotting this curve are given in GEN/1090/811.2)



6.1.2 Short Columns

The Euler formula of 6.1.1 cannot be used at the lower end of the L'/ρ scale (in the short column range) because primary bending failure occurs at stresses below those obtained thereby.

This is due to unavoidable eccentricities and to the reduction in slope of the stress-strain curve above the proportional limit of the material, so that, instead of E , it is necessary to use an effective value E' in the Euler formula.

For short columns this becomes:-

$$F_c = C \frac{\pi^2 E'}{(L'/\rho)^2} \quad \text{ANC 5, Equation 1.381 a.}$$

For short columns it is more convenient, however, to employ a transition curve, tangential to the Euler curve at some point B in Fig. 6.1.2-1 and having the value F_{CO} , the column yield stress (upper limit of column stress for primary failure), at $L'/\rho = 0$.

This transition curve is either a parabola, as shown in Fig. 6.1.2-1, or a straight line, depending on the material, the general expression for F_c over the short column range, that is, to the left of B, being:-

$$F_c = F_{CO} \left\{ 1 - K \left[\frac{L'/\rho}{\pi/E/F_{CO}} \right]^n \right\} \quad \text{ANC 5 Equation 1.382}$$

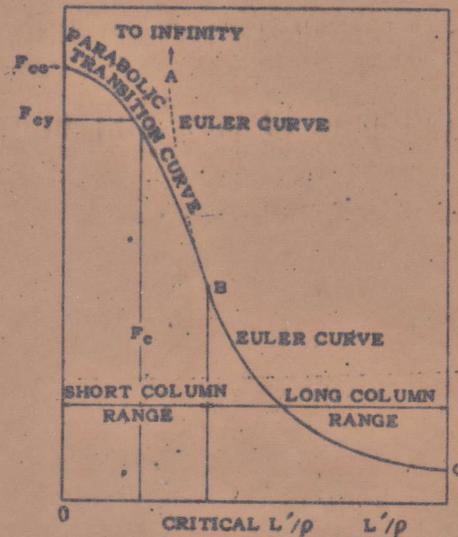


Fig. 6.1.2-1

Here F_{CO} is defined above, K is a coefficient and the index n can be 2.0 (Johnson Parabola), 1.5 (Parabola) or 1.0 (straight line), as the case may be.

Expressions for F_c and for the critical value of L'/ρ corresponding to point B, to the left of which columns are "short", are given for $n = 2.0, 1.5$ and 1.0 in the tabular summary that follows and are proved in GEN/1090/811.3.



F_c = Allowable compressive stress.
 F_{co} = Column yield stress (upper limit of column stress for primary failure), as shown in Fig. 6.1.2-1
 F_{cy} = Compressive yield stress.

n	SHORT COLUMN FORMULA	ANC-5 EQUATION	USED ON COLUMN CURVE FOR THESE MATERIALS	F_{cy} k.s.i.	F_{co} k.s.i.	CRITICAL L'/ρ , BELOW WHICH VALUE COLUMN IS SHORT
2.0	$F_{co} \left\{ 1 - \frac{F_{co} (L'/\rho)^2}{47^2 E} \right\}$	1.383	Aluminum alloy 75S-T6 Plain carbon steel 1025 Heat treated steel $F_{tu}=125$ ksi Heat treated steel $F_{tu}=150$ ksi Heat treated steel $F_{tu}=180$ ksi	70	1.075 $F_{cy} = 75.3$	51 124 75.6 65.0 58.9
1.5	$F_{co} \left\{ 1 - 0.3027 \frac{L'/\rho}{\pi \sqrt{E/F_{co}}} \right\}^{1.5}$	1.384	Magnesium alloy sheet FS-1h Alloy steel 4130	(0.025-0.064) = 25 (0.065-0.260) = 24	1.05 $F_{cy} = 26.25$ $F_{cy} = 25.2$	70
1.0	$F_{co} \left\{ 1 - 0.385 \frac{L'/\rho}{\pi \sqrt{E/F_{co}}} \right\}$	1.385	Aluminum alloy 24S-T3 Aluminum alloy 61S-T6	41.5 35.6	79.5 $F_{cy} \left[1 + \frac{F_{cy}}{200} \right]$	91.5 1.528 $\pi \sqrt{\frac{E}{F_{co}}}$ 1.732 $\pi \sqrt{\frac{E}{F_{co}}}$

See GEN/1090/811.3 for proofs of short column formulas for F_c and critical L'/ρ .
 Long column formula in each case is $F_c = \frac{\pi^2 E}{(L'/\rho)^2}$

For Magnesium alloy extruded shapes use the expression $F_c = K \frac{(F_{cy})^n}{(L'/\rho)^m}$ as in ANC-5, Table 4.21 (a), p.117.



6.1.3 Column Stress Curves

Fig. 6.1.3-1 Aluminum Alloy 24S-T3

Fig. 6.1.3-2 Aluminum Alloy 61S-T6

Fig. 6.1.3-3 Aluminum Alloy 75S-T6

Fig. 6.1.3-4 Steel 1025

Fig. 6.1.3-5 Heat Treated Steel $F_{tu} = 125$ k.s.i.Fig. 6.1.3-6 Heat Treated Steel $F_{tu} = 150$ k.s.i.Fig. 6.1.3-7 Heat Treated Steel $F_{tu} = 180$ k.s.i.

Fig. 6.1.3-8 Alloy Steel 4130

Fig. 6.1.3-9 Magnesium Alloy Sheet FS-1h

The values of F_{cy} and F_{co} used in obtaining these curves are given in the tabular summary of 6.1.2. Detailed calculations appear in GEN/1090/811.4.

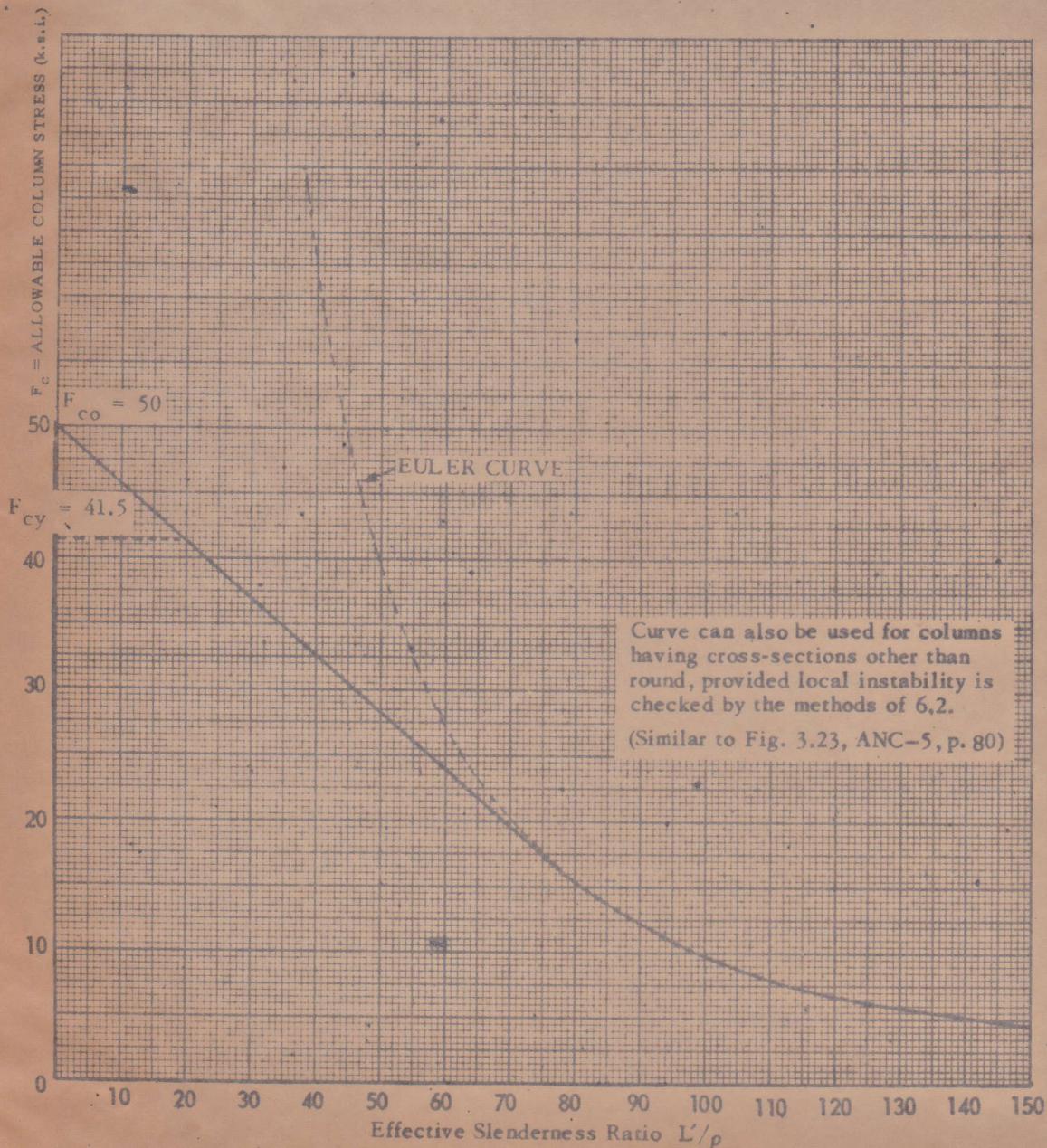


Fig. 6.1.3-1 Allowable Column Stress for 24S-T3



6.1.3 Column Stress Curves (cont'd)

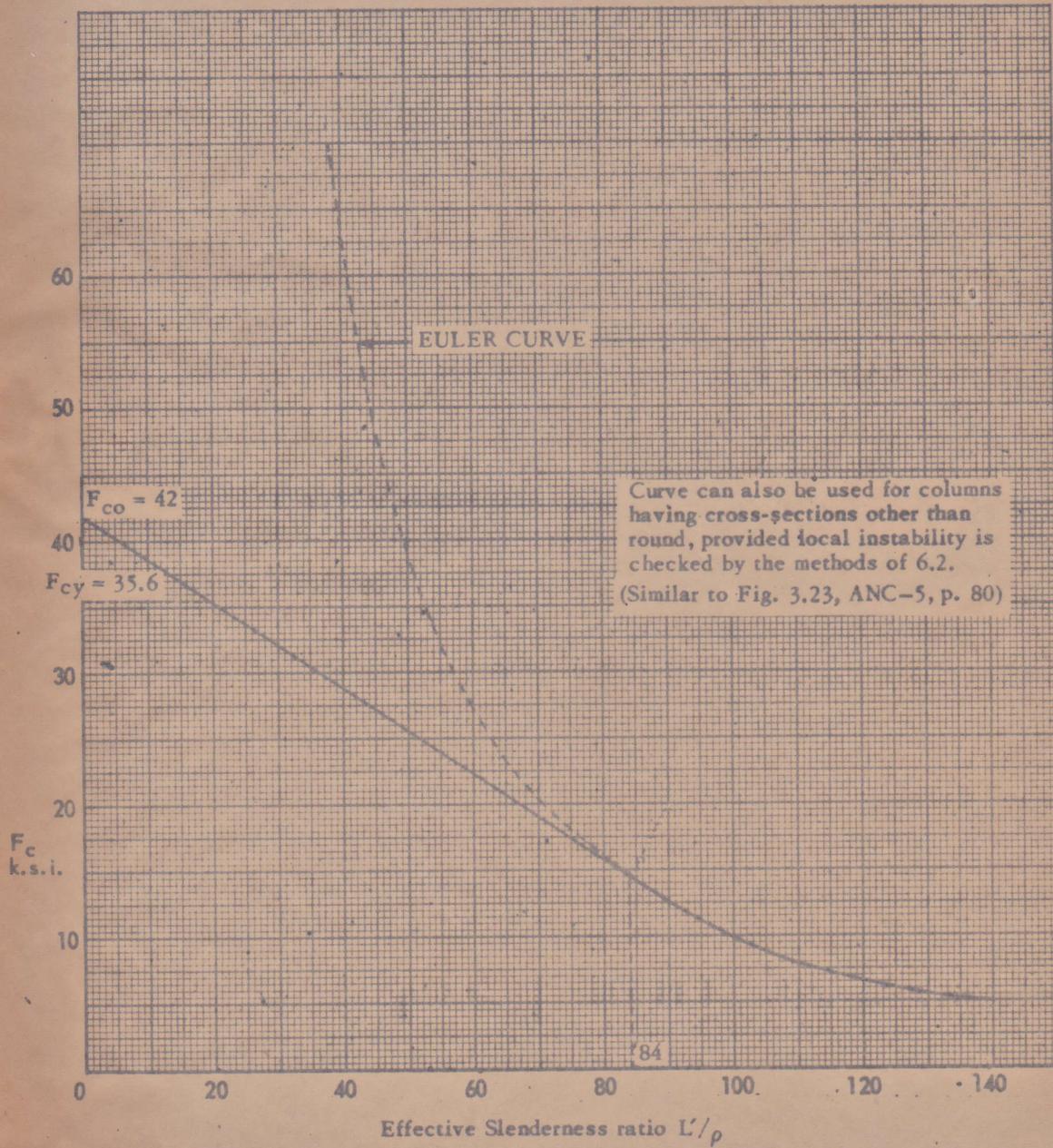


Fig. 6.1.3-2 Allowable Column Stress for 61S-T6



6.1.3 Column Stress Curves (cont'd)

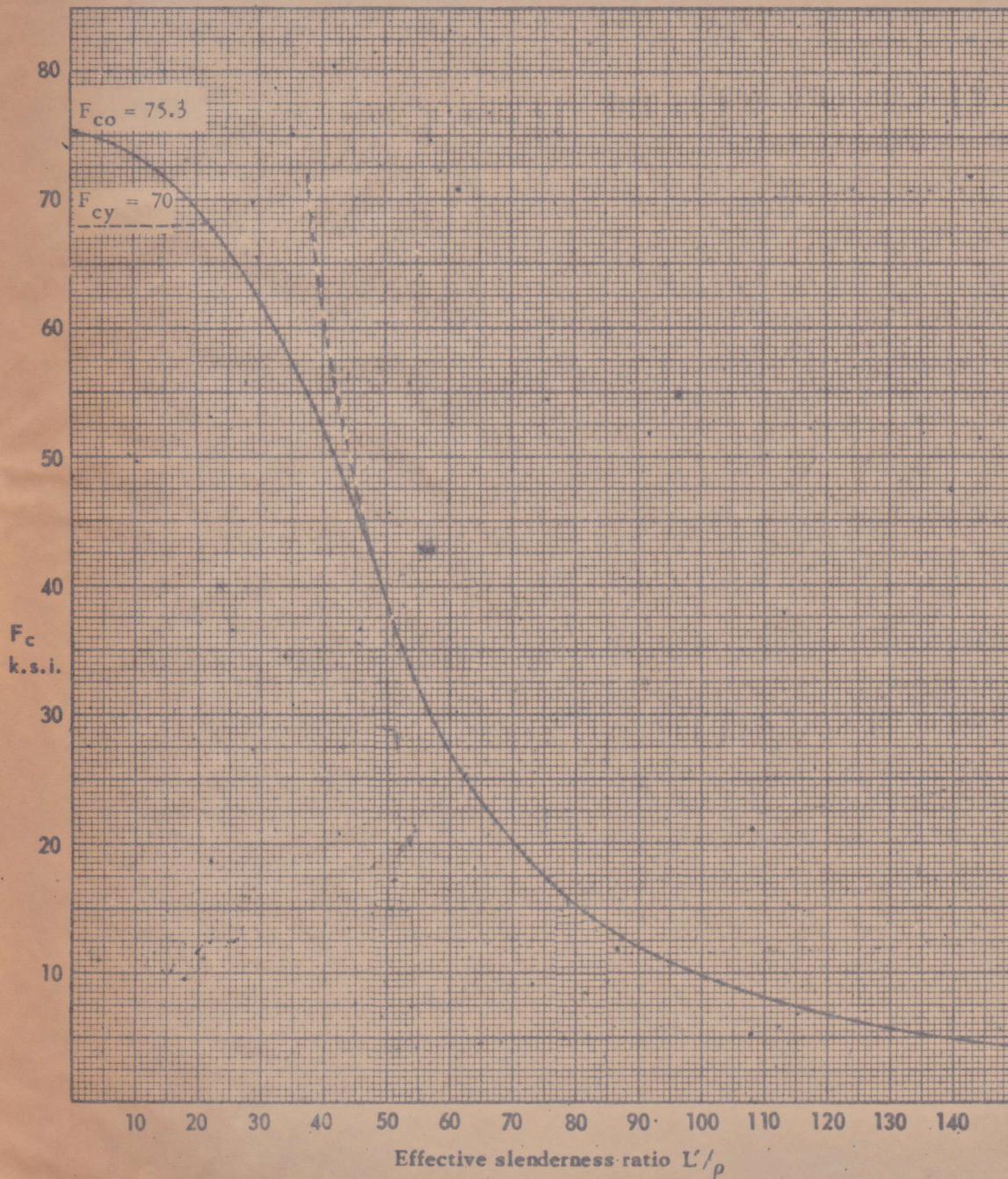


Fig 6.1.3-3 Fig. 6.1.3-3 Allowable Column Stress for 75S-T6



6.1.3 Column Stress Curves (cont'd)

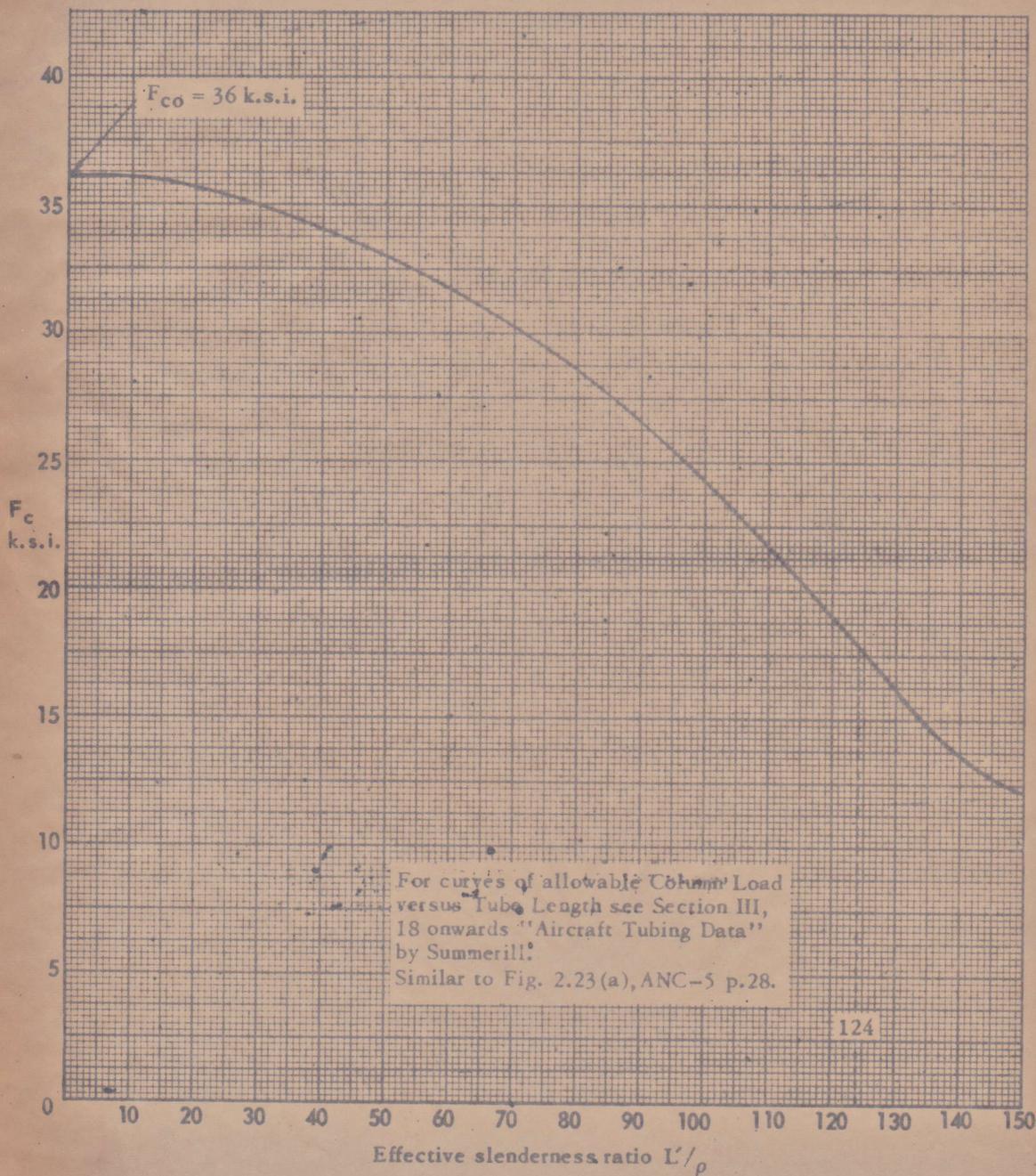


Fig. 6.1.3-4 Allowable Column Stress for Plain Carbon Steel 1025 Round Tubing



6.1.3 Column Stress Curves (cont'd)

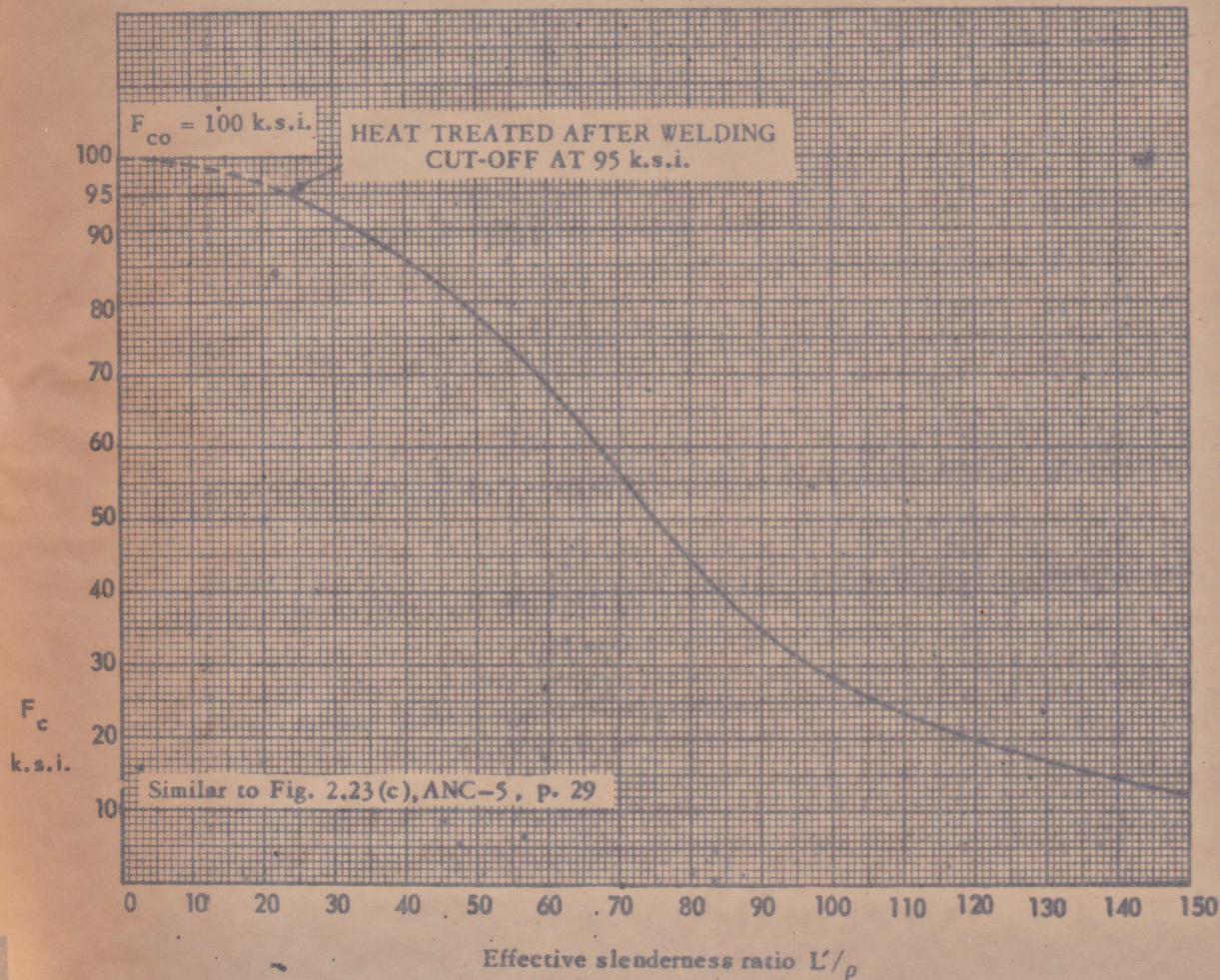


Fig. 6.1.3-5 Allowable Column Stress Curve for Heat-Treated Alloy Steel Round Tubing $F_{TU} = 125 \text{ k.s.i.}$



6.1.3 Column Stress Curves (cont'd)

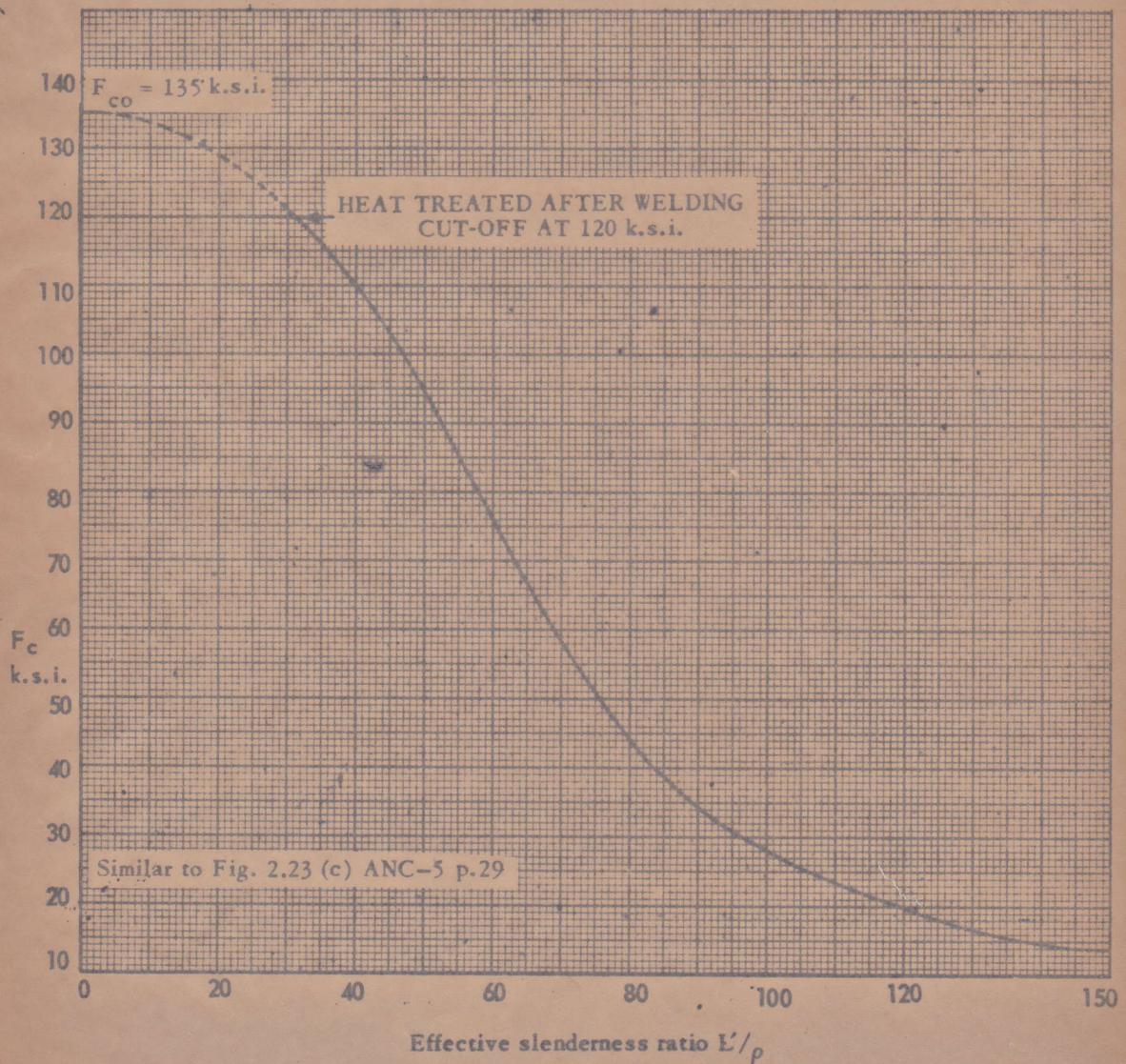


Fig. 6.1.3-6 Heat Treated Alloy Steel Round Tubing $F_{TU} = 150$ k.s.i.



6.1.3 Column Stress Curves (cont'd)

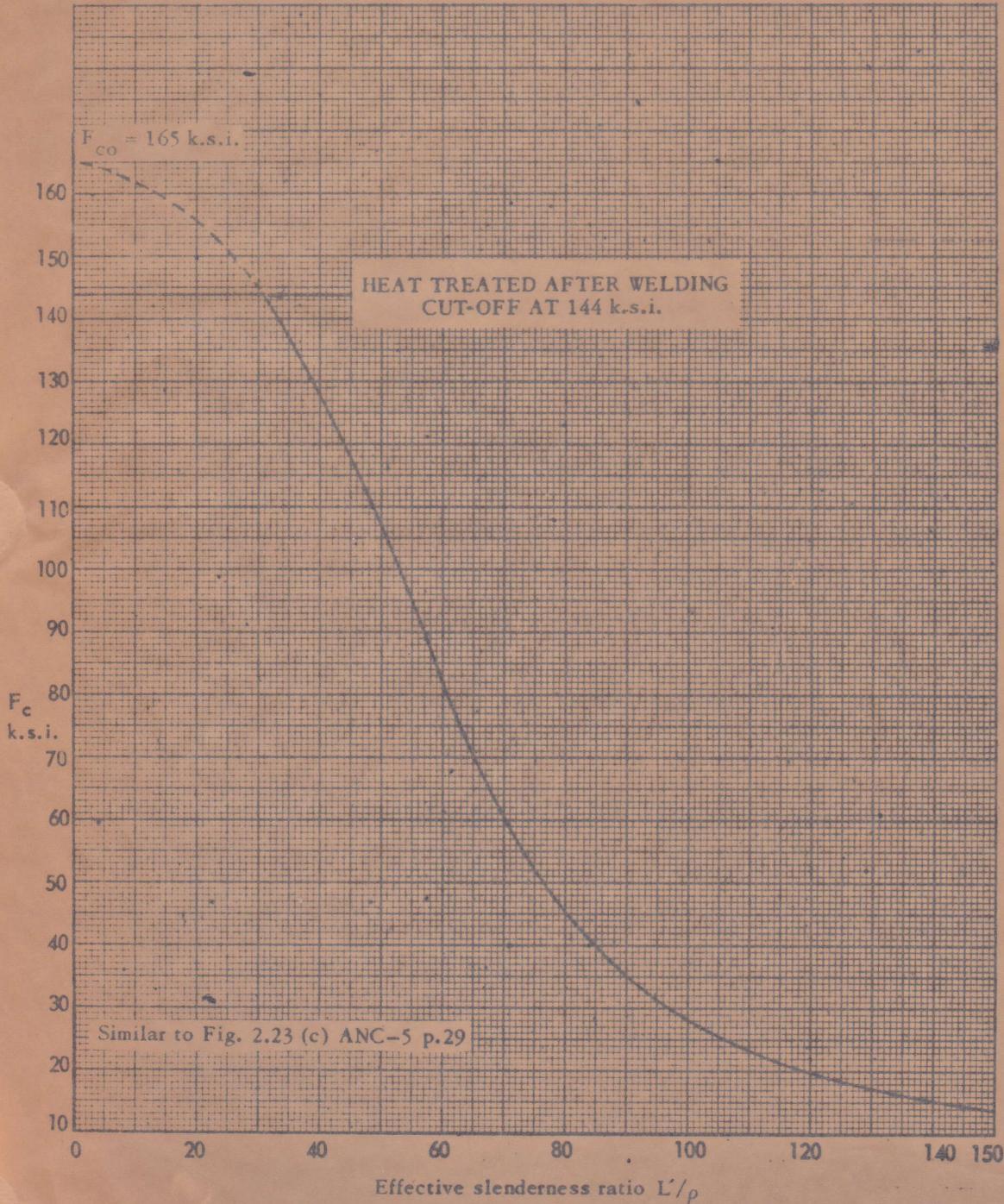


Fig. 6.1.3-7 Heat Treated Alloy Steel Round Tubing $F_{TU} = 180$ k.s.i.



6.1.3 Column Stress Curves (cont'd)

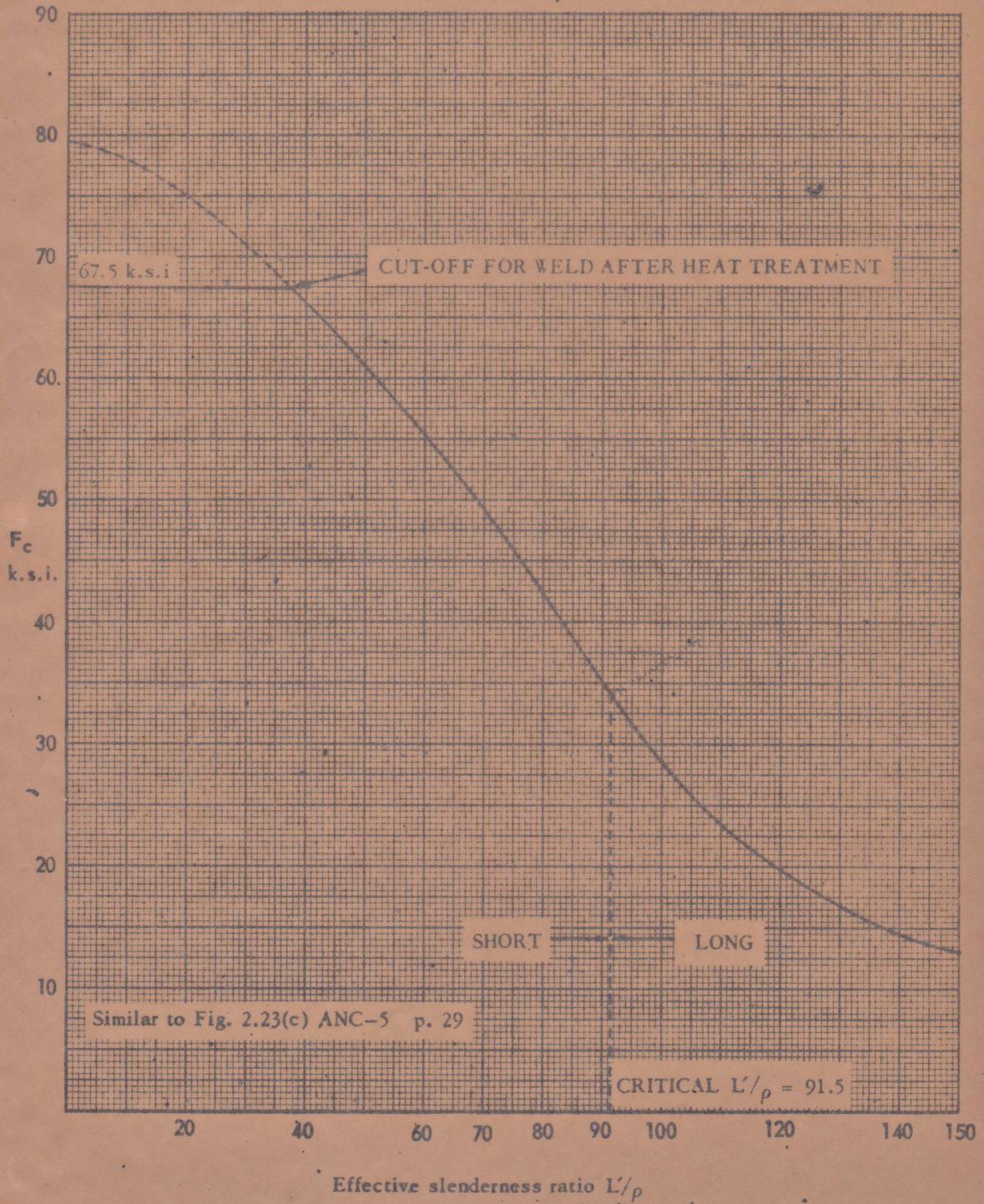


Fig. 6.1.3-8 Alloy Steel 4130 Round Tubing



6.1.3 Column Stress Curves (cont'd)

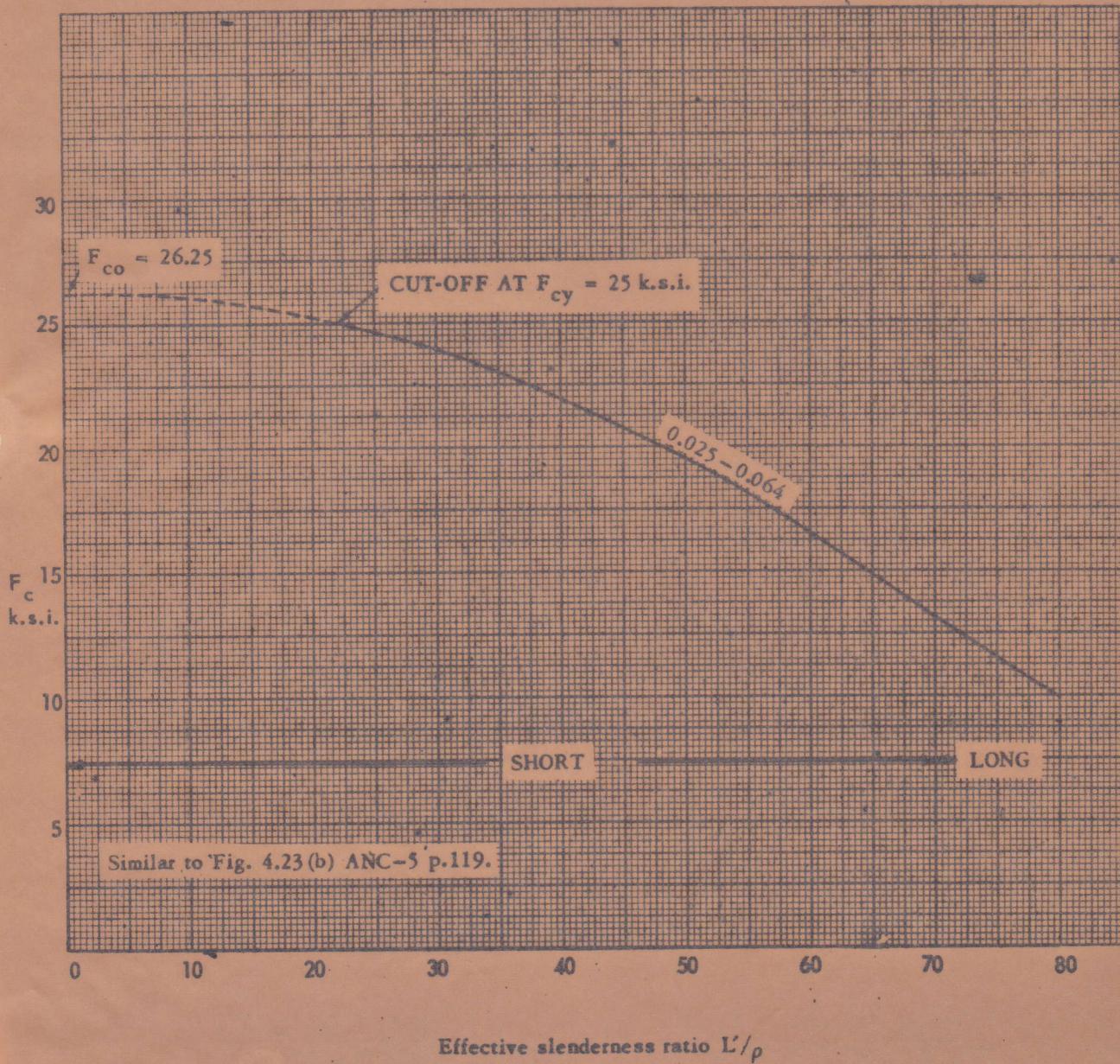


Fig. 6.1.3-9 Magnesium Alloy Sheet FS-1h



6.1.3 Column Stress Curves (cont'd)

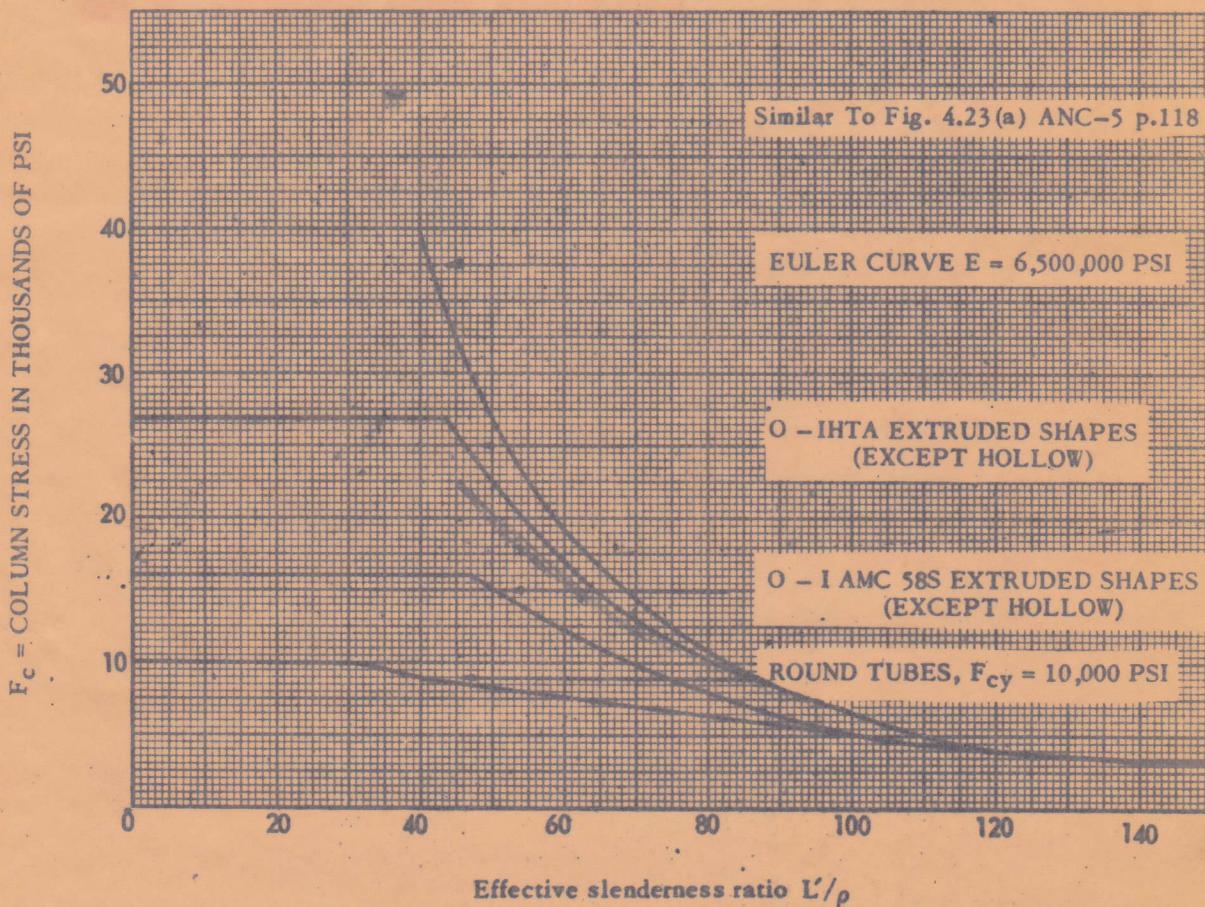


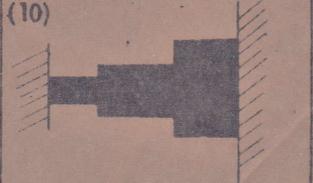
Fig. 6.1.3-10 Magnesium Alloy Columns



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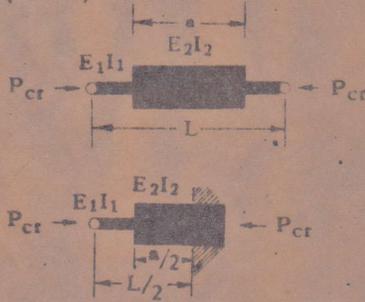
- 6.3.1 Standard Cases of Long Columns;
Tabular Summary
- 6.3.2 Approximate General Method for Pinned Ends
 - 6.3.2.1 Numerical Example

6.3.1 Long Columns of Varying Section

<p>(1) PINNED - PINNED</p> 	<p>See Fig. 6.3.1-1.</p>	<p>(6) PINNED - PINNED</p>  <p>STRAIGHT TAPER</p>	<p>See Roark, p. 298, Table XV</p>
<p>(2) PINNED - PINNED</p> 	<p>See Fig. 6.3.1-2</p>	<p>(7) FIXED - FIXED</p>  <p>STRAIGHT TAPER</p>	<p>See Roark, p. 298, Table XV</p>
<p>(3) PINNED - PINNED</p> 	<p>See Fig. 6.3.1-3</p>	<p>(8) PINNED - PINNED</p>  <p>ELLIPTICAL TAPER</p>	<p>See Fig. 6.3.1-4</p>
<p>(4) PINNED - FIXED</p> 	<p>See Fig. 6.3.1-1</p>	<p>(9)</p> 	<p>See J. Ae. Sc., April 54, and J. App. Mech. Dec. '51, 415.</p>
<p>(5) FREE - FIXED</p>  <p>(LIKE A MAST)</p>	$\tan \mu_1 l_1 \tan \mu_2 l_2 = \frac{\mu_1}{\mu_2}$ <p>See Numerical example in GEN/1090/811.5</p>	<p>(10)</p> 	<p>J. App. Mech. Dec. '51, 415</p>



6.3.1 Long Columns of Varying Section (cont'd)



$$\text{Buckling Load } P_{cr} = m \frac{E_2 I_2}{L^2}$$

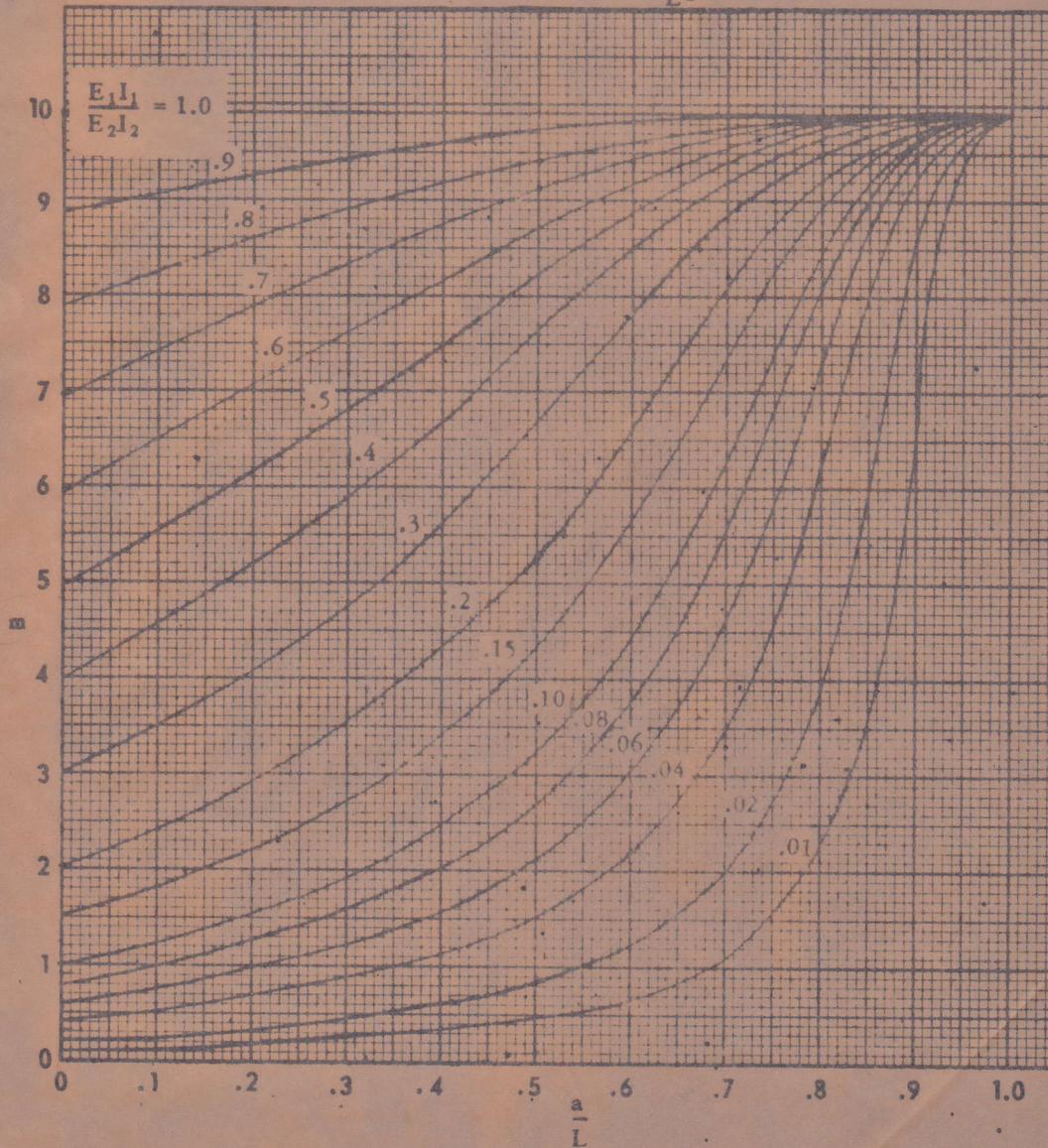


Fig. 6.3.1-1 Buckling Load for Column of Varying Section



6.3.1 Long Columns of Varying Section (cont'd)



$$\text{Buckling Load } P_{cr} = m \frac{E_1 I_1}{L^2}$$

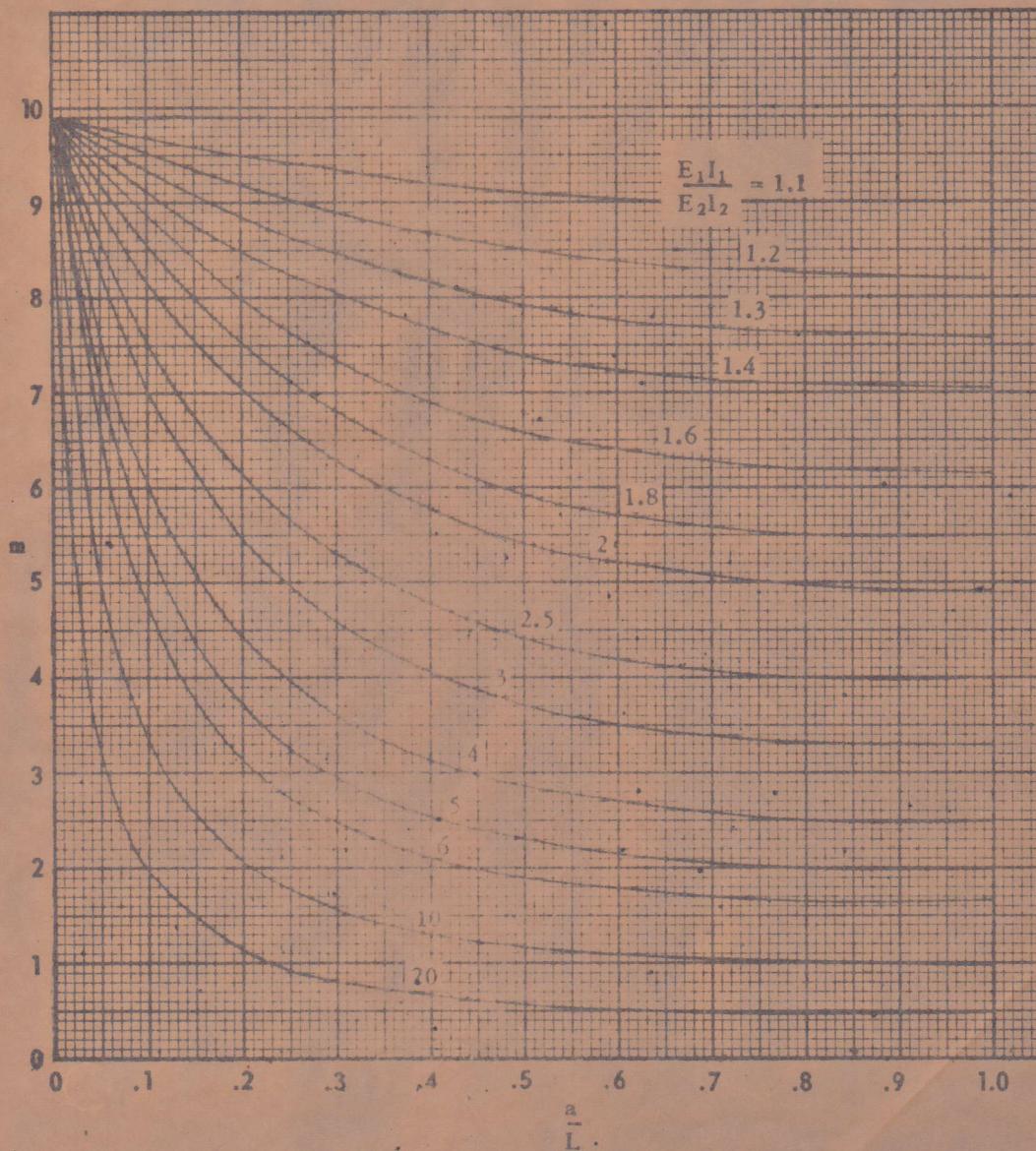


Fig. 6.3.1-2 Buckling Load for Column of Varying Section



6.3.1 Long Columns of Varying Section (cont'd)



$$\text{Buckling Load } P_{cr} = m \frac{E_2 I_2}{L^2}$$

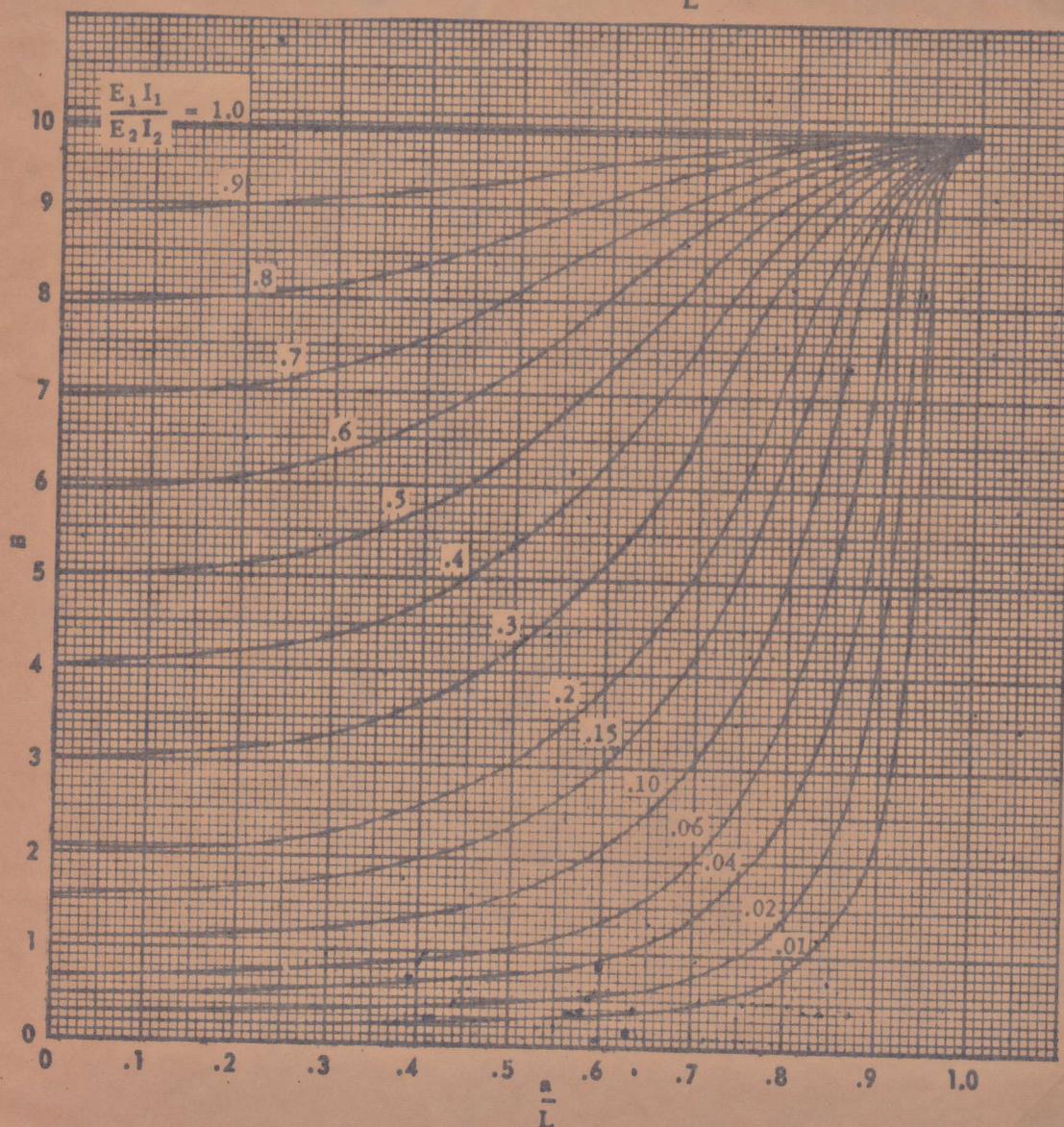
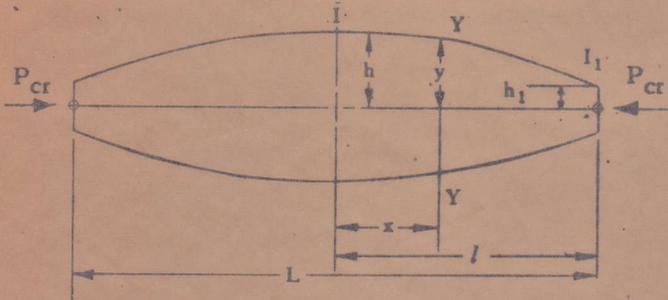


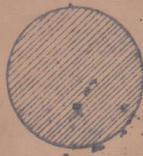
Fig. 6.3.1-3 Buckling Load for Column of Varying Section

6.3.1 Long Columns of Varying Section (cont'd)
Pinned-Pinned Strut with Elliptical Taper



SYMMETRICAL ABOUT MID-SPAN

SECTION YY



Equation to profile is $y^2 = h^2 - (h^2 - h_1^2) \left(\frac{x}{l}\right)^2$

See GEN/1090/812.1 for derivation.

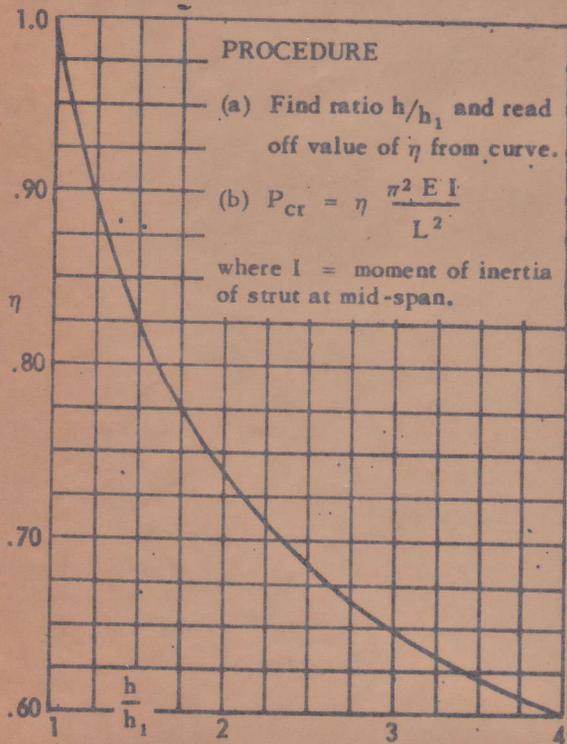


Fig. 6.3.1-4

Pinned-Pinned Strut with Elliptical Taper

6.3.2 Approximate General Method for Pinned Ends

Procedure

- (1) Divide the column length into 20 equal increments
- (2) Compute I at the mid-point of each increment

(3) Evaluate $\frac{k_1}{EI}$ (or $\frac{k_1}{I}$ if E is constant), the value of k_1 for each increment being obtained from Column 2 of the table below.

Note that these values of k_1 are the same for any pin-ended column and are independent of I and the length of the column. If the column is symmetrical about its mid-length, it is only necessary to consider 10 increments and double the value of $\sum \frac{k_1}{EI}$ so obtained.

INCREMENT	1	2	3	4	5
k_1	.020	.177	.476	.887	1.370
INCREMENT	20	19	18	17	16
INCREMENT	6	7	8	9	10
k_1	1.879	2.362	2.773	3.072	3.229
INCREMENT	15	14	13	12	11

(4) Critical Buckling Load

$$P_{cr} = \frac{320 \times (0.9)}{L^2 \sum_0^L \frac{k_1}{EI}}$$

$$= \frac{288 E}{L^2 \sum_0^L \frac{k_1}{I}} \quad \text{for constant } E.$$



6.3.2.1 Numerical Example

Pinned - pinned column with elliptical taper, shown in Fig. 6.3.2-1.

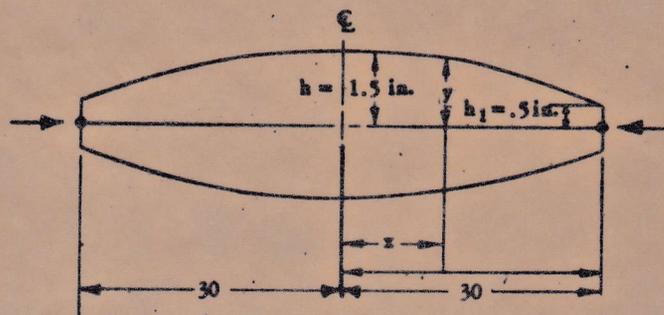


Fig. 6.3.2-1

From GEN/1090/812.1, the equation to an elliptical profile is:

$$y^2 = h^2 - (h^2 - h_1^2) \left(\frac{x}{l}\right)^2$$

Values of y and $I = \frac{\pi}{4} y^4$ are given in the table below and plotted in Fig. 6.3.2-2.

$$(h^2 - h_1^2) = (2.25 - .25) = 2.0$$

x	0	5	10	15	20	25	30
(x/l)	0	.167	.33	.50	.66	.83	1.0
(x/l) ²	0	.0278	.11	.25	.442	.687	1.0
(h ² - h ₁ ²) (x/l) ²	0	.0556	.22	.50	.884	1.374	2.0
y ²	2.25	2.194	2.03	1.75	1.366	.876	.25
y	1.5	1.481	1.426	1.322	1.170	.937	.50
y ⁴	5.07	4.80	4.12	3.06	1.865	.765	.0625
I	3.98	3.77	3.24	2.40	1.464	.601	.049

The half column length is now divided into ten equal increments and the value of I at the centre of each increment obtained from Fig. 6.3.2-2 and tabulated.



6.3.2.1 Numerical Example (cont'd)

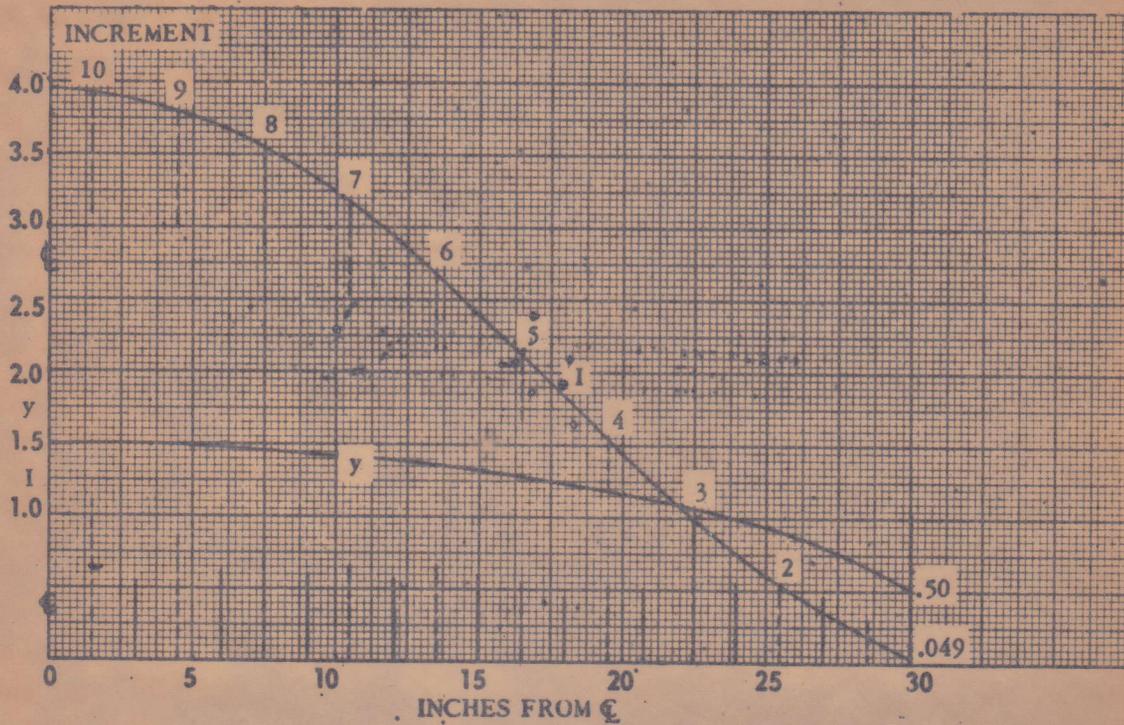


Fig. 6.3.2-2 Plot of y and I

INCREMENT	k_1	I	k_1/I
1	.02	.20	.100
2	.177	.50	.354
3	.476	1.0	.476
4	.887	1.56	.568
5	1.370	2.12	.646
6	1.879	2.66	.706
7	2.362	3.16	.747
8	2.773	3.56	.778
9	3.072	3.78	.812
10	3.229	3.93	.822

$$\sum \frac{k_1}{I} = 6.009$$

$$\sum_0^L \frac{k_1}{I} = 2 \times 6.009 = 12.018$$

$$P_{cr} = \frac{288E}{L^2 \sum_0^L \frac{k_1}{I}} = \frac{288 \times 10^7}{3600 \times 12.018}$$

$$= \underline{67000 \text{ lb. (approximately)}}$$

Check from Fig. 6.3.1-4

$$\text{For } \frac{h}{h_1} = 3.0, \quad \eta = .649$$

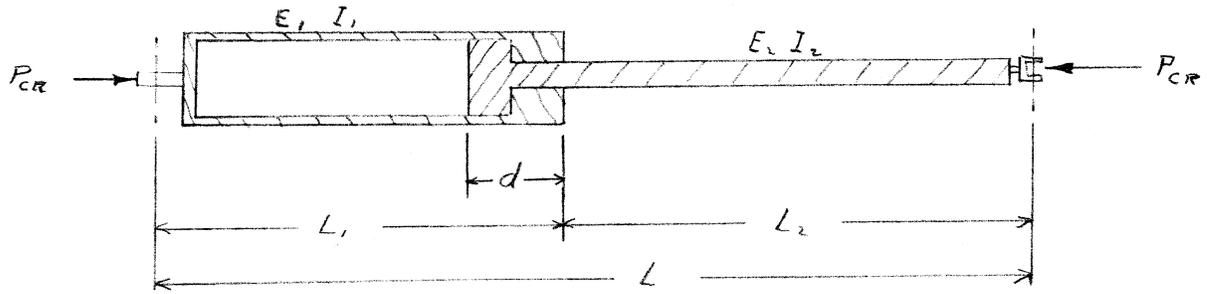
$$P_{cr} = .649 \frac{\pi^2 EI}{L^2}$$

$$= .649 \frac{9.85 \times 10^7 \times 3.98}{3600}$$

$$= .649 \times 109,000$$

$$= \underline{70800 \text{ lb.}}$$

FLUID COLUMN ANALYSIS (REF: LR 7249, PG. 10)



$$P_{CR} = \frac{\frac{3L}{L_1 L_2} K_1 K_2}{K_1 \beta_2 + K_2 \beta_1 + \frac{K_1 K_2}{K_C}} \quad \text{--- (A)}$$

WHERE $K_1 = \frac{E_1 I_1}{L_1}$, $K_2 = \frac{E_2 I_2}{L_2}$,

$$\beta_1 = \frac{3 \left(1 - \frac{L_1}{J_1} \cot \frac{L_1}{J_1} \right)}{\left(\frac{L_1}{J_1} \right)^2} , \quad \beta_2 = \frac{3 \left(1 - \frac{L_2}{J_2} \cot \frac{L_2}{J_2} \right)}{\left(\frac{L_2}{J_2} \right)^2} ,$$

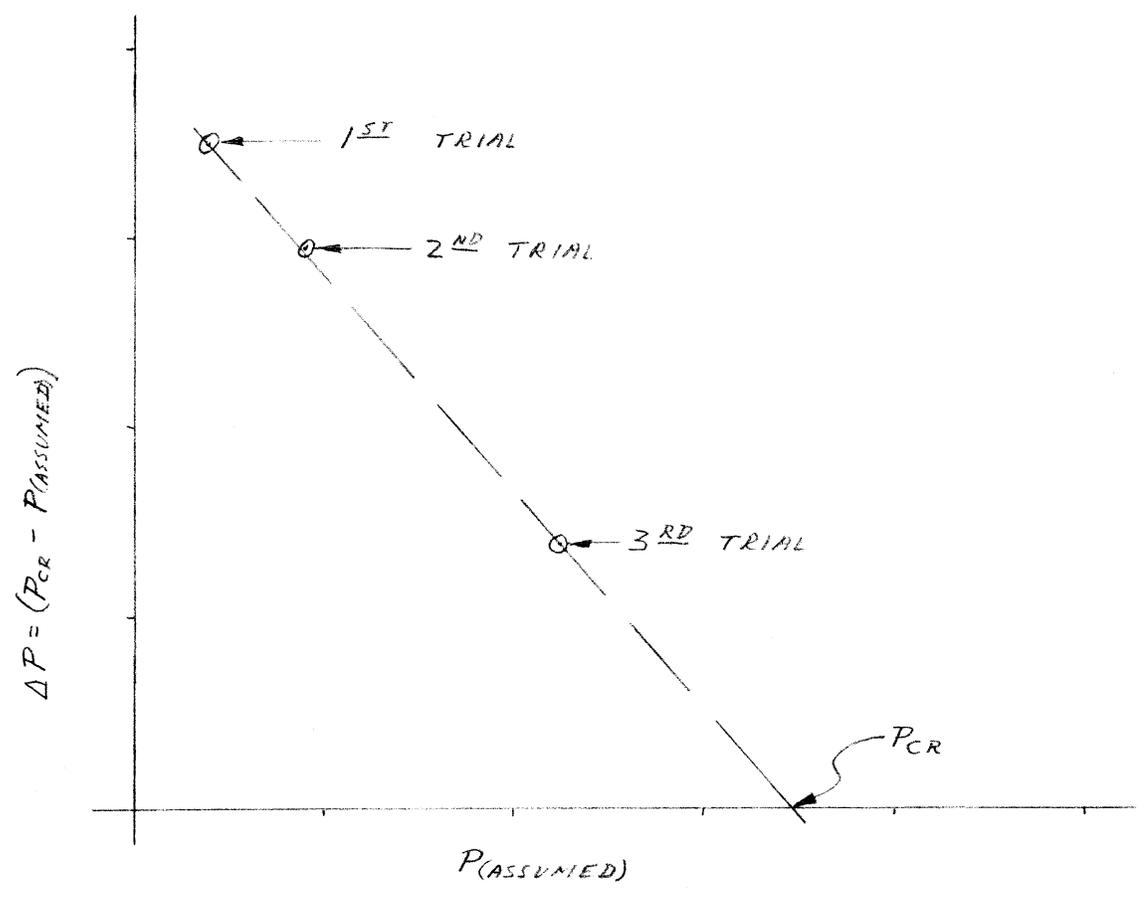
$$J_1 = \sqrt{\frac{E_1 I_1}{P}} , \quad J_2 = \sqrt{\frac{E_2 I_2}{P}} , \quad (\text{"P" IS ASSUMED})$$

$$K_C = \frac{E_2 I_2}{d}$$

THE FORMULA MUST BE SOLVED BY TRIAL & ERROR, SINCE β_1 & β_2 DEPEND UPON J_1 & J_2 WHICH DEPEND ON "P ASSUMED". ONLY WHEN "P_{CR}" IS EQUAL TO "P ASSUMED" IS THE VALUE OBTAINED FOR "P_{CR}" CORRECT.

FLUID COLUMN ANALYSIS (CONT'D.)

" P_{CR} " CAN BE FOUND GRAPHICALLY AFTER SEVERAL TRIALS BY EQUATION (A), AS SHOWN BELOW.





6.4

BUCKLING LOAD OF CONTINUOUS COLUMNS

TABLE OF CONTENTS

6.4.1 Two Span

6.4.1.1 Pinned at ends: E, I and P constant

6.4.1.2 Pinned at ends: E, I and P not constant

6.4.1.3 Pinned at one end, fixed at other end: E, I and P constant

6.4.1.4 Fixed at both ends: E, I and P constant

6.4.2 Three Span:

6.4.2.1 Pinned at ends: E, I and P constant

6.4.2.2 Pinned at ends: E, I and P not constant

This section is concerned with the determination of the buckling load of columns, continuous over two or three spans, carrying axial load but no lateral load.

6.4.1 Two Span

6.4.1.1 Two Span, Pinned at Ends. E, I and P constant. (Fig. 6.4.1-1).

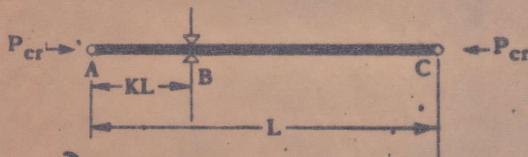


Fig. 6.4.1-1

Buckling load $P_{cr} = \beta^2 \frac{EI}{L^2}$

where L = overall length of column

β = a coefficient, plotted in Fig. 6.4.1-2 against K

K = ratio $\frac{\text{length AB}}{\text{length AC}}$

Fig. 6.4.1-2 is similar to Curve (1), Fig. 2, J. App. Mech., Dec. 1942, p. A-190. The method of obtaining $P_{cr} = 33.5 \frac{EI}{L^2}$ and hence $\beta = 5.8$ for the particular case in which one span is twice the other ($K = \frac{1}{3}$) is given in the numerical example of GEN/1090/811.6.

Clearly when $K = \frac{1}{2}$ the set up is the same as for the second column mode (b) given in GEN/1090/811.6 and consequently $P_{cr} = 4\pi^2 \frac{EI}{L^2}$

and $\beta = 4\pi^2$

$\beta = 2\pi = 6.28$, as in Fig. 6.4.1-2.

Also when $K = 0$ or $K = 1.0$, the problem reduces to the standard case of a column with one end pinned and the other end fixed.

$P_{cr} = 2.047 \pi^2 \frac{EI}{L^2}$ from 6.1.1

$\beta^2 = 2.047 \pi^2$

$\beta = 4.5$



6.4.1 Two Span (cont'd)

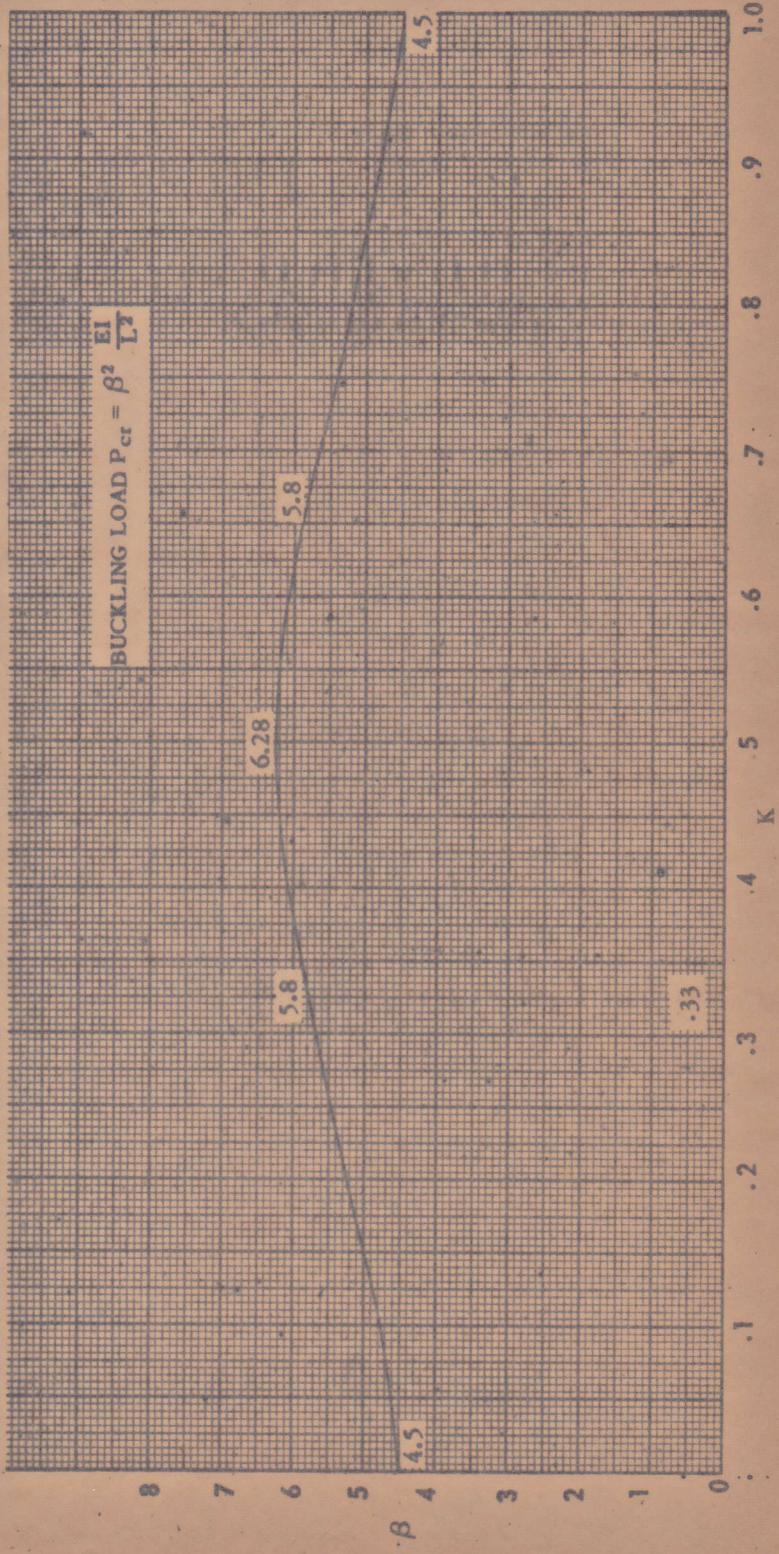
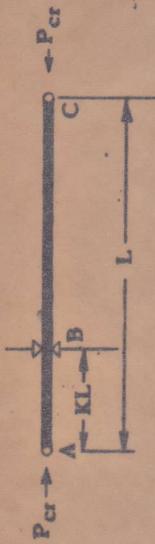


Fig. 6.4.1-2 Buckling Load for Two Span Continuous Column Pinned at Ends
E, I and P Constant



6.4.1.2 Two Span: Pinned at Ends: E.I. and P not constant.

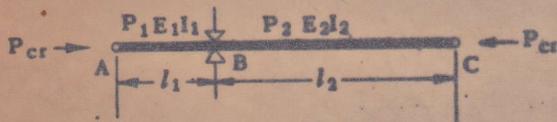


Fig. 6.4.1-3

The following method is from "Buckling of Struts and Frameworks", Ratzersdorfer, p. 280 (in German).

Let

- P_1 = applied end load in span AB
- P_2 = applied end load in span BC
- I_1 = moment of inertia of cross section of span AB
- I_2 = moment of inertia of cross section of span BC
- E_1 = modulus of elasticity of span AB
- E_2 = modulus of elasticity of span BC
- l_1 = span AB such that $\frac{l_1}{E_1 I_1}$ is less than $\frac{l_2}{E_2 I_2}$
- l_2 = span BC

Equivalent pin-jointed length of span AB, $\bar{l}_1 = \frac{l_1}{C_1}$

Equivalent pin-jointed length of span BC, $\bar{l}_2 = \frac{l_2}{C_2}$

where C_1 and C_2 are obtained from Fig. 6.4.1-4,

knowing the ratio $\frac{l_1}{l_2} \cdot \frac{E_2 I_2}{E_1 I_1}$ for a particular value

of $\lambda = \frac{P_2}{P_1} \cdot \frac{E_2 I_2}{E_1 I_1}$

The critical loads are:

$$P_{1cr} = \frac{\pi^2 E_1 I_1}{\bar{l}_1^2}$$

$$P_{2cr} = \frac{\pi^2 E_2 I_2}{\bar{l}_2^2}$$

ILLUSTRATIVE EXAMPLE

$$E_1 = E_2, I_1 = I_2, l_1 = 0.5 l_2 \text{ and } P_1 = P_2$$

$$\frac{l_1}{l_2} \cdot \frac{E_2 I_2}{E_1 I_1} = 0.5$$

$$\lambda = \frac{P_2}{P_1} \cdot \frac{E_2 I_2}{E_1 I_1} = 1.0$$

$$\text{From Fig. 6.4.1-4 } C_1 = 0.613 \\ C_2 = 1.23$$

$$\bar{l}_1 = \frac{l_1}{C_1} = 1.63 l_1 \quad \bar{l}_1^2 = 2.645$$

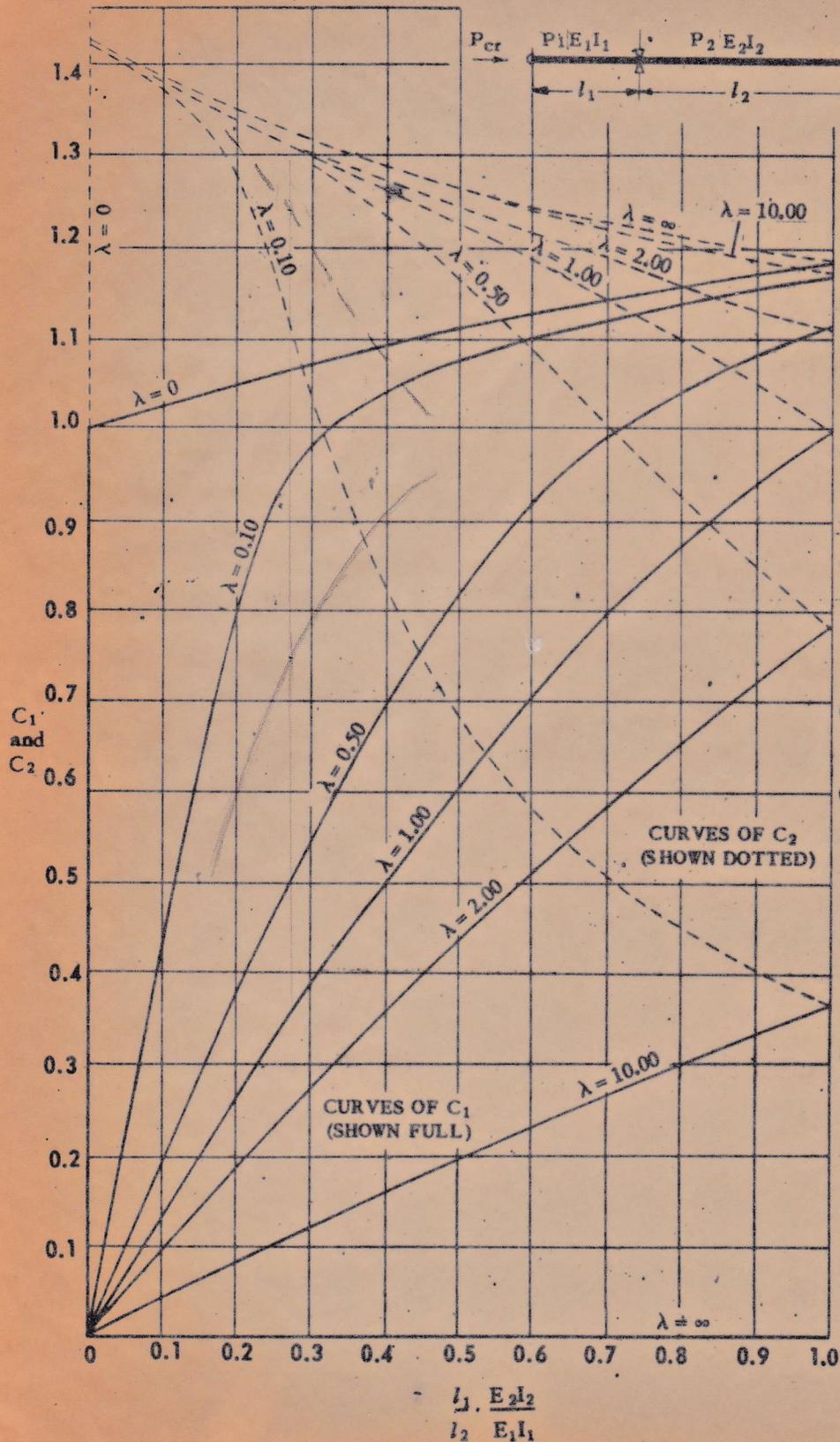
$$\bar{l}_2 = \frac{l_2}{C_2} = \frac{l_2}{1.23} = 0.812 l_2$$

$$P_{1cr} = \frac{\pi^2 EI}{\bar{l}_1^2} = \frac{\pi^2 EI}{2.645 l_1^2} = 3.72 \frac{EI}{l_1^2} = 33.5 \frac{EI}{L^2}$$

This is the same result as quoted in 6.4.1.1.



6.4.1.2 Two Span (cont'd)



Procedure

- (1) Calculate $\frac{l_1}{l_2} \cdot \frac{E_2 I_2}{E_1 I_1}$
and $\lambda = \frac{P_2}{P_1} \cdot \frac{E_2 I_2}{E_1 I_1}$
- (2) Read off C_1 from the full curves and C_2 from the dotted curves
- (3) $P_{1cr} = \frac{\pi^2 E_1 I_1}{\bar{l}_1^2}$ where $\bar{l}_1 = \frac{l_1}{C_1}$
 $P_{2cr} = \frac{\pi^2 E_2 I_2}{\bar{l}_2^2}$ where $\bar{l}_2 = \frac{l_2}{C_2}$

Fig. 6.4.1-4 Buckling Load for Two Span Continuous Column, Pinned at Ends. E, I and P not constant



6.4.1.3 Two Span - Pinned at A: Fixed at C
 (Fig. 6.4.1-5) E, I and P constant

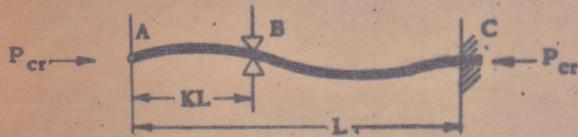


Fig. 6.4.1-5

$$\text{Buckling load } P_{cr} = \beta^2 \frac{EI}{L^2}$$

Clearly when $K = 0$, the column is fixed - fixed

$$\text{and } P_{cr} = 4\pi^2 \frac{EI}{L^2} \text{ (from 6.4.1.1 and 6.1.1)}$$

$$\beta^2 = 4\pi^2$$

$$\beta = 2\pi = 6.28$$

When $K = L$ the column is pinned at A and fixed at C without an intermediate support and

$$P_{cr} = 2.047 \pi^2 \frac{EI}{L^2} \text{ from 6.1.1}$$

$$\beta^2 = 2.047 \pi^2$$

$$\beta = 4.5.$$

Values of β are plotted against K in Fig. 6.4.1-6.

6.4.1.4 Two Span - Fixed at Ends A and C
 (Fig. 6.4.1-7) E, I and P constant

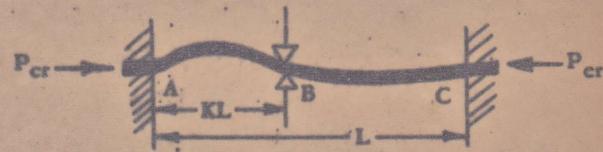


Fig. 6.4.1-7

$$\text{Buckling load } P_{cr} = \beta^2 \frac{EI}{L^2}$$

When $K = 0$ or $K = 1.0$ the column is a single span one with fixed ends

$$P_{cr} = 4\pi^2 \frac{EI}{L^2} \text{ and } \beta = 6.28$$

β is plotted against K in Fig. 6.4.1-8.



6.4.1.3 Two Span (cont'd)

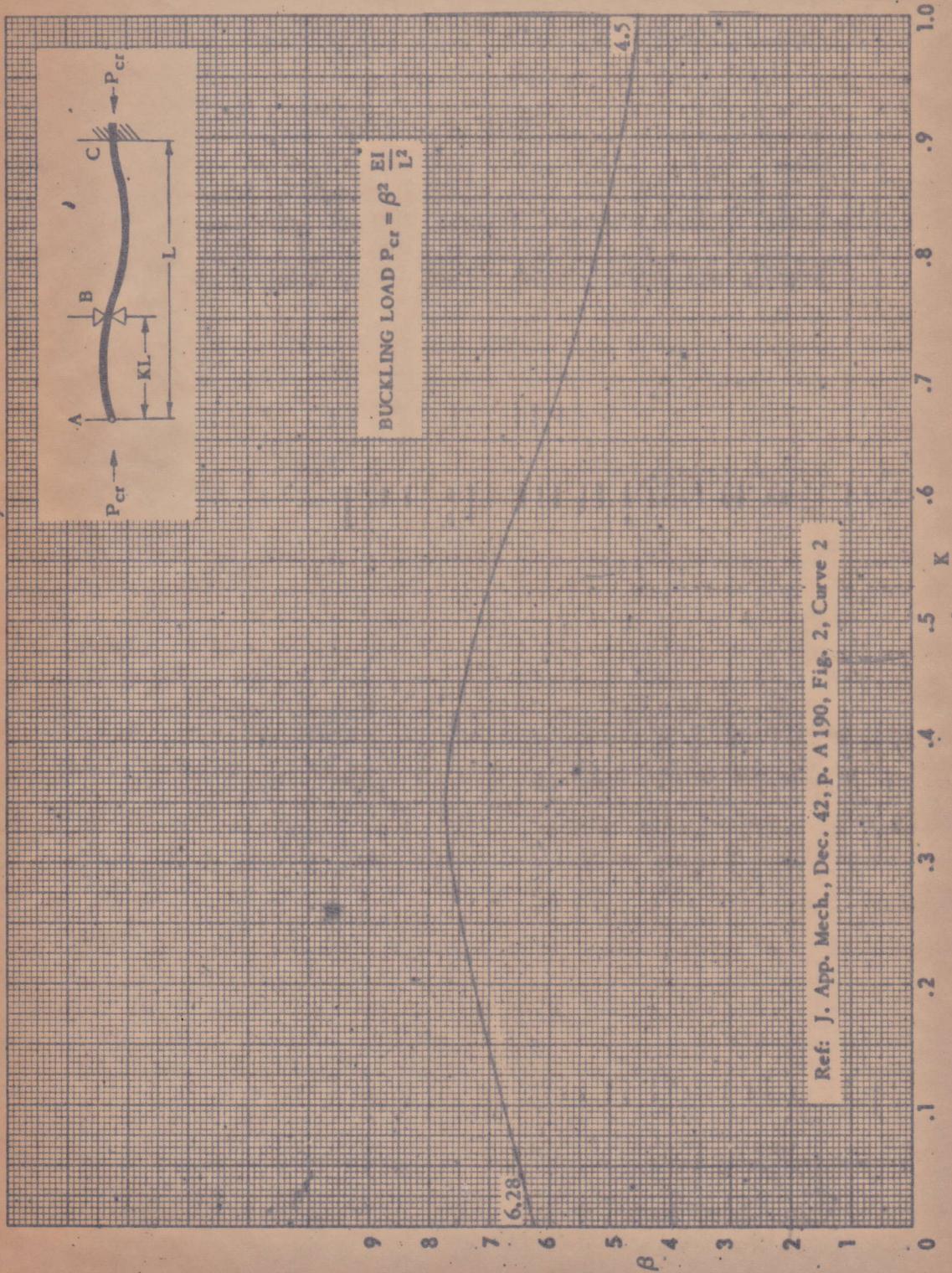


Fig. 6.4.1-6 Buckling Load for Two Span Continuous Column -- Pinned at A Fixed at C
E, I and P constant



6.4.1.3 Two Span (cont'd)

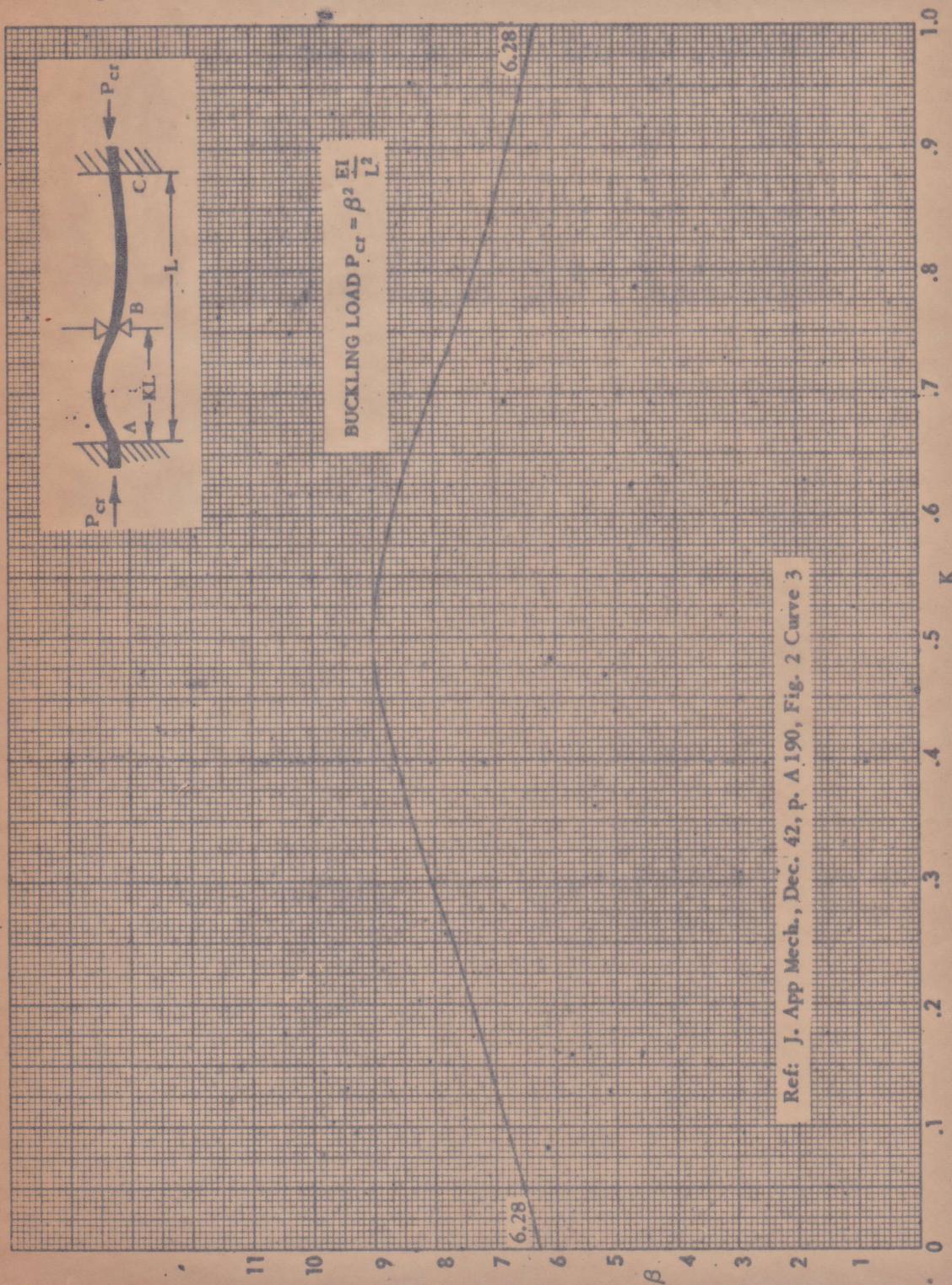


Fig. 6.4.1-8 Buckling Load for Two Span Continuous Column: Fixed at A and C
 E, I and P constant



6.4.2 Three Span

6.4.2.1 Three Span, Pinned at Ends E, I and P constant

See Figs. 3 and 4 in J. App. Mech., Dec. 1942
p. A-190.

6.4.2.2 Three Span, Pinned at Ends E, I and P not constant

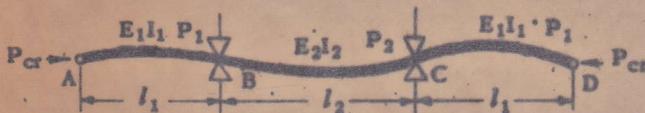


Fig. 6.4.2-1

The following method is taken from "Buckling of Struts and Frameworks", Ratzersdorfer, p. 280 (in German).

Let P_1 = applied end load in spans AB and CD

P_2 = applied end load in span BC

I_1 = moment of inertia of cross section of spans AB and CD

I_2 = moment of inertia of cross section of span BC

E_1 = modulus of elasticity of spans AB and CD

E_2 = modulus of elasticity of span BC

l_1 = span AB = span CD

l_2 = span BC

Equivalent pin-jointed length of span AB, $\bar{l}_1 = \frac{l_1}{C_1}$

Equivalent pin-jointed length of span BC, $\bar{l}_2 = \frac{l_2}{C_2}$

where C_1 and C_2 are obtained from Fig. 6.4.2-2,

knowing the ratio $\frac{l_1}{l_2} \cdot \frac{E_2 I_2}{E_1 I_1}$ for a particular value of

$$\lambda = \frac{P_2}{P_1} \frac{E_2 I_2}{E_1 I_1}$$

The critical loads are:

$$P_{1cr} = \frac{\pi^2 E_1 I_1}{\bar{l}_1^2}$$

$$P_{2cr} = \frac{\pi^2 E_2 I_2}{\bar{l}_2^2}$$

ILLUSTRATIVE EXAMPLE

Continuous beam of constant E, I and P in which
 $l_1 = l_2 = 10$ in. and $AD = 30$ in.

From Fig. 6.4.2-2,

$$\frac{l_1}{l_2} \frac{E_2 I_2}{E_1 I_1} = 1.0$$

$\lambda = 1.0$ since P is constant

and $C_1 = 1.0$

$$\bar{l}_1 = l_1 = 10$$

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \underline{0.985 EI}$$

From Fig. 3, Curve (1), J. App. Mech., Dec. 1942
p. A-190 for $j = 1/3$, $\beta = 9.42$ and $\beta^2 = 88.7$.

$$P_{cr} = \beta^2 \frac{EI}{L^2} = 88.7 \frac{EI}{30^2} = \underline{0.985 EI} \text{ as above.}$$



6.4.2 Three Span (cont'd)

Procedure

- (1) Calculate $\frac{l_1}{l_2} \frac{E_2 I_2}{E_1 I_1}$ and $\lambda = \frac{P_2}{P_1} \frac{E_2 I_2}{E_1 I_1}$
- (2) Read off C_1 from the full curves and C_2 from the dotted curves
- (3) $P_{1cr} = \frac{\pi^2 E_1 I_1}{\bar{l}_1^2}$ where $\bar{l}_1 = \frac{l_1}{C_1}$
 $P_{2cr} = \frac{\pi^2 E_2 I_2}{\bar{l}_2^2}$ where $\bar{l}_2 = \frac{l_2}{C_2}$

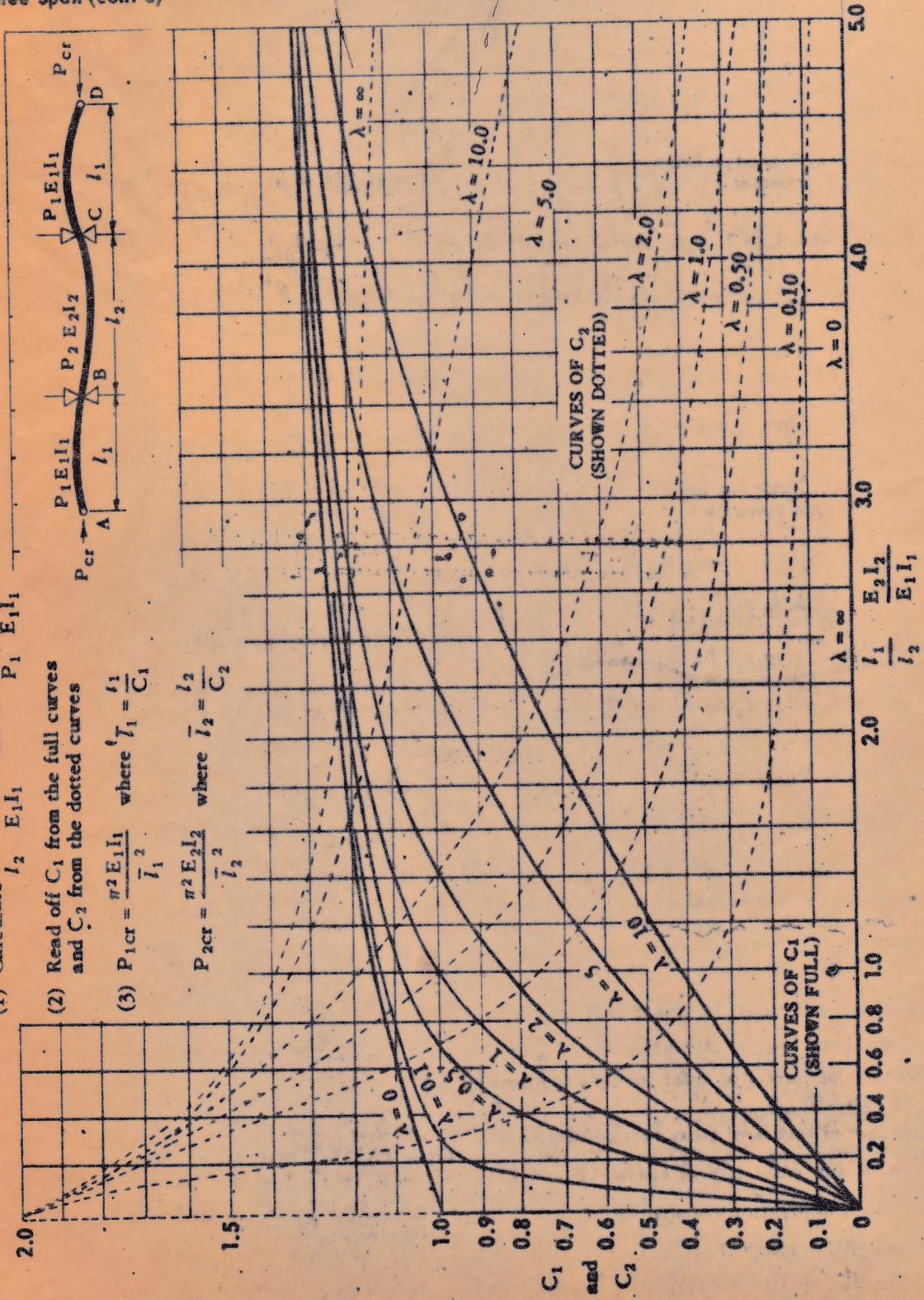


Fig. 6.4.2.2 Buckling Load for Three Span Continuous Column, Pinned at Ends
E, I and P not constant

STRAIGHT BEAM-COLUMNS

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6.5.1 Introduction

6.5.2 Tabular Summaries: Constant I Beam-Columns

6.5.2.1 Cantilever

6.5.2.2 Simply Supported

6.5.2.3 Fixed Ends

6.5.2.4 Propped Cantilever

6.5.3 Perry's Approximation

6.5.4 Numerical Examples

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6.5.4.2 Triangular Load

6.5.5 Polar Diagrams for Straight Compression Beam-Columns

6.5.6 Tapered Beam-Columns: General Method

6.5.7 Stiffness and Carry-over Charts (including effect of end load)

6.5.1 Introduction

Methods are given herein for finding the distribution of bending moment over the length of a beam-column, that is, a member subjected to end load (compression or tension) and lateral load. Note that the formulas quoted are only valid for values of P less than the buckling load under compression alone (no lateral load).

Only straight beam-columns are dealt with in this particular section: for those with slight initial bow, see 6.6.

When the beam-column reduces to a standard case, a quick solution is given directly by the tabular summaries of 6.5.2.

Knowing the end moments, the distribution of bending moment can in many cases be found by

constructing a Polar Diagram (6.5.5), except where the beam column has varying I and a complex loading system, under which circumstances the general treatment of 6.5.6 can be used.

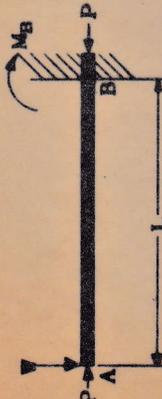
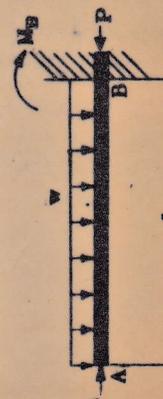
The Perry Approximation of 6.5.4 is useful for a quick estimate of bending moments, even when the loading is complex.

Stiffness and carry-over charts, to include the effect of end load, are given in 6.5.7.

6.5.2 Tabular Summaries: Constant I Beam-Columns

Common cases of Beam-Columns are given in the sections that follow.



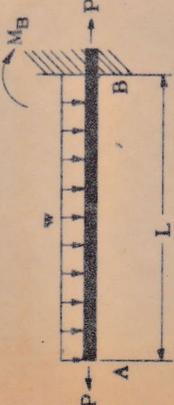
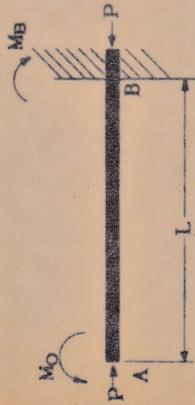
CANTILEVER BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(1)</p> <p>AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD AT TIP.</p> 	$M_B = -Wj \tan U$ <p>This is the max. B.M.</p>	$\text{Max. } y = -\frac{W}{P} (j \tan U - L) \text{ at } A$ $\theta = \frac{W}{P} \frac{(1 - \cos U)}{\cos U} \text{ at } A$
<p>(1A)</p> <p>AXIAL TENSION + CONCENTRATED LATERAL LOAD AT TIP.</p> 	$M_B = -Wj \tanh U$ <p>This is the max. B.M.</p>	$\text{Max. } y = -\frac{W}{P} (L - j \tanh U) \text{ at } A$
<p>(2)</p> <p>AXIAL COMPRESSION + UNIFORMLY DISTRIBUTED LATERAL LOAD.</p> 	$M_B = -wj [j(1 - \sec U) + L \tan U]$ <p>This is the max. B.M.</p>	$\text{Max. } y = -\frac{wj}{P} \left[j \left(1 + \frac{1}{2} U^2 - \sec U \right) + L (\tan U - U) \right] \text{ at } A$ $\theta = \frac{w}{P} \left[\frac{L}{\cos U} - j \frac{(1 - \cos 2U)}{\sin 2U} \right] \text{ at } A$

6.5.2.1 Cantilever Beam-Columns: I Constant

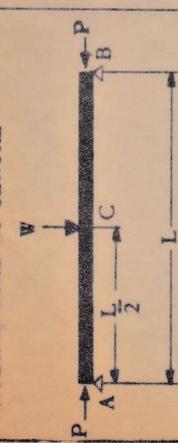
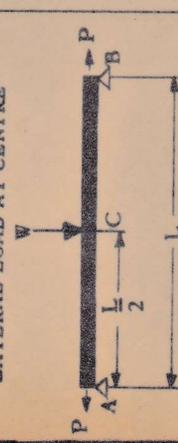
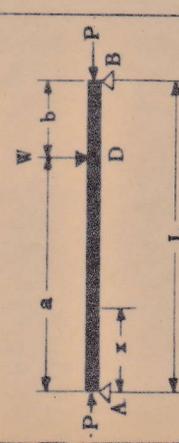
$$j^2 = \frac{EI}{P}$$

$$U = \frac{L}{j}$$



CANTILEVER BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(2A)</p> <p>AXIAL TENSION + UNIFORMLY DISTRIBUTED LATERAL LOAD.</p> 	$M_B = -wj [L \tanh U - j (1 - \operatorname{sech} U)]$ <p>This is the max. B.M.</p>	$\text{Max. } y = -\frac{wj}{P} \left[j \left(1 - \frac{1}{2} U^2 - \operatorname{sech} U \right) - L (\tanh U - U) \right] \text{ at A}$
<p>(3)</p> <p>AXIAL COMPRESSION + END COUPLE AT TIP.</p> 	$M_B = -M_0 \operatorname{sec} U$	

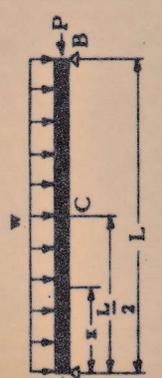
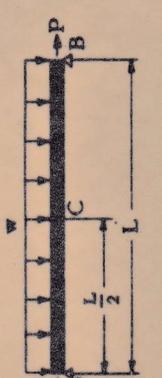
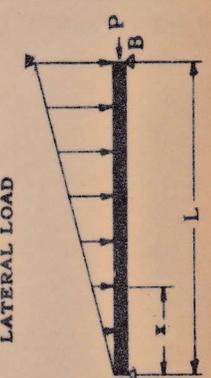


SIMPLY SUPPORTED BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(1A)</p> <p>AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD AT CENTRE</p> 	$M_A = M_B = 0$ $\text{Max. } M = \frac{1}{2} Wj \tan \frac{U}{2} \text{ at C}$ <p>See also Perry Approximation in 6.5.3</p>	$\text{Max. } y = -\frac{1}{2} \frac{Wj}{P} \left(\tan \frac{U}{2} - \frac{U}{2} \right) \text{ at C}$ $\theta = -\frac{W}{2P} \frac{(1 - \cos \frac{U}{2})}{\cos \frac{U}{2}} \text{ at A}$
<p>(1B)</p> <p>AXIAL TENSION + CONCENTRATED LATERAL LOAD AT CENTRE</p> 	$M_A = M_B = 0$ $\text{Max. } M = \frac{1}{2} Wj \tanh \frac{U}{2} \text{ at C}$	$\text{Max. } y = -\frac{W}{P} \left(\frac{L}{4} - \frac{j}{2} \tanh \frac{U}{2} \right) \text{ at C}$
<p>(2)</p> <p>AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD ANYWHERE ON SPAN</p> 	$M_A = M_B = 0$ $(A \text{ to } D) \ M = \frac{Wj \sin \frac{x}{j} \sin \frac{L-x}{j}}{\sin U}$ $(D \text{ to } B) \ M = \frac{Wj \sin \frac{a}{j} \sin \frac{(L-x)}{j}}{\sin U}$ <p>Max. M occurs at $x = \frac{1}{2} \pi j$ if $\frac{1}{2} \pi j < a$</p> <p>$x = (L - \frac{1}{2} \pi j)$ if $(L - \frac{1}{2} \pi j) > a$</p> <p>$x = a$ if $\frac{1}{2} \pi j > a$ and $(L - \frac{1}{2} \pi j) < a$</p>	<p>(A to C)</p> $y = \frac{Wj}{P} \left[\frac{\sin \frac{b}{j} \sin \frac{x}{j}}{\sin U} - \frac{bx}{Lj} \right]$ <p>(C to B)</p> $y = \frac{Wj}{P} \left[\frac{\sin \frac{a}{j} \sin \frac{L-x}{j}}{\sin U} - \frac{a(L-x)}{Lj} \right]$ $\theta = -\frac{W}{P} \left[\frac{b}{L} + \frac{\sin \frac{a}{j}}{\tan U} - \cos \frac{a}{j} \right] \text{ at A}$ $\theta = \frac{W}{P} \left[\frac{\sin \frac{a}{j}}{L} - \frac{1}{\sin U} \right] \text{ at B}$

6.5.2.2 Simply Supported Beam-Columns: I Constant

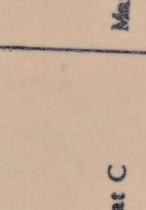
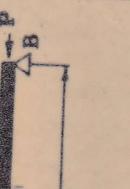
$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$



SIMPLY SUPPORTED BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(3A)</p> <p>AXIAL COMPRESSION + UNIFORMLY DISTRIBUTED LATERAL LOAD</p> 	$M_A = M_B = 0$ $M = A \cos \frac{x}{j} + B \sin \frac{x}{j} - wj^2$ <p>where $A = wj^2$</p> $B = wj^2 \tan \frac{U}{2}$ $\text{Max. } M = wj^2 (\sec \frac{U}{2} - 1) \text{ at } C$ <p>See also Perry approximation in 6.5.3</p>	$\text{Max. } y = -\frac{wj^2}{P} \left(\sec \frac{U}{2} - 1 - \frac{1}{8} U^2 \right) \text{ at } C$ $\theta = -\frac{wj}{P} \left[\frac{1 - \cos U}{\sin U} - \frac{U}{2} \right] \text{ at } A$
<p>(3B)</p> <p>AXIAL TENSION + UNIFORMLY DISTRIBUTED LATERAL LOAD</p> 	$M_A = M_B = 0$ $\text{Max. } M = wj^2 (1 - \text{sech} \frac{U}{2}) \text{ at } C$	$\text{Max. } y = -\frac{w}{P} \left[\frac{L^2}{8} - j^2 (1 - \text{sech} \frac{U}{2}) \right] \text{ at } C$
<p>(4)</p> <p>AXIAL COMPRESSION + TRIANGULAR LATERAL LOAD</p>  <p>$w = \frac{1}{2} wL$</p>	$M_A = M_B = 0$ $M = wj^2 \left[\frac{\sin \frac{x}{j}}{\sin U} - \frac{x}{L} \right]$ <p>Max. M occurs at</p> $x = j \arccos \left(\frac{\sin U}{U} \right)$ <p>See Worked Example in 6.5.4</p>	$y = -\frac{w}{P} \left(\frac{x^3}{6L} + \frac{j^2 \sin \frac{x}{j}}{\sin U} - \frac{jx}{U} - \frac{1}{6} Lx \right)$ $\theta = -\frac{w}{P} \left(\frac{j}{\sin U} - \frac{j}{U} - \frac{L}{6} \right) \text{ at } A$ $\theta = \frac{w}{P} \left(\frac{j}{U} - \frac{j}{\tan U} - \frac{L}{3} \right) \text{ at } B$

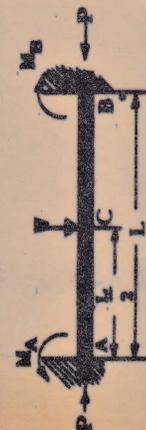
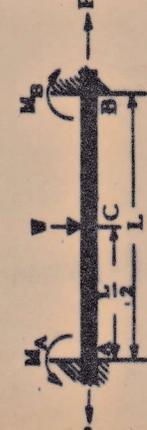
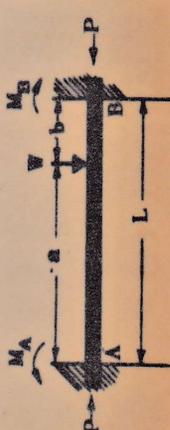
$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$



SIMPLY SUPPORTED BEAM-COLUMN	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(5)</p> <p>AXIAL COMPRESSION + IRREGULAR LATERAL LOAD</p> 	<p>BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]</p> <p>Use Perry Approximation of 6.5.3 in the absence of a better method</p>
<p>(6)</p> <p>AXIAL COMPRESSION + EQUAL COUPLE AT EACH END. NO LATERAL LOAD.</p> 	<p>Max. $M = M_A \sec \frac{U}{2}$ at C</p> <p>This is Southwell's secant formula, proved in GEN/1090/811.9.</p> <p>B.M. at any station can be found by polar diagram (6.5.5)</p> $\text{Max. } y = - \frac{M_A}{P} \left(\frac{1 - \cos \frac{U}{2}}{\cos \frac{U}{2}} \right) \text{ at C}$ $\theta = - \frac{M_A}{Pj} \tan \frac{U}{2} \text{ at A}$
<p>(7)</p> <p>AXIAL COMPRESSION + UNEQUAL COUPLES AT ENDS. NO LATERAL LOAD.</p>  <p>$M_B > M_A$</p>	<p>$M = \frac{(M_B - M_A \cos U)}{\sin U} \sin \frac{x}{j} + M_A \cos \frac{x}{j}$</p> <p>Max. M occurs at $x = j \arctan \frac{(M_B - M_A \cos U)}{M_A \sin U}$</p> <p>B.M. at any station can be found by polar diagram (6.5.5)</p> $y = \frac{1}{P} \left[M_A + (M_B - M_A) \frac{x}{L} - \frac{(M_B - M_A \cos U)}{\sin U} \sin \frac{x}{j} - M_A \cos \frac{x}{j} \right]$ $\theta = \frac{1}{P} \left[\frac{M_B - M_A}{L} - \frac{M_B - M_A \cos U}{j \sin U} \right] \text{ at A}$ $\theta = \frac{1}{P} \left[\frac{M_B - M_A}{L} - \frac{M_B - M_A \cos U}{j \sin U} \cos U + \frac{M_A}{j} \sin U \right] \text{ at B}$ <p style="text-align: right;">$j^2 = \frac{EI}{P}$ $U = \frac{L}{j}$</p>

6.5.2.2 Simply Supported Beam-Columns: I Constant (cont'd)



FIXED-END BEAM COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(1A)</p> <p>AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD AT CENTRE</p> 	$M_A = -\frac{WL}{8} C_A \quad M_B = -\frac{WL}{8} C_B$ <p>where coefficients C_A and C_B are given in Fig. 6.5.2.3 - 1 for $\frac{a}{L} = \frac{b}{L} = 0.50$</p> $M_C = \frac{WL}{8} \left(\frac{\tan \frac{U}{4}}{\frac{U}{4}} \right)$ <p>Use polar diagram of 6.5.5 for finding intermediate B.M. See also Perry approximation in 6.5.3</p>	$\text{Max. } y = -\frac{WL^3}{192 EI} \gamma$ $\text{where } \gamma = 3 \left[\frac{\frac{U}{4} - \frac{U}{4}}{\left(\frac{U}{4}\right)^3} \right]$
<p>(1B)</p> <p>AXIAL TENSION + CONCENTRATED LATERAL LOAD AT CENTRE</p> 	$M_A = M_B = \frac{1}{2} Wj \left(\frac{\cosh \frac{U}{2} - 1}{\sin h \frac{U}{2}} \right)$ $\text{Max. } + M = \frac{1}{2} Wj \left(\frac{1 - \cos h \frac{U}{2}}{\sin h \frac{U}{2}} + \tan h \frac{U}{2} \right) \text{ at C}$ $= \frac{1}{2} Wj \left(\frac{U}{2 \cos h \frac{U}{2}} + \tan h \frac{U}{2} \right)$ <p>Use polar diagram of 6.5.5 for finding intermediate B.M.</p>	$\text{Max. } y = -\frac{Wj}{2P} \left[\frac{U}{2} - \tan h \frac{U}{2} - \frac{(1 - \cos h \frac{U}{2})^2}{\sin h \frac{U}{2}} \frac{U}{2} \right]$
<p>(2A)</p> <p>AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD ANYWHERE ON SPAN</p> 	<p>See Fig. 6.5.2.3-1 for values of M_A and M_B.</p> <p>Use polar diagram of 6.5.5 for finding intermediate B.M.</p>	

$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$

6.5.2.3 Fixed End Beam-Columns: I Constant



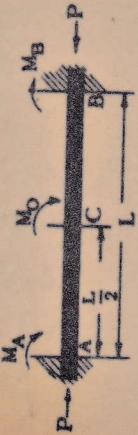
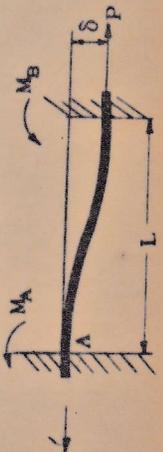
FIXED-END BEAM COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(2B)</p> <p>AXIAL TENSION + CONCENTRATED LATERAL LOAD ANYWHERE ON SPAN</p>	<p>See Fig. 6.5.2.3-1 for values of M_A and M_B</p> <p>Use polar diagram of 6.5.5 for finding intermediate B.M.</p>	
<p>(3A)</p> <p>AXIAL COMPRESSION + UNIFORMLY DISTRIBUTED LATERAL LOAD</p>	$M_A = M_B = -\frac{wL^2}{C}$ <p>where coefficient C is given in Fig. 6.5.2.3-5</p> $M_C = wj^2 \left(\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1 \right)$ <p>Proof in GEN/1090/811.7.4</p> <p>See also Perry Approximation in 6.5.3</p>	$\text{Max. } y = -\frac{1}{384} \frac{wL^3}{EI} \gamma \text{ at C}$ <p>where $\gamma = 3 \left[\frac{\tan \frac{U}{4} - \frac{U}{4}}{\left(\frac{U}{4}\right)^3} \right]$</p>
<p>(3B)</p> <p>AXIAL TENSION + UNIFORMLY DISTRIBUTED LATERAL LOAD</p>	$M_A = M_B = wj^2 \left(\frac{U}{2} \cot h \frac{U}{2} - 1 \right)$ <p>Max. + M = $wj^2 \left(1 - \frac{U}{2} \operatorname{cosec} h \frac{U}{2} \right)$ at C</p>	$\text{Max. } y = -\frac{wj^2}{8P} \left[\frac{4U(1 - \cos h \frac{U}{2})}{\sin h \frac{U}{2}} + U^2 \right] \text{ at C}$



FIXED-END BEAM COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]
<p>(4A)</p> <p>AXIAL COMPRESSION + UNIFORMLY DISTRIBUTED LATERAL LOAD OVER PART OF SPAN</p>	<p>See Fig. 6.5.2.3-2 for values of M_A and M_B.</p> <p>Use polar diagram of 6.5.5 for finding intermediate B.M.</p>
<p>(4B)</p> <p>AXIAL TENSION + UNIFORMLY DISTRIBUTED LATERAL LOAD OVER PART OF SPAN</p>	
<p>(5A)</p> <p>AXIAL COMPRESSION + TRIANGULAR LATERAL LOADING</p>	<p>See Fig. 6.5.2.3-3 for values of M_A and M_B</p>
<p>(5B)</p> <p>AXIAL TENSION + TRIANGULAR LATERAL LOADING</p>	<p>See Fig. 6.5.2.3-4 for values of M_A and M_B</p>

6.5.2.3 Fixed End Beam-Columns: J Constant (cont'd) $j^2 = \frac{EI}{P}$ $U = \frac{L}{j}$



FIXED-END BEAM COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]	DEFLECTION (y) [POSITIVE UPWARD] AND SLOPE (θ) [POSITIVE WHEN UPWARD TO THE RIGHT]
<p>(6)</p> <p>AXIAL COMPRESSION: NO LATERAL LOAD BUT MOMENT APPLIED AT CENTRE</p> 	$M_A = + \frac{M_0}{4} \frac{\alpha''}{\beta''}$ $M_B = - \frac{M_0}{4} \frac{\alpha''}{\beta''}$ <p>where $\alpha'' = 6 \left[\frac{\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1}{\left(\frac{U}{2}\right)^2} \right]$</p> <p>and $\beta'' = 3 \left[\frac{1 - \frac{U}{2} \cot \frac{U}{2}}{\left(\frac{U}{2}\right)^2} \right]$</p> <p>(Ref. Lockheed Report No. 5048)</p>	<p>$y = 0$ at C</p> <p>See Lockheed Report Extract in GEN/1090/811</p>
<p>(7A)</p> <p>AXIAL COMPRESSION: NO LATERAL LOAD. SUPPORT B SINKS</p> 	$M_A = M_B = - \frac{6EI}{L^2} \delta \cdot C$ <p>C = a coefficient, values of which are given in Fig. 6.5.2.3-6</p> <p>For proof, see GEN/1090/811.7.6</p>	
<p>(7B)</p> <p>AXIAL TENSION: NO LATERAL LOAD SUPPORT B SINKS</p> 	$M_A = M_B = - \frac{6EI}{L^2} \delta \cdot C$ <p>C = a coefficient, values of which are given in Fig. 6.5.2.3-6</p> <p>For proof, see GEN/1090/811.7.6</p>	

6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)

$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$



6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)

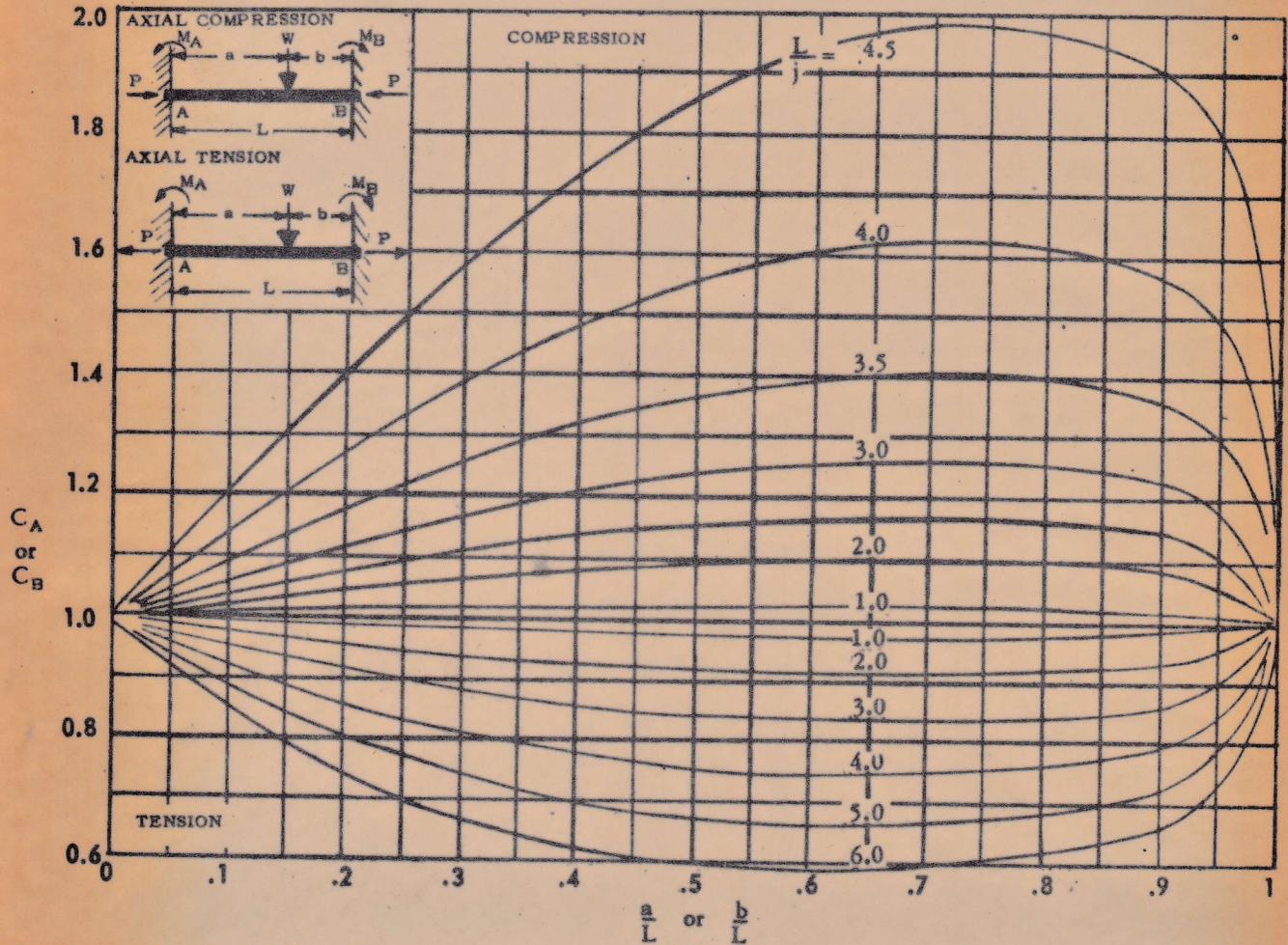


Fig. 6.5.2.3-1 Fixed End Moment Coefficients for Beam-Columns carrying Concentrated Lateral Load anywhere on Span [From N.A.C.A., T.N. 534]

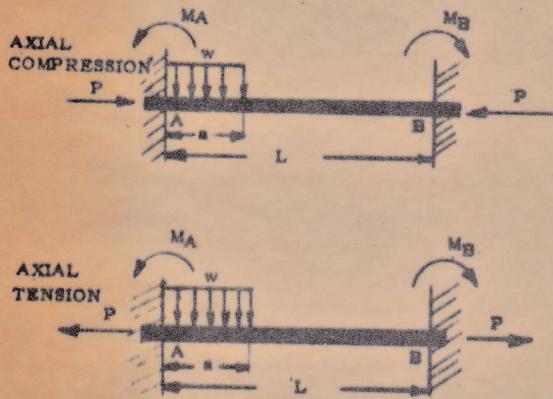
Procedure

To find M_A
(i) Calculate $\frac{a}{L}$ and read off value of C_A from the appropriate curve of $\frac{L}{j}$, tension or compression, where $j^2 = \frac{EI}{P}$.
(ii) $M_A = -\frac{Wab}{L^2} \cdot bC_A$
For W at mid-span $\frac{a}{L} = .50$, $M_A = -\frac{WL}{8} C_A$

To find M_B
(i) Calculate $\frac{b}{L}$ and read off value of C_B from the appropriate curve of $\frac{L}{j}$, tension or compression.
(ii) $M_B = -\frac{Wab}{L^2} \cdot aC_B$
For W at mid-span $\frac{b}{L} = .50$, $M_B = -\frac{WL}{8} C_B$



6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)



Procedure

To find M_A
(i) Calculate $\frac{a}{L}$ and read off value of C_A from the appropriate curve of $\frac{L}{j}$, tension or compression, where $j^2 = \frac{EI}{P}$
(ii) $M_A = -\frac{wa^2}{12} \left[6 - \frac{8a}{L} + 3 \left(\frac{a}{L} \right)^2 \right] C_A$
To find M_B
(i) Calculate $\frac{a}{L}$ and read off value of C_B from the appropriate curve of $\frac{L}{j}$, tension or compression.
(ii) $M_B = -\frac{wa^2}{12} \left[4 \frac{a}{L} - 3 \left(\frac{a}{L} \right)^2 \right] C_B$
When $\frac{a}{L}$ is > 0.5 , find the fixed end moments for a uniformly distributed load over the entire span (Case 3A or 3B) and subtract the fixed end moments that would be caused by a uniform load over the part of the span that is not loaded.

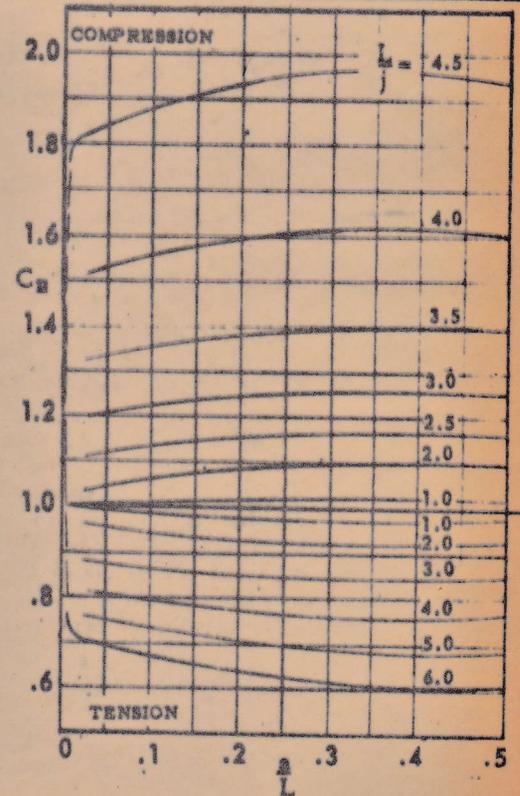
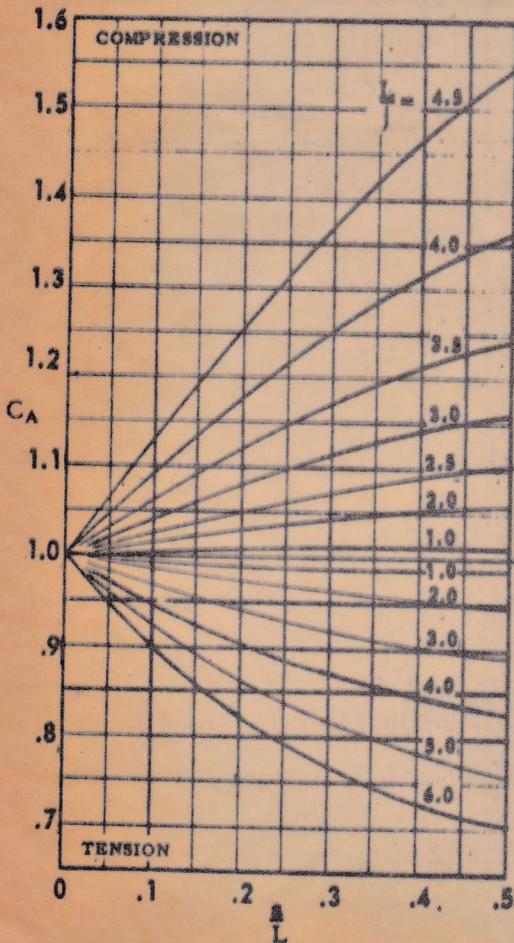


Fig. 6.5.2.3-2 Fixed End Moment Coefficients for Beam-Columns carrying Uniformly Distributed Lateral Load over part of Span [From N.A.C.A., T.N. 534]

6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)

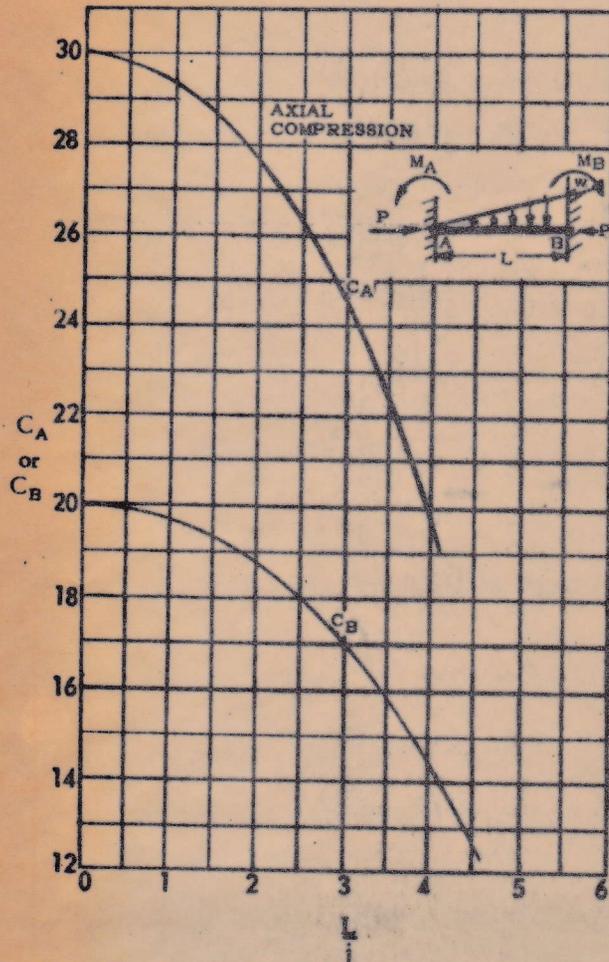


Fig. 6.5.2.3-3 Fixed End Moment Coefficients for Compression Beam Column Carrying Triangular Lateral Loading [From N.A.C.A., T.N. 534]

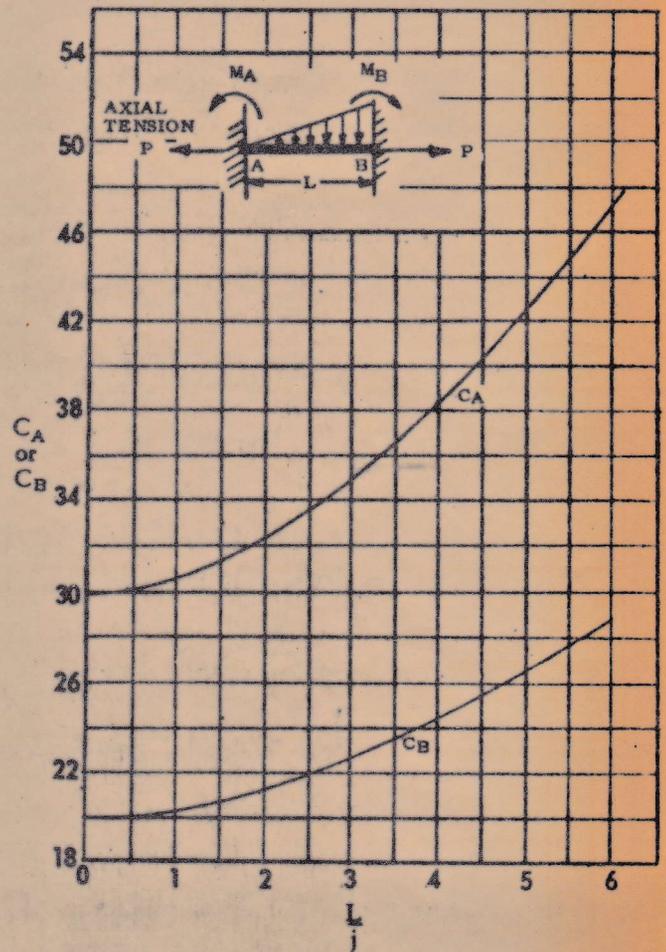


Fig. 6.5.2.3-4 Fixed End Moment Coefficients for Tension Beam Column Carrying Triangular Lateral Loading [From N.A.C.A., T.N. 534]

Procedure

To find M_A	
(i)	Calculate $\frac{L}{j}$ where $j^2 = \frac{EI}{P}$ and read off the value of C_A .
(ii)	$M_A = -\frac{wL^2}{C_A} = -\frac{2WL}{C_A}$ since $W = \frac{1}{2} wL$
To find M_B	
(i)	Calculate $\frac{L}{j}$ and read off the value of C_B
(ii)	$M_B = -\frac{wL^2}{C_B} = -\frac{2WL}{C_B}$

Procedure

To find M_A	
(i)	Calculate $\frac{L}{j}$ where $j^2 = \frac{EI}{P}$ and read off the value of C_A .
(ii)	$M_A = \frac{wL^2}{C_A} = \frac{2WL}{C_A}$ since $W = \frac{1}{2} wL$
To find M_B	
(i)	Calculate $\frac{L}{j}$ and read off the value of C_B .
(ii)	$M_B = \frac{wL^2}{C_B} = \frac{2WL}{C_B}$



6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)

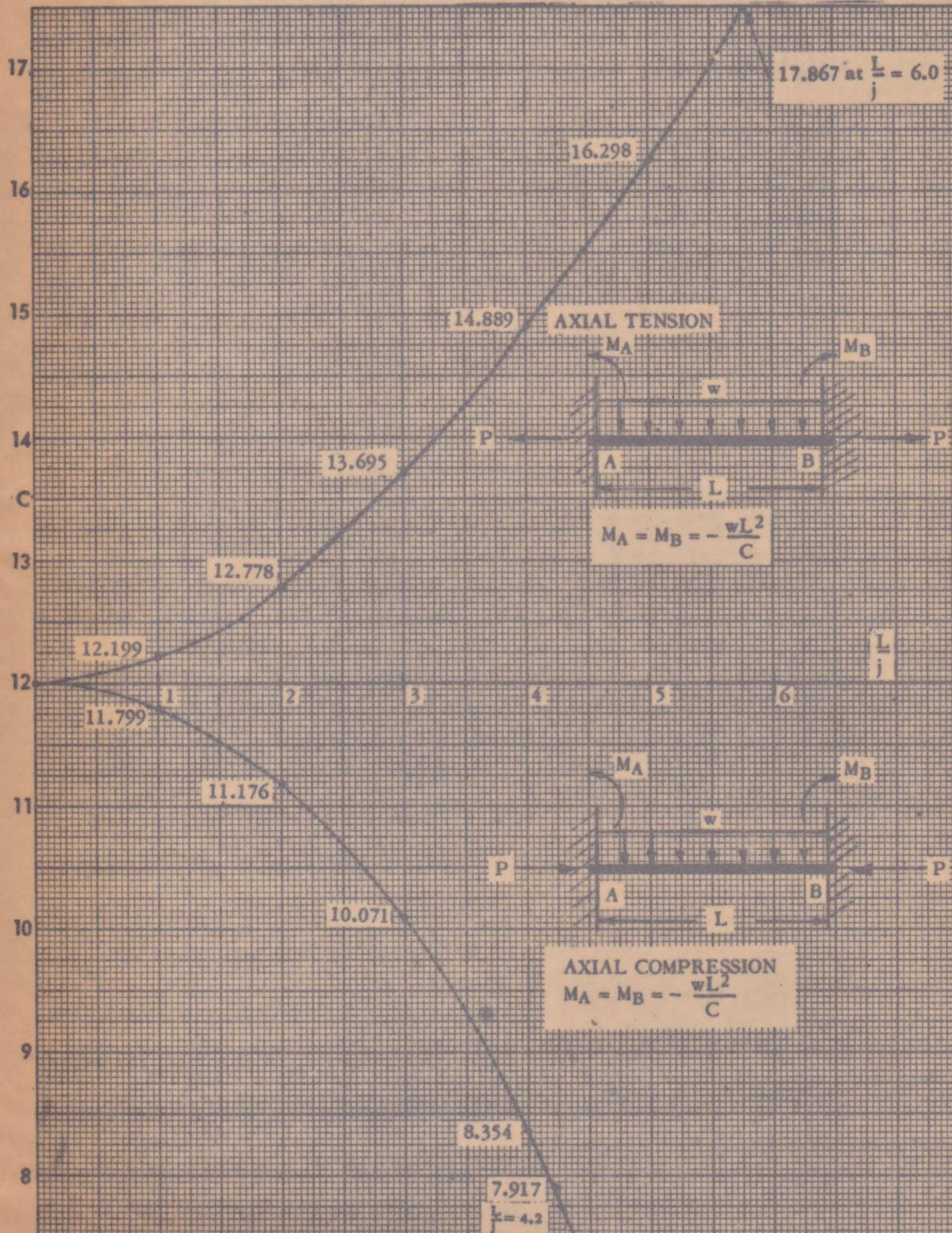


Fig. 6.5.2.3-5 End Moments for Beam Columns carrying a Uniformly Distributed Load over whole Span



6.5.2.3 Fixed End Beam-Columns: I Constant (cont'd)

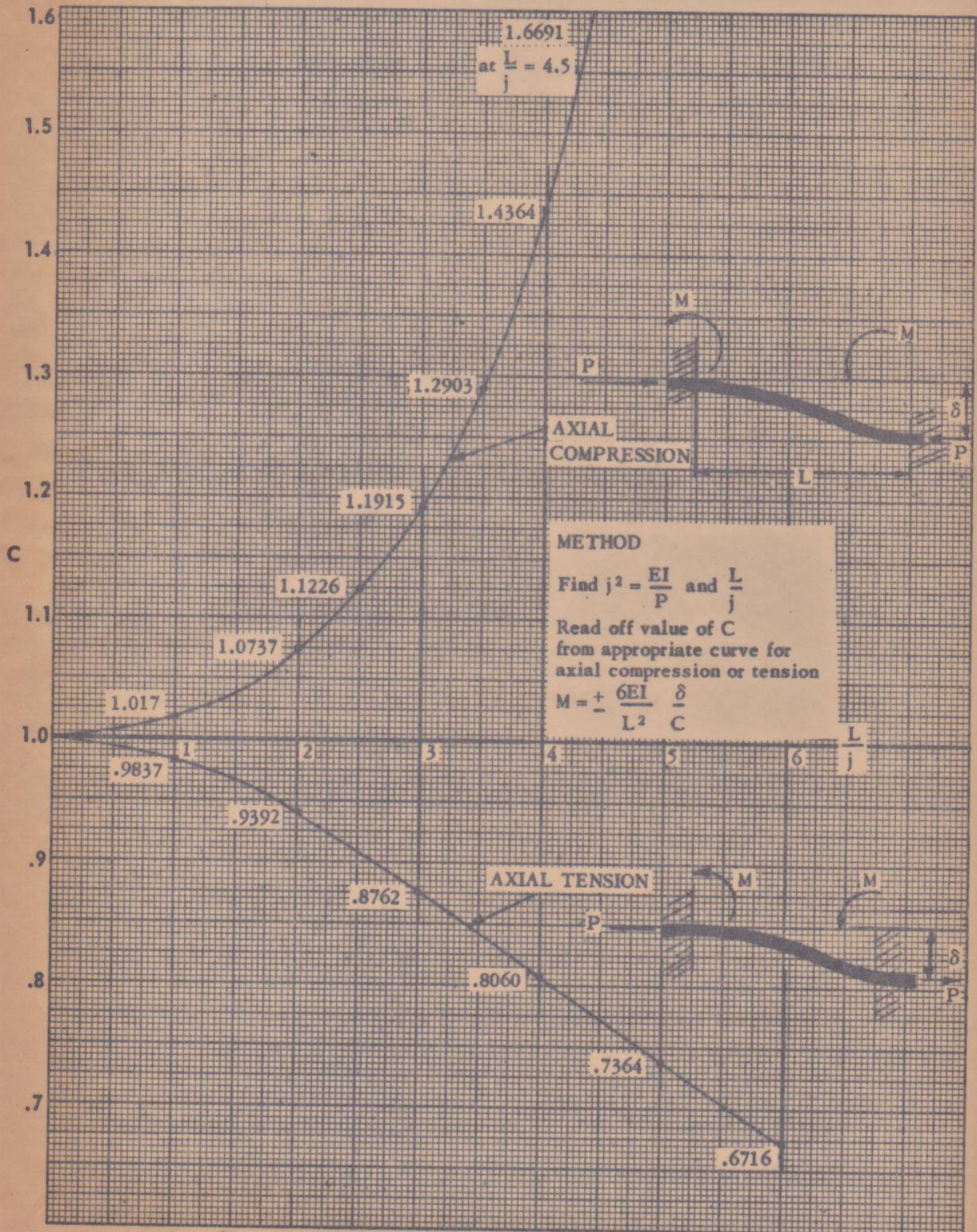
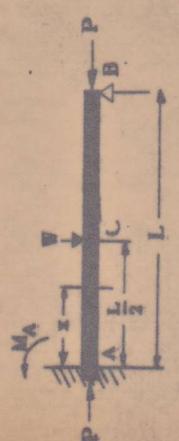
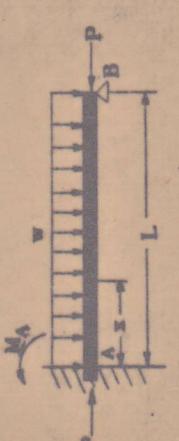
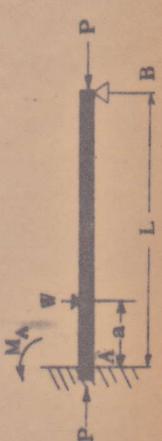
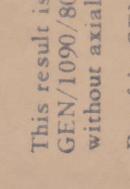


Fig. 6.5.2.3-6 End Moments for Beam-Column with Sinking Support



PROPPED CANTILEVER BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]
<p>(1) AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD AT CENTRE</p>  <p style="text-align: center;">$R_B = \frac{W}{2} - \frac{M_A}{L}$</p>	<p>C to B</p> $M = M_A \left(\frac{\sin \frac{x}{j}}{\tan U} - \cos \frac{x}{j} \right) + Wj \left(\sin \frac{U}{2} \cos \frac{x}{j} - \sin \frac{U}{2} \sin \frac{x}{j} \frac{\tan U}{\tan U} \right)$ $\text{Max. } M = M_A = -\frac{WL}{2} \left[\frac{j \tan U (\sec \frac{U}{2} - 1)}{j \tan U - L} \right]$ <p>Alternatively, adopt the treatment of Case 3 below For y, see Roark, p.133, Case 11.</p>
<p>(2) AXIAL COMPRESSION + UNIFORMLY DISTRIBUTED LATERAL LOAD</p>  <p style="text-align: center;">$R_B = \frac{1}{2} wL - \frac{M_A}{L}$</p>	$M = M_A \left(\cot U \sin \frac{x}{j} - \cos \frac{x}{j} \right) + wj^2 \left[\frac{\sin \frac{x}{j}}{\sin U} (1 - \cos U) + \cos \frac{x}{j} - 1 \right]$ $\text{Max. } M = M_A = -wLj \left[\frac{\tan U (\tan \frac{U}{2} - \frac{U}{2})}{\tan U - U} \right]$ <p>Alternatively, adopt the treatment of Case 3 below For y, see Roark, p.133, Case 12</p>
<p>(3) AXIAL COMPRESSION + CONCENTRATED LATERAL LOAD ANYWHERE ON SPAN</p> 	<p>First consider as a beam-column fixed at both ends (Case 2A, Section 6.5.2.3) and find M_A and M_B. Then release B (by making $M_B = 0$) and find the carry-over moment to A by using the carry-over factor for the appropriate L/j value, given in 6.5.7. This gives M_A.</p>

PROPPED CANTILEVER BEAM-COLUMN	BENDING MOMENT (M) [POSITIVE WHEN PUTTING TOP FLANGE IN COMPRESSION]
<p>(4)</p> <p>AXIAL COMPRESSION: NO LATERAL LOAD BUT END COUPLE</p> 	$M_A = -M_0 \frac{\alpha}{2\beta}$ <p>where $\alpha = 6 \frac{(U \operatorname{cosec} U - 1)}{U^2}$</p> $\beta = 3 \frac{(\tan U - U)}{U^2 \tan U}$
<p>(5)</p> <p>AXIAL COMPRESSION: NO LATERAL LOAD BUT SUPPORT B SINKS</p> 	$M_A = -\frac{3EI\delta}{L^2\beta}$ <p>where $\beta = 3 \frac{(\tan U - U)}{U^2 \tan U}$</p> <p>This result is analogous to that in GEN/1090/803, Special Case (b) without axial load.</p> <p>Proof in GEN/1090/811.7.5.</p>

6.5.3 Perry's Approximation

The bending moment M at any section X of a constant I beam-column pinned at both ends and carrying a uniformly distributed lateral load is proved in GEN/1090/811.7.1 to be:-

$$M = \frac{Q}{Q - P} M' \quad (\text{approx.})$$

where M' = the bending moment at section X due to the lateral load alone that is, ignoring any load effect,

P = end load (less than Q)

and Q = Euler failing load for pinned ends = $\frac{\pi^2 EI}{L^2}$.

This approximate formula gives results which are in close agreement with the secant formula of 6.5.2.2, Case 6, except when P approaches the value of Q . (See worked example in 6.5.4).

OTHER CASES

By writing the expression in the form

$$M = \frac{Q}{Q - CP} M'$$

an approximate expression for M for other loading cases and different end conditions results, C being a coefficient having values as below:

END CONDITION	LATERAL LOAD	C	REMARKS
Both Ends Pinned	Uniformly Distributed	1.0	B.M. at centre of Beam
Both Ends Pinned	Central Load	0.894	B.M. at centre of Beam
Both Ends Fixed	Uniformly Distributed	0.276	B.M. at centre of Beam
Both Ends Fixed	Uniformly Distributed	0.172	Fixed end moment
Both Ends Fixed	Central Load	0.212	Fixed end moments and B.M. at centre of beam



6.5.4 Worked Examples

6.5.4.1 Uniformly Distributed Load

Beam column shown in Fig. 6.5.4-1 for which $EI = 10^6$

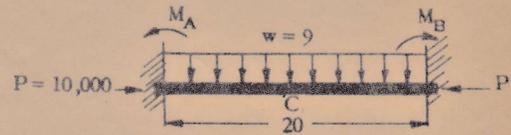


Fig. 6.5.4-1

END MOMENTS

$$M_A = M_B = -\frac{WL}{12} \beta'' \text{ from 6.5.2.3, Case 3A}$$

$$\beta'' = 3 \left[\frac{1 - \frac{U}{2} \cot \frac{U}{2}}{\left(\frac{U}{2}\right)^2} \right] = \frac{3 \times .358}{1.0} = 1.074$$

$$M_A = M_B = -300 \times 1.074 = -322.2 \text{ lb. in.}$$

This result was obtained in 3.3.3.

CHECK BY PERRY APPROXIMATION of 6.5.3

$$M = \frac{Q}{Q - CP} M'$$

$$= 1.075 \times 300 = \underline{322.5 \text{ lb. in.}}$$

CENTRE MOMENT

$$M_C = \frac{WL}{24} \alpha''$$

$$= 150 \times 1.1298 = \underline{169.5 \text{ lb. in.}}$$

CHECK BY PERRY APPROXIMATION

$$M_C = 1.126 \times 150 = \underline{169 \text{ lb. in.}}$$

$$j^2 = \frac{EI}{P} = 100 \quad j = 10$$

$$\frac{U}{2} = \frac{L}{2j} = \frac{20}{20} = 1.0 \text{ radian} = 57.3 \text{ deg.}$$

$$\cot \frac{U}{2} = .6420$$

$$(1 - \cot \frac{U}{2}) = .358$$

$$W = 9 \times 20 = 180$$

$$\frac{WL}{12} = \frac{180 \times 20}{12} = 300$$

$$Q = \frac{\pi^2 EI}{L^2} = \frac{9.85 \times 10^6}{400} = 24625$$

$$C = .172 \quad CP = .172 \times 10000 = 1720$$

$$Q - CP = 22905 \quad \frac{Q}{Q - CP} = \frac{24625}{22905} = 1.075$$

$$\frac{U}{2} = 1.0 \text{ rad.} = 57.3 \text{ deg.}$$

$$\operatorname{cosec} \frac{U}{2} = 1.1883$$

$$\left(\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1\right) = .1883$$

$$\alpha'' = 6 \left[\frac{\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1}{\left(\frac{U}{2}\right)^2} \right]$$

$$= 6 \times .1883 = 1.1298$$

$$C = .276 \quad CP = 2760$$

$$Q - CP = 21865 \quad \frac{Q}{Q - CP} = 1.126$$

$$M' = \frac{WL}{24} = 150$$



6.5.4.2 Triangular Load

Simply supported beam column shown in Fig. 6.5.4-2. carrying triangular load.

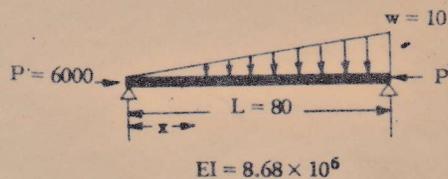


Fig. 6.5.4-2

By the formula of 6.5.2.2, Case 4,

$$M = wj^2 \left[\frac{\sin \frac{x}{j}}{\sin U} - \frac{x}{L} \right]$$

$$j^2 = \frac{EI}{P} = \frac{8.68 \times 10^6}{6000} = 1.449 \times 10^3$$

$$j = 38 \quad U = \frac{L}{j} = \frac{80}{38} = 2.105$$

$$= 120.7^\circ$$

$$\sin U = .8599$$

Max. M occurs at $x = j \arccos \left(\frac{\sin U}{U} \right)$

$$\frac{\sin U}{U} = \frac{.8599}{2.105} = .411$$

The angle whose cosine is .411 is $65.7^\circ = 1.146$ rad.

$$x = 38 \times 1.146 = \underline{43.6 \text{ in.}}$$

Values of M at this and other stations are evaluated in the table below.

x	10	20	30	40	43.6	50	60	70	80
$\frac{x}{j}$ (rad.)	.263	.526	.789	1.052	1.146	1.315	1.578	1.841	2.104
$\frac{x}{j}$ (deg.)	15.1	30.2	45.3	60.4	65.7	75.5	90.6	105.7	120.8
$\sin \frac{x}{j}$.2605	.503	.711	.869	.911	.968	1.0	.963	.859
$\frac{\sin x/j}{\sin U}$.305	.585	.825	1.012	1.062	1.128	1.162	1.120	1.0
$\frac{x}{L}$.125	.25	.375	.50	.544	.625	.75	.875	1.0
$\left[\frac{\sin x/j}{\sin U} - \frac{x}{L} \right]$.18	.335	.45	.512	.518	.503	.412	.245	0
M	2600	4850	6510	7440	7510	7300	5970	3550	0



6.5.5 Polar Diagrams for Straight Compression Beam-Columns

See GEN/1090/811.7.2 for explanation of theory.

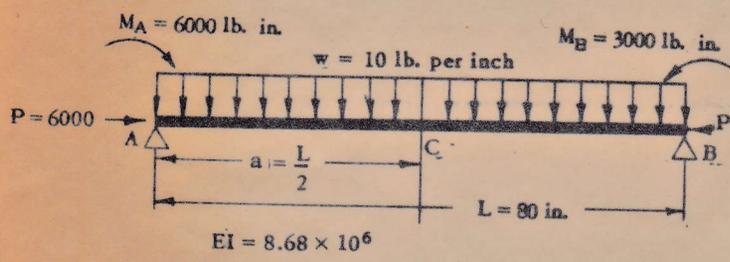
TYPE		
1A	Constant I Uniformly Distributed Lateral Load	plus aggravating end moments
1B	Constant I Uniformly Distributed Lateral Load	plus one aggravating and one relieving end moment
1C	Constant I Uniformly Distributed Lateral Load	plus both relieving end moments
2	Constant I Sudden change in Uniformly Distributed Lateral Load	plus relieving end moments
3A	Constant I Single Concentrated Lateral Load	plus relieving end moments
*3B	Constant I Single Concentrated Lateral Load	plus relieving end moments
3C	Constant I Single Concentrated Lateral Load	plus aggravating end moments
4	Constant I Two Concentrated Lateral Loads	plus aggravating end moments
5A	I Varying Stepwise Uniformly Distributed Lateral Load	plus one aggravating and one relieving end moment
5B	No Lateral Load	plus end moments

* Type 3A differs from Type 3B only in so far as the relative value of W , the lateral load.



6.5.5 Type 1A

CONSTANT I: COMPRESSIVE END LOAD: UNIFORMLY DISTRIBUTED LATERAL LOAD + AGGRAVATING END MOMENTS



Procedure

(i) Find $a = \frac{L}{2} = 40$

$$\mu^2 = \frac{P}{EI} = \frac{6000}{8.68 \times 10^6} = 6.9 \times 10^{-4}$$

$$\mu = .0263$$

$$\mu a = .0263 \times 40 = 1.052 \text{ radians} = 60 \text{ degrees}$$

$$\frac{w}{\mu^2} = \frac{10}{6.9 \times 10^{-4}} = \underline{14500 \text{ lb. in.}}$$

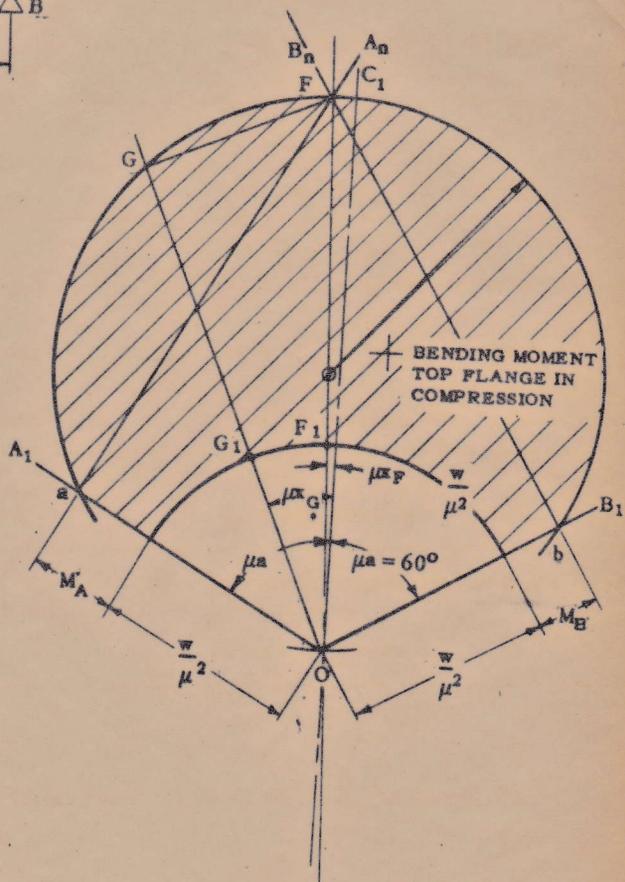
(ii) Draw OA_1 and OB_1 at 60 degrees to OC_1 and markoff points a and b. Draw normals aA_n and bB_n to meet at F. Construct a circle on OF as diameter.

(iii) Draw arc of radius $\frac{w}{\mu^2}$. Resultant B.M. is shown shaded. Max. M is given by FF_1 and equals 24,000 lb. in.

It occurs at $\mu x_F = 2.25$ degrees that is, at a distance $\frac{2.25}{60} \times 40 = 1.5$ in. to the left of C.

B.M. at any station G is $GG_1 = \underline{21700 \text{ lb. in.}}$ which occurs at an angle μx_G from OC_1

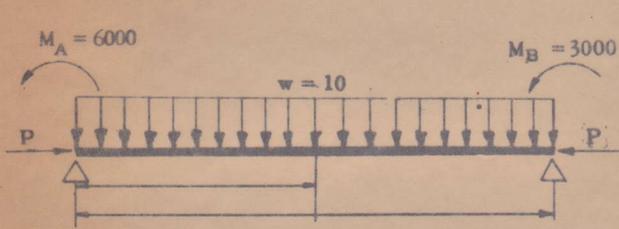
Shear at G is $FG \times \mu = 13350 \times .0263 = \underline{351 \text{ lb.}}$



Type 1A

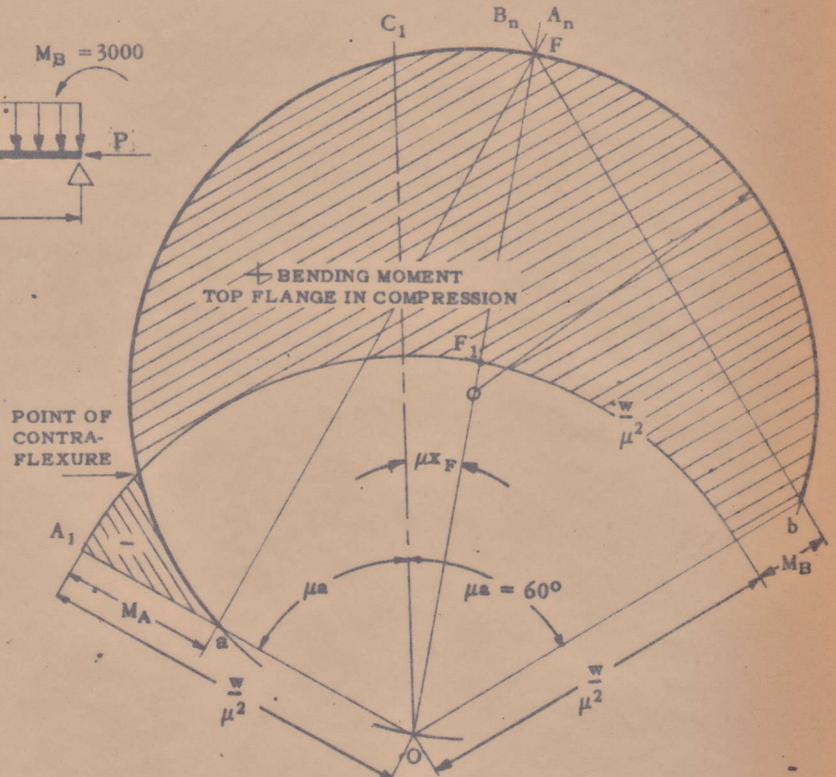
Type 1B

THE SAME AS TYPE 1A BUT M_A IS A RELIEVING MOMENT.



Procedure

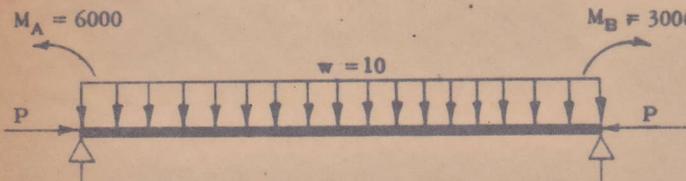
Similar to Type 1A except that point a is positioned differently
 Max. $M = FF_1 = 11700 \text{ lb. in.}$
 at $\mu x_F = 11.5$ degrees to the right of C



Type 1B

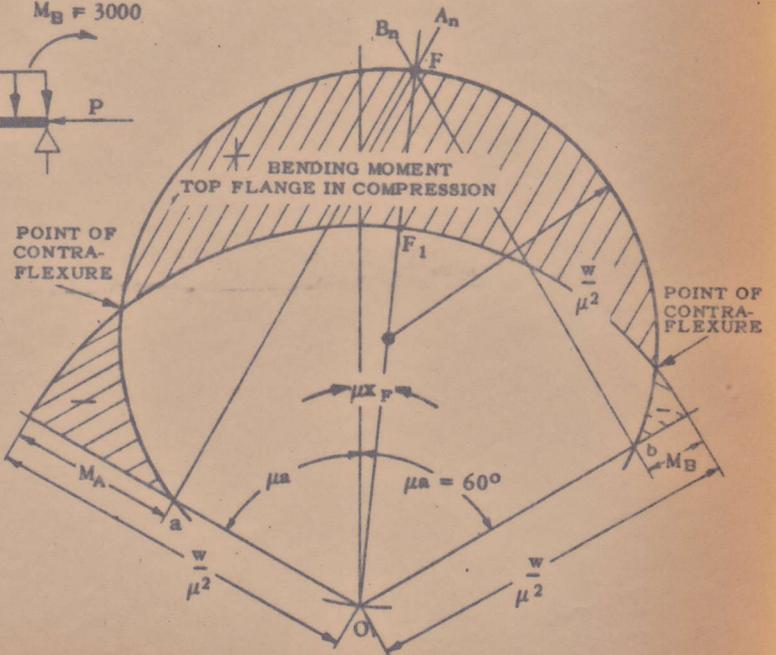
Type 1C

THE SAME AS TYPE 1A BUT M_A AND M_B ARE BOTH RELIEVING MOMENTS.



Procedure

Similar to Type 1A
 Max. $M = FF_1 = 6100 \text{ lb. in.}$
 At $\mu x_F = 6^\circ$ to the right of C.

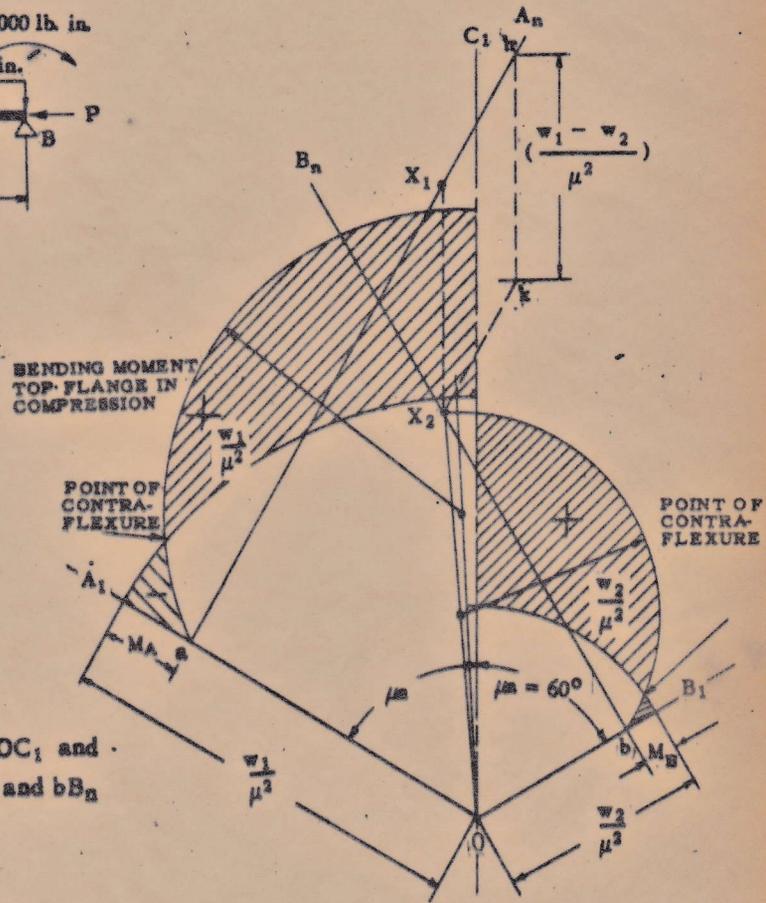
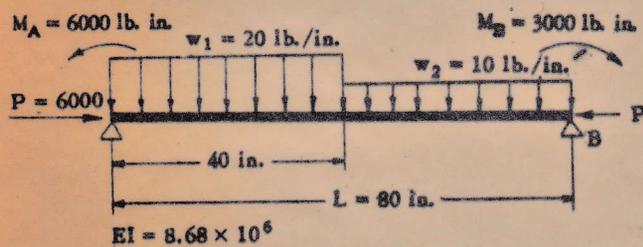


Type 1C



6.5.5 Type 2

CONSTANT 1. COMPRESSIVE END LOAD. SUDDEN CHANGE IN UNIFORMLY DISTRIBUTED LATERAL LOAD. + RELIEVING END MOMENTS.



Procedure

(i) $\mu\alpha = 60$ degree as for Case 1 A

$$\frac{w_1}{\mu^2} = 29000 \quad \frac{w_2}{\mu^2} = 14,500$$

(ii) Draw OA_1 and OB_1 at 60 degrees to OC_1 and mark off points a and b. Draw normals aA_n and bB_n .

(iii) Draw arcs of radius $\frac{w_1}{\mu^2}$ and $\frac{w_2}{\mu^2}$.

From any point h on aA_n draw hk of length $\frac{(w_1 - w_2)}{\mu^2}$

parallel to OC_1 and from k draw kX_2 parallel to aA_n to meet normal bB_n in X_2 . Draw the part-circle on OX_2 as diameter. From X_2 draw X_2X_1 parallel to OC_1 to meet normal aA_n in X_1 . Construct part-circle on OX_1 as diameter.

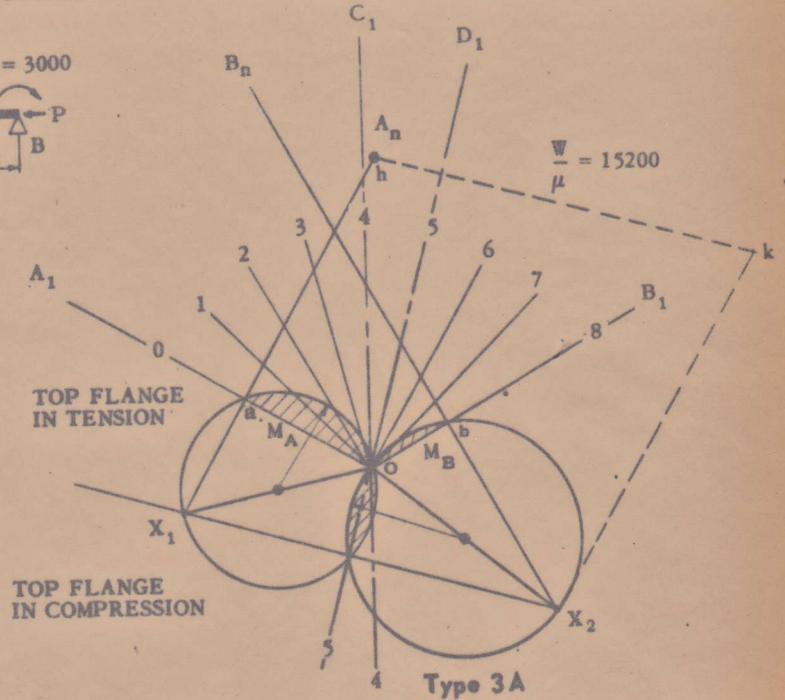
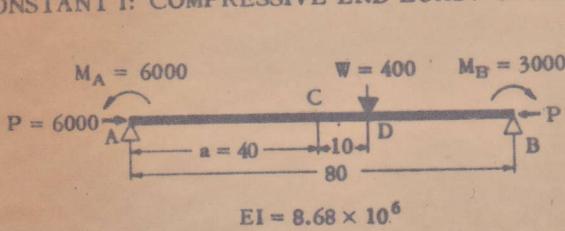
Resultant B.M. is shown shaded.

The same method can be employed for any combination of end moments.

Type 2

6.5.5 Type 3A

CONSTANT I: COMPRESSIVE END LOAD. CONCENTRATED LATERAL LOAD + RELIEVING END MOMENTS.



Procedure

(i) $\mu a = 60$ degree as for Case IA
 $\frac{W}{\mu} = \frac{400}{.0263} = 15200$

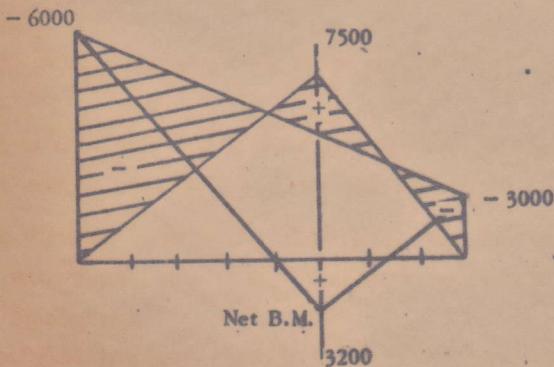
(ii) Draw OA_1 and OB_1 at 60 degree to OC_1 and draw OD_1 at 15 degree to OC_1 to represent the position of D (where W acts). Mark off points a and b on OA_1 and OB_1 and construct the normals aA_n and bB_n .

(iii) From any point h on A_n draw (to the right of A_n) a line hk of length $\frac{W}{\mu}$ at right angles to OD_1 ; from k draw kX_2 parallel to $A_n a$ to meet $B_n b$ produced in X_2 .

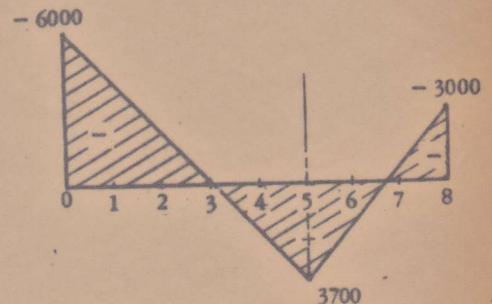
Construct a part circle on OX_2 as diameter.

From X_2 draw X_2X_1 parallel to kh to meet $A_n a$ produced in X_1 . Draw the part-circle on OX_1 as diameter. Resultant B.M. is shaded.

$$\text{Free BM} = \frac{Wab}{L} = \frac{400 \times 50 \times 30}{80} = 7500$$



B.M. Diagram without End Load on a Straight Base



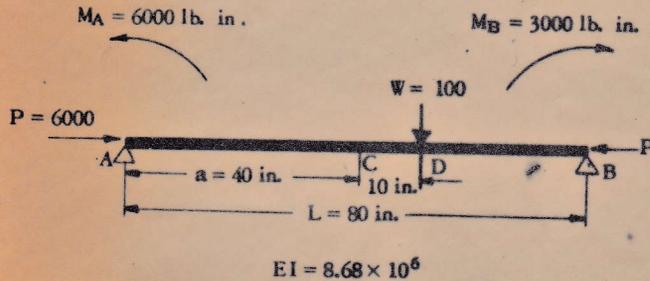
B.M. Diagram with End Load on a Straight Base



6.5.5 Type 3B

CONSTANT I, COMPRESSIVE END LOAD, CONCENTRATED LATERAL LOAD +
RELIEVING END MOMENTS

This case differs from 3A only in so far as W (and hence $\frac{W}{\mu}$) is smaller. This results in a different diagram, as will be understood by considering the analogous cases when there is no axial load.

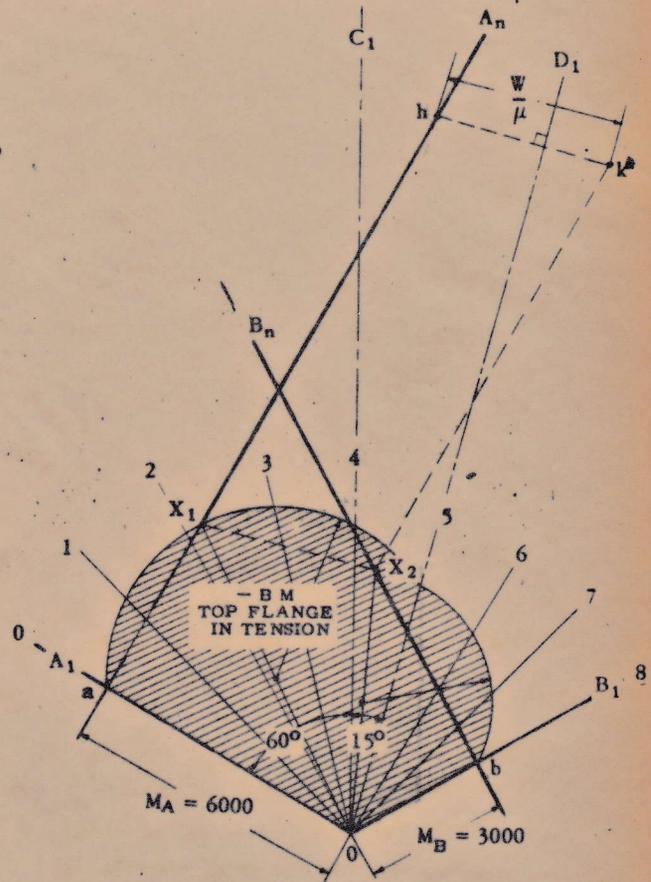


Procedure

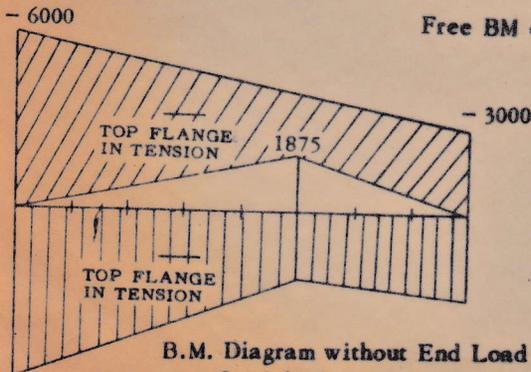
- (i) $\mu a = 60$ degree as for Case 1 A
 $\frac{W}{\mu} = \frac{100}{.0263} = 3800$
- (ii) Draw OA_1 and OB_1 at 60 degree to OC_1 and draw OD_1 at 15 degree to OC_1 to represent the position of D (where W acts). Mark off points a and b on OA_1 and OB_1 and construct the normals aA_n and bB_n .
- (iii) From any point h on aA_n draw (to the right of aA_n) a line hk of length $\frac{W}{\mu}$ at right angles to OD_1 ; from k draw kX_2 parallel to $A_n a$ to meet bB_n in X_2 . Construct a part-circle on OX_2 as diameter.

From X_2 draw X_2X_1 parallel to kh to meet aA_n in X_1 . Draw the part-circle on OX_1 as diameter.

Resultant B.M. is shaded.

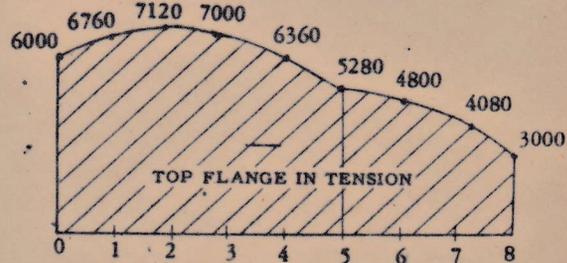


Type 3B



B.M. Diagram without End Load on a Straight Base

$$\text{Free BM} = \frac{Wab}{L} = \frac{100 \times 50 \times 30}{80} = 1875$$

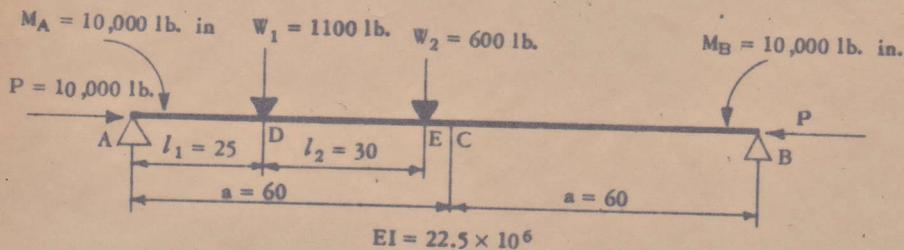


B.M. Diagram with End Load on a Straight Base



6.5.5 Type 4

CONSTANT I: COMPRESSIVE END LOAD. TWO CONCENTRATED LATERAL LOADS + AGGRAVATING END MOMENTS.



Procedure

$$(i) \mu = \sqrt{\frac{P}{EI}} = \sqrt{\frac{10^4}{22.5 \times 10^6}} = .02105$$

$$\mu l_1 = .02105 \times 25 = .527 \text{ radians} = 30^\circ 11'$$

$$\mu l_2 = .02105 \times 30 = .633 \text{ radians} = 36^\circ 12'$$

$$\mu a = .02105 \times 60 = 1.261 \text{ radians} = 72^\circ 24'$$

$$\frac{W_1}{\mu} = \frac{1100}{.02105} = 52250$$

$$\frac{W_2}{\mu} = \frac{600}{.02105} = 28500$$

(ii) Draw OC_1 vertically and mark off OA_1 and OB_1 on each side of it at $72^\circ 24'$. Then draw in OD_1 at $30^\circ 11'$ to OA_1 and OE_1 at $36^\circ 12'$ to OD_1 .

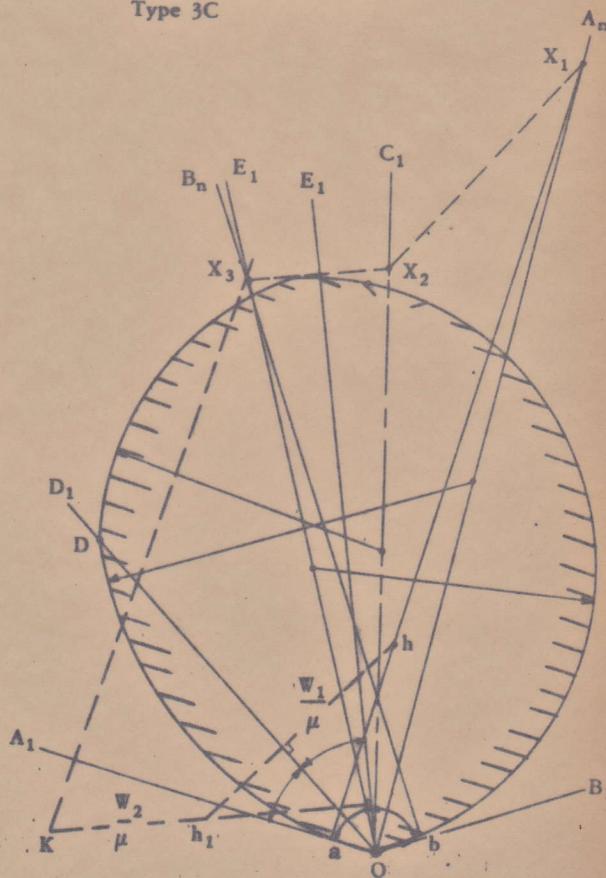
Mark off Oa to represent M_A and Ob to represent M_B and construct the normals aA_n and bB_n .

From any point h on aA_n draw hh_1 normal to OD_1 and equal to $\frac{W_1}{\mu}$. From h_1 draw h_1K equal to $\frac{W_2}{\mu}$ and normal to OE_1 . From K draw KX_3 parallel to aA_n and cutting bB_n in X_3 . From X_3 draw X_3X_2 parallel to Kh_1 to cut OC_1 in X_2 . From X_2 draw X_2X_1 parallel to h_1h to cut aA_n in X_1 .

Construct part-circles on OX_1 , OX_2 and OX_3 as diameters.

Resultant B.M. is shaded.

This diagram is similar in general characteristics to Type 3C



Type 4.

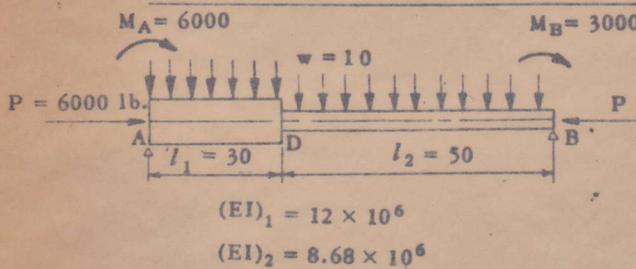
$$\text{ANGLE } A_1OC_1 = \text{ANGLE } B_1OC_1 = 72^\circ 24'$$

$$\text{ANGLE } A_1OD_1 = 30^\circ 11'$$

$$\text{ANGLE } D_1OE_1 = 36^\circ 12'$$

6.5.5 Type 5A

I VARYING STEPWISE. COMPRESSIVE END LOAD. UNIFORMLY DISTRIBUTED LATERAL LOAD + ONE AGGRAVATING AND ONE RELIEVING END MOMENT



Procedure

$$(i) \mu_1^2 = \frac{P}{(EI)_1} = \frac{6000}{12 \times 10^6} = 5.0 \times 10^{-4}$$

$$\mu_2^2 = \frac{P}{(EI)_2} = \frac{6000}{8.68 \times 10^6} = 6.9 \times 10^{-4}$$

$$\mu_1 = .0224 \quad \mu_2 = .0263 \quad \frac{\mu_1}{\mu_2} = .852$$

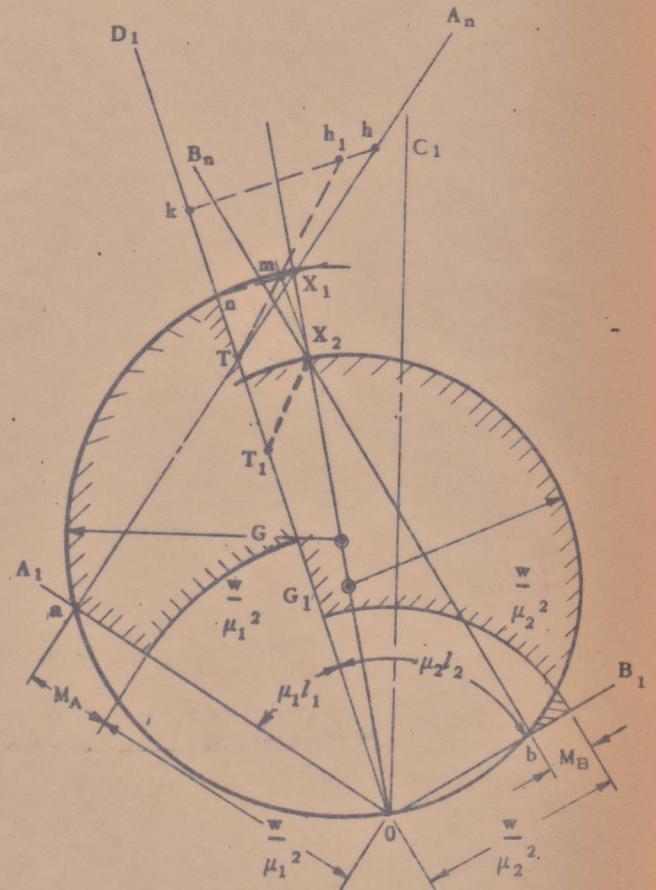
$$\mu_1 l_1 = .0224 \times 30 = .672 \text{ radians} = 38^\circ 30'$$

$$\mu_2 l_2 = .0263 \times 50 = 1.315 \text{ radians} = 75^\circ 30'$$

$$\frac{w}{\mu_1^2} = 20000 \quad \frac{w}{\mu_2^2} = 14500$$

(ii) Draw OC_1 vertically and OA_1 and OB_1 at $\frac{114}{2} = 57$ degrees on each side of it. Then put in OD_1 at an angle $\mu_1 l_1 = 38^\circ 30'$ to OA_1 . Mark off $\frac{w}{\mu_1^2}$ and M_A on OA_1 to give point a and $\frac{w}{\mu_2^2}$ and M_B on OB_1 to give point b. Draw normals aA_n and bB_n .

(iii) From any point h on aA_n draw hk normal to OD_1 to meet OD_1 in k. Choose a point h_1 on hk such that $\frac{hk}{h_1k} = \frac{\mu_2}{\mu_1}$ or $h_1k = \frac{\mu_1}{\mu_2} hk = .852 hk$.



Type 5A

Join h_1 to T, the point of intersection of aA_n and OD_1 . Mark off a length TT_1 on OD_1 equal to GG_1 that is, equal to $(\frac{w}{\mu_1^2} - \frac{w}{\mu_2^2})$, and from T_1 draw T_1X_2 parallel to Th_1 to cut bB_n in X_2 . Draw a part circle on OX_2 diameter. From X_2 draw X_2m parallel to OD_1 to cut Th_1 in m.

Construct the normal to OD_1 viz nmX_1 to cut aA_n in X_1 .

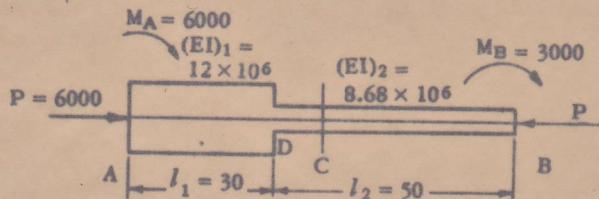
Draw the part circle on OX_1 as diameter.

Resultant B.M. is shaded.

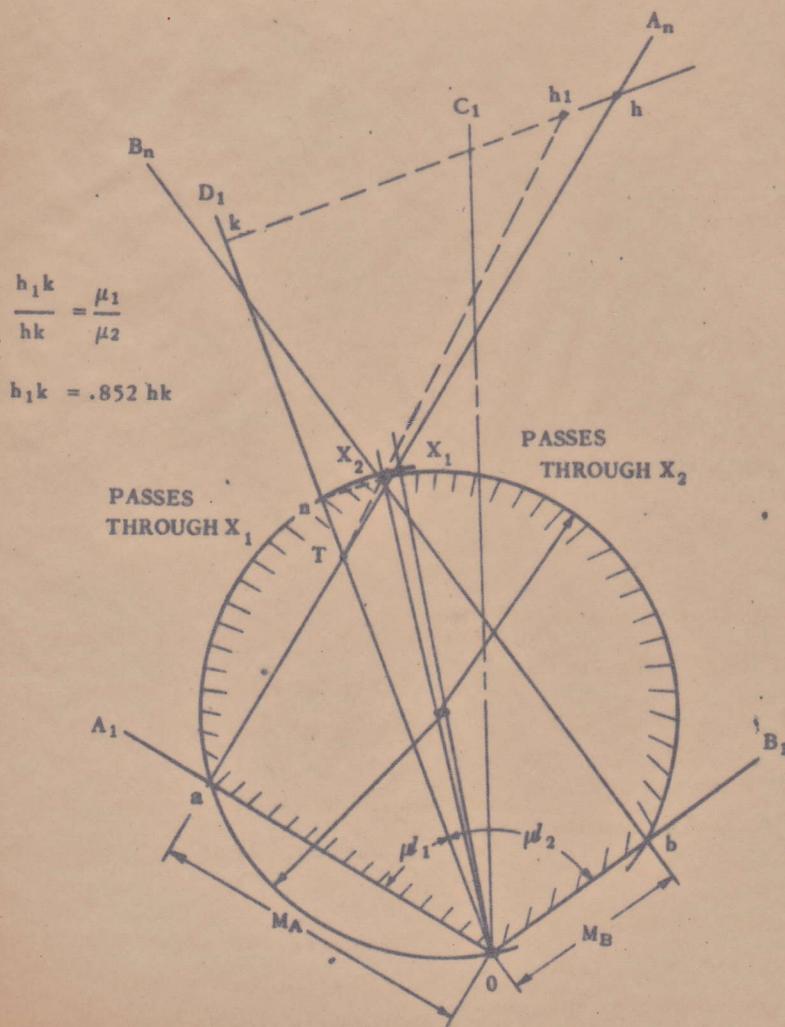


6.5.5 Type 5B

I VARYING STEPWISE. NO LATERAL LOAD BUT END MOMENTS.



Same construction as Type 5A, but since there is no lateral load X_2 coincides with m and T with T_1 .



$$\frac{h_1 k}{hk} = \frac{\mu_1}{\mu_2}$$

$$h_1 k = .852 hk$$

Type 5B

6.5.6 Tapered Beam-Columns: General Method

A general method for determining the distribution of bending moment, shear and deflection of a beam-column of varying I acted on by end moments and carrying a fairly complex lateral load is developed in "Aircraft Engineering", February 1953, p.56 by Gittleman.

Briefly the method is as follows:- Considering any segment KL of a deflected beam-column AB having ordinates x_K and x_L , such that $l = (x_L - x_K)$ Fig. 6.5.6-1(a), it is shown that if

M_K and M_L = bending moment
 θ_K and θ_L = slope
] at K and L respectively

$$\theta_L = -M_K \frac{\mu}{P} \sin \mu l + \theta_K \cos \mu l + a$$

$$M_L = M_K \cos \mu l + \theta_K \frac{P}{\mu} \sin \mu l + \beta$$

where

$$a = \frac{\Sigma W - Q}{P} (1 - \cos \mu l) - \frac{w}{P} \left(\frac{\sin \mu l}{\mu} + x_K \cos \mu l - x_L \right)$$

$$\beta = -(\Sigma W - Q) \frac{\sin \mu l}{\mu} - \frac{w}{\mu^2} + \frac{w}{\mu} \left(\frac{\cos \mu l}{\mu} - x_K \sin \mu l \right)$$

$$\mu^2 = \frac{P}{EI} \quad \text{and} \quad Q = R_A' + \frac{M_B - M_A}{L} \quad \text{where } R_A' \text{ is the free reaction at A.}$$

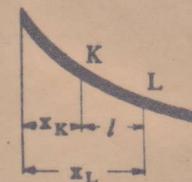


Fig. 6.5.6-1(a)

a and β depend solely upon the loading and beam geometry.

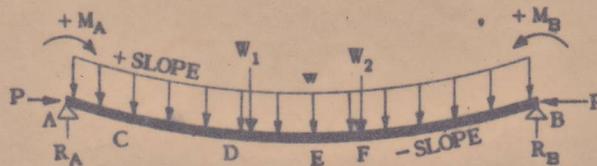


Fig. 6.5.6-1(b)

By first assuming that K coincides with A and that the slope there, viz $\theta_K = \theta_A$, is zero (which it may very well not be of course), it is possible to calculate a and β and find θ_L and M_L at C .

Having found these values for segment AC , consider the next segment CD and use the L values just found as K values for this second segment and find θ_L and M_L at D .

In this way θ_L at $B = \theta_B$ and M_L at $B = M_B$ are ultimately found (by tabulation).

A further relationship is:- $M_B = a \theta_A + b$ where a and b are constants.

The calculation is repeated for another value of θ_A , two trial solutions being necessary.



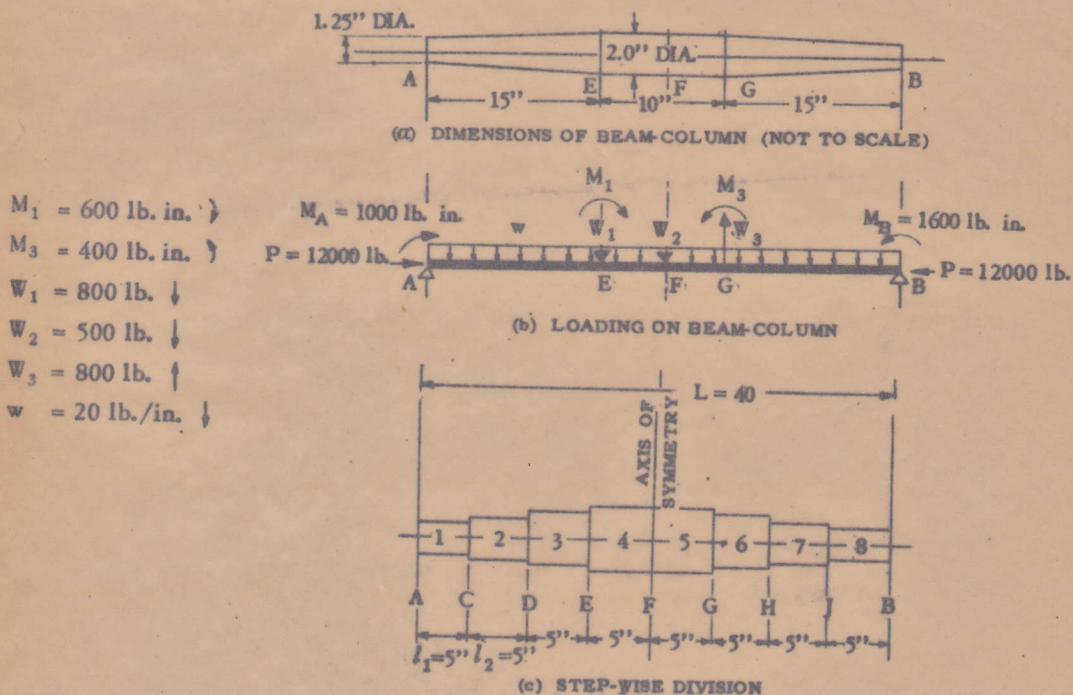
6.5.6 Tapered Beam-Columns: General Method (cont'd)

NUMERICAL EXAMPLE

The problem considered is the same as in the reference quoted above.

Although this particular beam-column is symmetrical about its mid-span, it is not necessary for this to be so, for the method is a general one in this respect.

Fig. 6.5.6-2(a) shows the dimensions of the beam (not to scale), (b) shows the complex loading and (c) the method of step-wise division, referred to below



$$M_1 = 600 \text{ lb. in. } \rightarrow$$

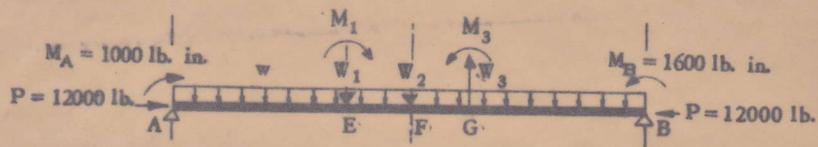
$$M_3 = 400 \text{ lb. in. } \rightarrow$$

$$W_1 = 800 \text{ lb. } \downarrow$$

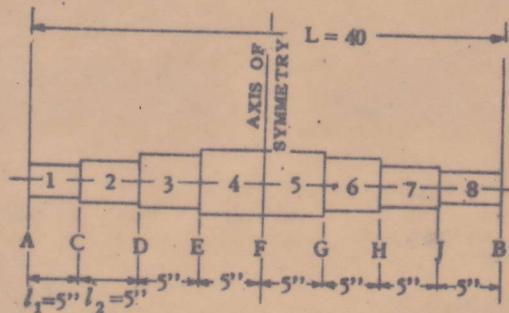
$$W_2 = 500 \text{ lb. } \downarrow$$

$$W_3 = 800 \text{ lb. } \uparrow$$

$$w = 20 \text{ lb./in. } \downarrow$$



(b) LOADING ON BEAM-COLUMN



(c) STEP-WISE DIVISION

Fig. 6.5.6-2

PROCEDURE

(1) Divide the span into convenient segments. In this case each is 5 in. long, such that $l = 5$ constant, but it is neither necessary nor desirable that l should be constant in some instances. In fact, the subdivision into segments must be guided by the rule that no concentrated load or moment must be applied within the length of a segment. Note, for example, that the concentrated loads and moments occur at E, F and G at one end of a segment.

The diameter of cross-section of each segment is considered as uniform over its length and equal to that at the mid-section, that is, mid-way between A and C, C and D, etc.

(2) Consider the beam-column as a beam pinned at the ends and calculate the free reaction R_A' at A.

By moments about B,

$$R_A' \times 40 + 600 - 400 = \frac{20}{2} \times 40^2 + 800 \times 25 - 800 \times 15 + 500 \times 20$$

$$R_A' = 845 \text{ lb.}$$

$$Q = R_A' + \frac{M_B - M_A}{L}$$

$$= 845 + \frac{1600 - 1000}{40} = 845 + 15 = \underline{860 \text{ lb.}}$$

(3) Find the sum of the concentrated loads ΣW at any station and hence $(\Sigma W - Q)$.

For each segment calculate the values in Table 6.5.6-1 and then α (Table 6.5.6-2) and β (Table 6.5.6-3).



6.5.6 Tapered Beam-Columns: General Method (cont'd)

TABLE 6.5.6-1

SEGMENT	1	2	3	4	5	6	7	8
MEAN DIAMETER D	1.375	1.625	1.875	2.0	2.0	1.875	1.625	1.375
D^4	3.56	6.96	12.25	16.0	16.0	12.25	6.96	3.56
$I = \frac{\pi}{64} D^4$.1748	.3415	.602	.786	.786	.602	.3415	.1748
$EI \times 10^{-6}$	5.244	10.245	18.06	23.58	23.58	18.06	10.245	5.244
$\mu^2 \times 10^4$	22.9	11.71	6.65	5.08	5.08	6.65	11.71	22.9
μ	.0477	.0341	.0257	.0226	.0226	.0257	.0341	.0477
μl (RADIANS)	.2385	.1705	.1285	.1130	.1130	.1285	.1705	.2385
(DEGREES)	13.7	9.77	7.36	6.48	6.48	7.36	9.77	13.7
$\sin \mu l$.2368	.1696	.1282	.1126	.1126	.1282	.1696	.2368
$\cos \mu l$.9717	.9855	.9917	.9936	.9936	.9917	.9855	.9717
ΣW	-	-	-	800	1300	500	500	500
$(\Sigma W - Q)$	-860	-860	-860	-60	440	-360	-360	-360

TABLE 6.5.6-2 FOR FINDING α

$\frac{(\Sigma W - Q)}{P}$	-0.7167	-0.7167	-0.7167	-0.005	.03667	-.030	-.030	-.030
$(1 - \cos \mu l)$.0283	.0145	.0083	.0064	.0064	.0083	.0145	.0283
$10^3 \times \frac{(\Sigma W - Q)}{P} (1 - \cos \mu l)$	-2.028	-1.039	-.595	-.032	-.235	-.249	-.435	-.849
$\frac{\sin \mu l}{\mu}$	4.9539	4.9736	4.9883	4.9823	4.9823	4.9883	4.9736	4.9539
x_k	0	5	10	15	20	25	30	35
$x_k \cos \mu l$	0	4.9275	9.917	14.904	19.872	24.7925	29.565	34.0095
x_L	5	10	15	20	25	30	35	40
$(\frac{\sin \mu l}{\mu} + x_k \cos \mu l - x_L)$	-.0461	-.0989	-.0947	-.1137	-.1457	-.2192	-.4614	-1.0366
$\frac{w}{P}$	CONSTANT AT			1.666×10^{-3}				
$10^4 \frac{w}{P} (\frac{\sin \mu l}{\mu} + x_k \cos \mu l - x_L)$	-.77	-1.65	-1.58	-1.90	-2.43	-3.65	-7.69	-17.28
$10^4 \alpha$	-19.51	-8.74	-4.37	1.58	4.78	1.16	3.34	8.79



6.5.6 Tapered Beam-Columns: General Method (cont'd)

TABLE 6.5.6-3 FOR FINDING β

	1	2	3	4	5	6	7	8
$(\sum w - Q)$	- 860	- 860	- 860	- 60	440	- 360	- 360	- 360
$\frac{\sin \mu l}{\mu}$	4.9539	4.9736	4.9883	4.9823	4.9823	4.9883	4.9736	4.9539
$(\sum w - Q) \frac{\sin \mu l}{\mu}$	-4260	-4277	-4290	- 299	2192	-1796	-1790	-1783
$\frac{w}{\mu^2}$	8790	17199	30280	39159	39159	30280	17199	8790
$\frac{\cos \mu l}{\mu}$	20.3711	28.9003	38.5875	43.9646	43.9646	38.5875	28.9003	20.3711
$x_K \sin \mu l$	0	.8480	1.2820	1.6890	2.2520	3.2050	5.0880	8.2705
$\frac{\cos \mu l}{\mu} - x_K \sin \mu l$	20.3711	28.0523	37.3055	42.2756	41.7126	35.3825	23.8123	12.1006
$\frac{w}{\mu} - (\cos \frac{\mu l}{\mu} - x_K \sin \mu l)$	8542	16453	29031	37414	36916	27535	13966	5074
β	4012	3531	3041	-1446	-4435	- 949	-1443	-1933

(4) Assume that K coincides with A and take $M_K = 1000$ (the known fixing moment at A) and $\theta_K = 0$, as an initial assumption.

Then for segment AC

$$\theta_L = -M_K \frac{\mu}{P} \sin \mu l + a$$

$$= -1000 \times .939 \times 10^{-6} - 19.51 \times 10^{-4}$$

$$= -9.39 \times 10^{-4} - 19.51 \times 10^{-4} = -28.90 \times 10^{-4}$$

$$M_L = 1000 \cos \mu l + \beta$$

$$= 971.7 + 4012 = \underline{4984}$$

Assume now that K coincides with C and L with D

$$M_K = M_L \text{ above} = 4984$$

$$\theta_K = \theta_L \text{ above} = -28.9 \times 10^{-4}$$

Hence determine θ_L and M_L for segment CD and for all segments, by tabulation, as shown in Tables 6.5.6-4 and -5.

It is found that M_L at B = $M_B = -2867$



6.5.6 Tapered Beam-Columns: General Method (cont'd)

TABLE 6.5.6-4

	1	2	3	4	5	6	7	8
$\frac{\mu}{P} \sin \mu l \times 10^{-6}$.939	.482	.275	.212	.212	.275	.482	.939
M_K	1000	4984	8271	<u>11476</u>	9432	<u>3879</u>	2152	-116
$-M_K \frac{\mu}{P} \sin \mu l \times 10^{-4}$	-9.39	-24.02	-22.75	-24.33	-20.00	-10.67	-10.37	-1.09
$\theta_K \times 10^{-4}$	0	-28.9	-61.24	-87.85	-110.04	-124.56	-133.05	-138.15
$\cos \mu l$.9717	.9855	.9917	.9936	.9936	.9917	.9855	.9717
$\theta_K \cos \mu l \times 10^{-4}$	0	-28.48	-60.73	-87.29	-109.34	-123.54	-131.12	-134.24
$10^4 \alpha$	-19.51	-8.74	-4.37	1.58	4.78	1.16	3.34	8.79
$10^4 \theta_L$	-28.9	-61.24	-87.85	-110.04	-124.56	-133.05	-138.15	-124.36

TABLE 6.5.6-5

	1	2	3	4	5	6	7	8
M_K	1000	4984	8271	<u>11476</u>	9432	<u>3879</u>	2152	-116
$M_K \cos \mu l$	972	4912	8202	11403	9372	3847	2121	-113
$\frac{P}{\mu} \sin \mu l$	59446	59683	59860	59788	59788	59860	59683	59446
$\theta_K \frac{P}{\mu} \sin \mu l$	0	-172	-367	-525	-658	-746	-794	-821
β	4012	3531	3041	-1446	-4435	-949	-1443	-1933
M_L	4984	8271	<u>10876</u>	9432	<u>4279</u>	2152	-116	-2867

Note that when transferring M_L from column 3 to M_K in column 4, it is necessary to add on the applied moment of 600 lb.in. at E, thus making $10876 + 600 = 11476$ as indicated by the dotted line in Table 6.5.6-5.

Similarly when transferring $M_L = 4279$ from column 5 to column 6 it is reduced by 400 to 3879, because of the applied moment at G.



6.5.6 Tapered Beam-Columns: General Method (cont'd)

The deflected shape of the beam, *relative to A*, is shown diagrammatically in Fig. 6.5.6-3. (It is thus because all the values of θ_L are negative).

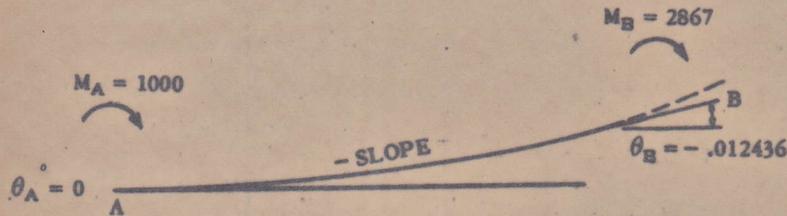


Fig. 6.5.6-3

$$\begin{aligned} \text{From the relationship } M_B &= a\theta_A + b \\ -2867 &= 0 + b \\ b &= -2867 \end{aligned}$$

SECOND APPROXIMATION

Assume a second value $\theta_A = 0.10$ and take $M_A = 1000$ again. Carry out a similar calculation (not shown here or in the original reference) to give $M_B = 34968$

$$\begin{aligned} M_B &= a\theta_A + b \\ 34968 &= a \times 0.10 - 2867 \\ 0.10a &= 37835 \\ a &= 378,350 \end{aligned}$$

It is known that the actual value of $M_B = 1600$

$$\begin{aligned} \text{Hence } 1600 &= 378,350 \theta_A - 2867 \\ \text{and } \theta_A &= 0.011807 \end{aligned}$$

FINAL SOLUTION

Start with $\theta_A = 0.011807$ and $M_A = 1000$ and work through as in Table 6.5.6-6 to find θ_L and M_L for all segments

$$\begin{aligned} \text{Values at B are } \theta_B &= -0.010075 \\ M_B &= 1598 \end{aligned}$$

SHEAR AND DEFLECTION

Refer to the original article for the method to be used.



6.5.6 Tapered Beam-Columns: General Method (cont'd)

TABLE 6.5.6-6

	1	2	3	4	5	6	7	8
$\frac{\mu}{P} \sin \mu l \times 10^{-6}$.939	.482	.275	.212	.212	.275	.482	.939
M_K	1000	5686	9647	13498	12068	7096	5904	4083
$-M_K \frac{\mu}{P} \sin \mu l \times 10^{-4}$	-9.39	-27.41	-26.53	-28.62	-25.59	-19.51	-28.46	-38.34
$\theta_K \times 10^{-4}$	118.07	85.83	48.44	17.14	-10.01	-30.76	-48.86	-73.27
$\cos \mu l$.9717	.9855	.9917	.9936	.9936	.9917	.9855	.9717
$\theta_K \cos \mu l \times 10^{-4}$	14.73	84.59	48.04	17.03	-9.95	-30.51	-48.15	-71.20
$10^4 \alpha$	-19.51	-8.74	-4.37	1.58	4.78	1.16	3.34	8.79
$10^4 \theta_L$	85.83	48.44	17.14	-10.01	-30.76	-48.86	-73.27	-100.75

TABLE 6.5.6-7

M_K	1000	5686	9647	13498	12068	7096	5904	4083
$M_K \cos \mu l$	972	5604	9567	13412	11991	7037	5818	3967
$\frac{P}{\mu} \sin \mu l$	59446	59683	59860	59788	59788	59860	59683	59446
$\theta_K \frac{P}{\mu} \sin \mu l$	702	512	290	102	-60	-184	-292	-436
β	4012	3531	3041	-1446	-4435	-949	-1443	-1933
M_L	5686	9647	12898	12068	7496	5904	4083	1598



6.5.7 Beam-Columns

STIFFNESS AND CARRY-OVER CHARTS (to include effect of End Load)

Stiffness and carry-over factors, required for carrying out moment distribution calculations of beams with axial load and lateral load, are given herein in chart form.

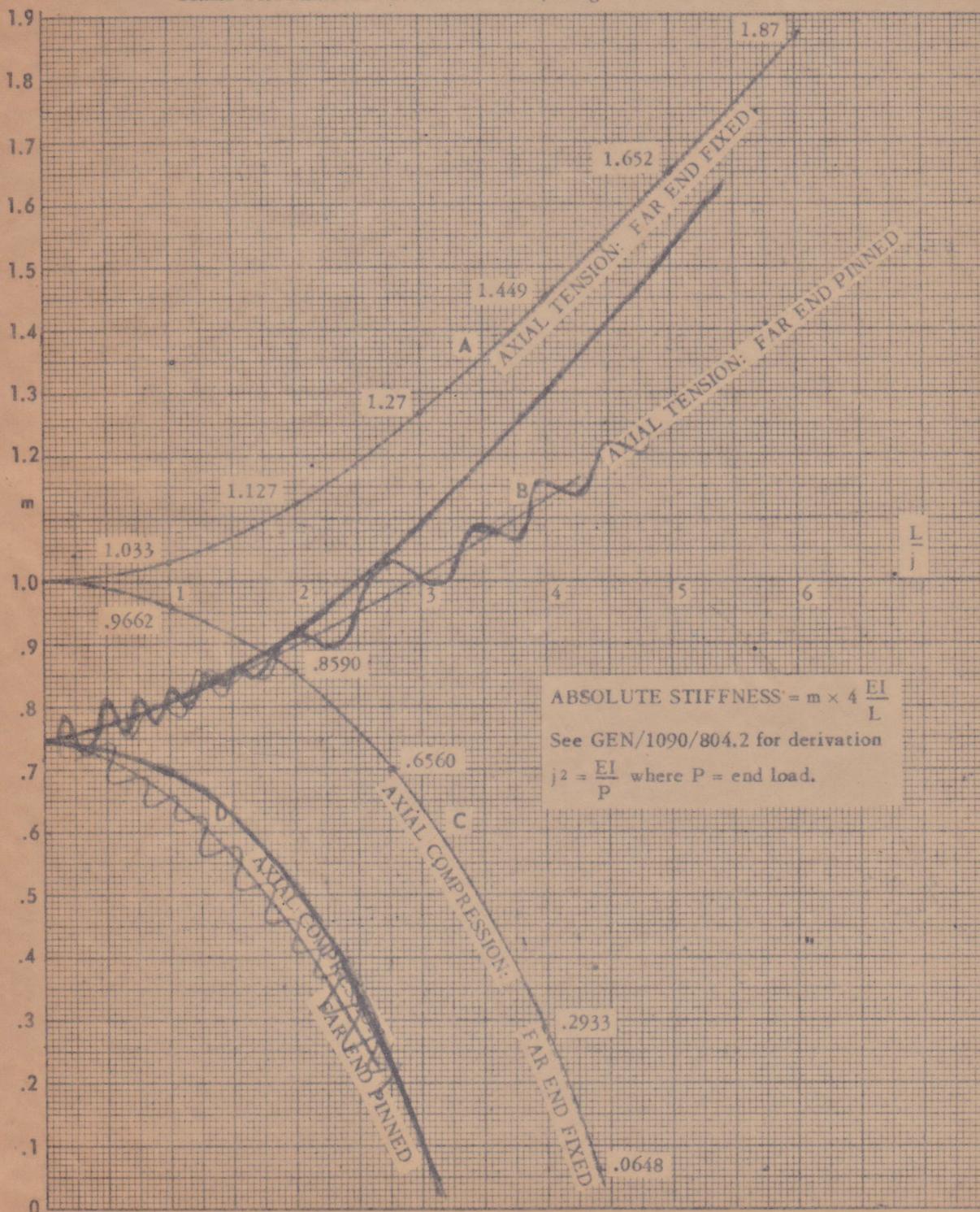


Fig. 6.5.7-1 Stiffness Factor m for Constant I Beam-Columns (From N.A.C.A., T.N. 534)



6.5.7 Beam-Columns (cont'd)

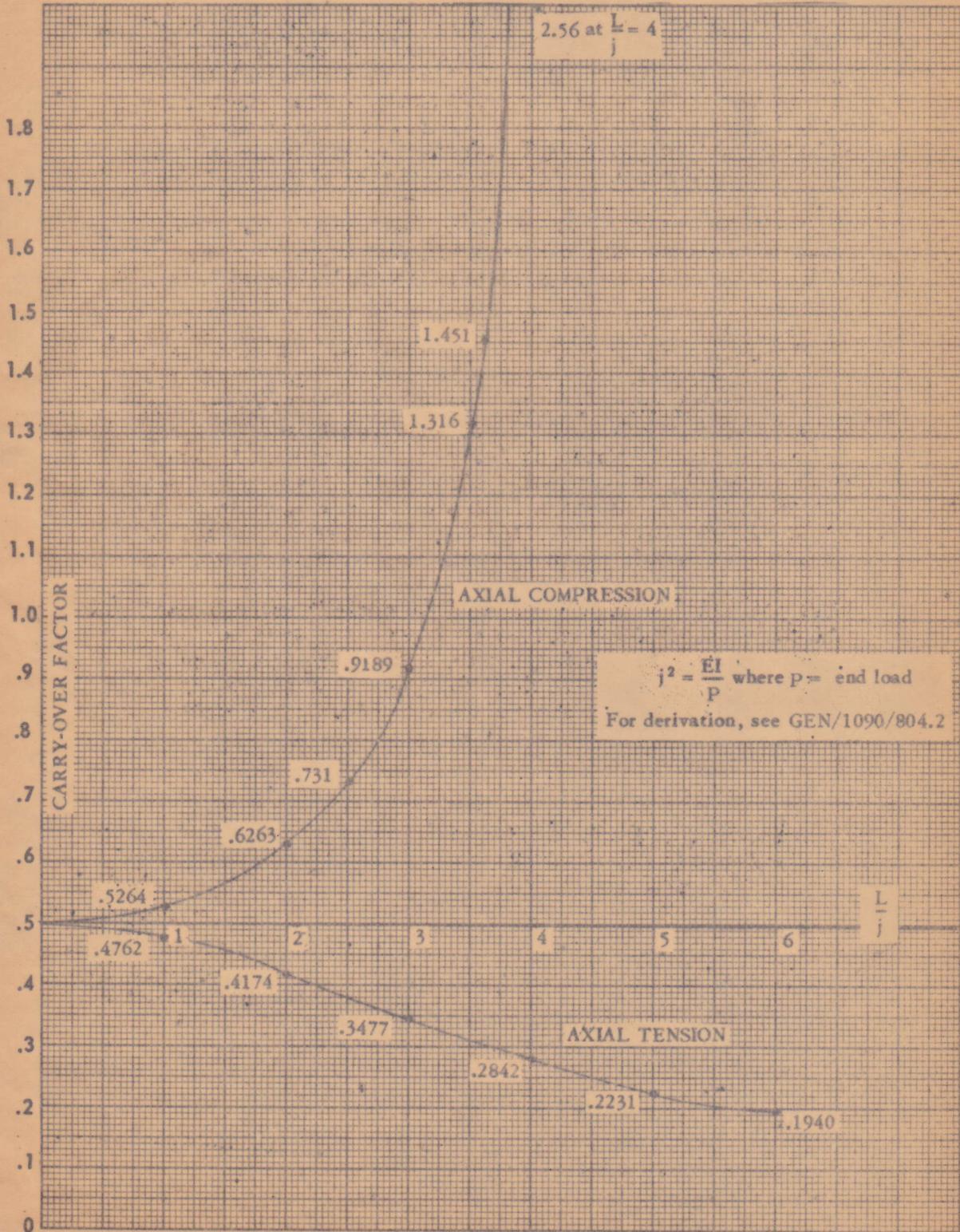


Fig. 6.5.7-2 Carry-Over Factors for Constant I Beam-Columns (From N.A.C.A., T.N. 534)



6.6

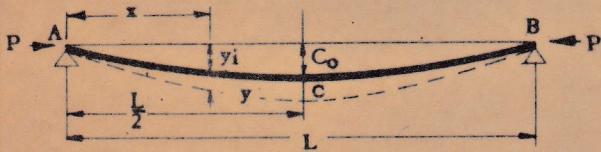
COLUMNS WITH SMALL INITIAL BOW

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- 6.6.1 Tabular Summary of Standard Cases
- 6.6.2 Numerical Example
- 6.6.3 Polar Diagrams

6.6.1 Columns with Small Initial Bow

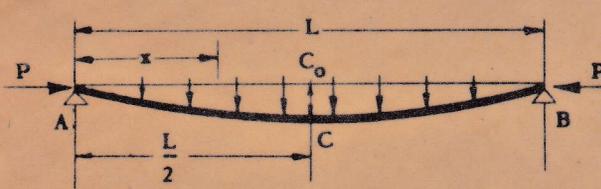
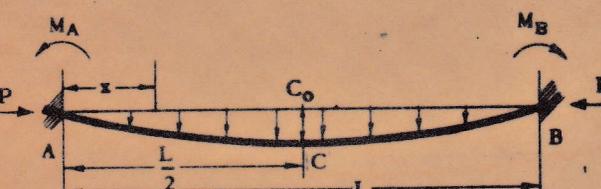
$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$

COLUMNS WITH SMALL INITIAL BOW	+ B.M. PUTS TOP FLANGE IN COMPRESSION
<p>(1)</p> <p style="text-align: center;">SIMPLY SUPPORTED AT ENDS COMPRESSIVE END LOAD NO LATERAL LOAD</p>  <p style="text-align: center;">(Deflected shape shown dotted)</p> <p>INITIAL BOW (SINUSOIDAL) $y_i = C_0 \sin \frac{\pi x}{L}$</p> <p style="text-align: center;">Where C_0 = initial bow at centre C</p>	<p>At any station distant x from A,</p> $M = \frac{Q}{Q-P} P \cdot C_0 \sin \frac{\pi x}{L} \quad \text{(Perry approximation)}$ <p>where Q = Euler failing load for pinned ends = $\frac{\pi^2 EI}{L^2}$</p> $\text{Max. } M = \frac{Q}{Q-P} P \cdot C_0 \text{ at } C$ <p>Deflection relative to line AB is</p> $y + y_i = \frac{Q}{Q-P} C_0 \sin \frac{\pi x}{L}$ $= \frac{Q}{Q-P} C_0 \text{ at } C \text{ where it is a max.}$ <p>For derivation see GEN/1090/811.8</p>



6.6.1 Columns with Small Initial Bow (cont'd)

$$j^2 = \frac{EI}{P} \quad U = \frac{L}{j}$$

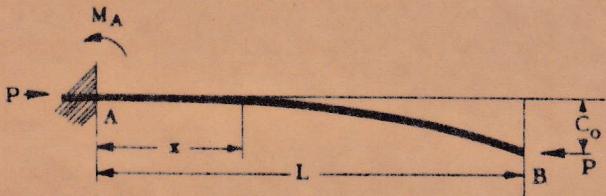
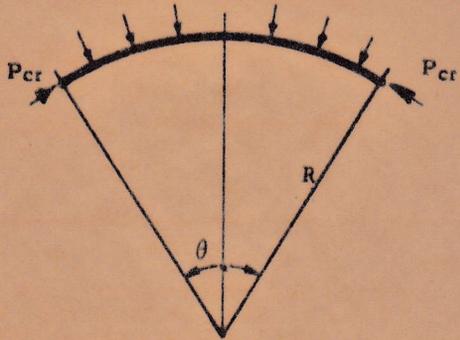
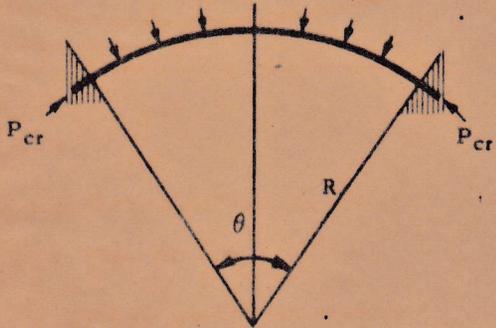
COLUMNS WITH SMALL INITIAL BOW	+ B.M. PUTS TOP FLANGE IN COMPRESSION
<p>(2)</p> <p>SIMPLY SUPPORTED AT ENDS COMPRESSIVE END LOAD + UNIFORMLY DISTRIBUTED LATERAL LOAD (INCREASING INITIAL BOW)</p>  <p>INITIAL BOW (PARABOLIC) $y_i = Kx(L - x)$</p> <p>Where $k = \frac{4C_o}{L^2}$</p> <p>and $C_o =$ initial bow at centre C</p>	<p>At any station distant x from A,</p> $M = A \cos \frac{x}{j} + B \sin \frac{x}{j} - j^2 (w + 2Pk)$ <p>where $A = j^2 (w + 2Pk)$,</p> $B = j^2 (w + 2Pk) \tan \frac{U}{2}$ <p>Alternatively, use a polar diagram, as in 6.6.3</p> $\text{Max. } M = j^2 (w + 2Pk) \left(\sec \frac{U}{2} - 1 \right) \text{ at C}$ <p>These expressions are basically similar to those in 6.5.2.2, Case 3A with the addition of a term involving k, due to the initial bow.</p> <p>When the lateral load <i>decreases</i> the initial bow, use $(w - 2Pk)$ instead of $(w + 2Pk)$ throughout.</p>
<p>(3)</p> <p>FIXED AT ENDS. COMPRESSIVE END LOAD + UNIFORMLY DISTRIBUTED LATERAL LOAD (INCREASING INITIAL BOW)</p>  <p>INITIAL BOW (PARABOLIC) $y_i = Kx(L - x)$</p> <p>Where $k = \frac{4C_o}{L^2}$</p> <p>and $C_o =$ initial bow at centre C</p>	$M_A = M_B = -j^2 (w + 2Pk) \left[1 - \frac{U}{2} \cot \frac{U}{2} \right]$ <p>At any station distant x from C,</p> $M = A \cos \frac{x}{j} + B \sin \frac{x}{j} - j^2 (w + 2Pk)$ <p>where $A = j^2 (w + 2Pk) - M_A$</p> $B = j^2 \left[(w + 2Pk) - M_A \right] \tan \frac{U}{2}$ <p>Alternatively, use a polar diagram, as in 6.6.3.</p> $\text{Max. } +M = j^2 (w + 2Pk) \left[\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1 \right] \text{ at C.}$ <p>When the lateral load <i>decreases</i> the initial bow, use $(w - 2Pk)$ instead of $(w + 2Pk)$ throughout.</p>



6.6.1 Columns with Small Initial Bow (cont'd)

$$j^2 = \frac{EI}{P}$$

$$U = \frac{L}{j}$$

COLUMNS WITH SMALL INITIAL BOW	+ B.M. PUTS TOP FLANGE IN COMPRESSION
<p>(4) CANTILEVER</p>  <p>INITIAL BOW $y_i = C_0 \sin \frac{\pi x}{L}$</p>	$M_A = - \frac{Q}{Q-P} P \cdot C_0$ <p>where Q = Euler failing load for pinned ends</p> $= \frac{\pi^2 EI}{L^2}$
<p>(5) ARCH PINNED AT ENDS EXTERNAL RADIAL PRESSURE</p> 	$P_{cr} = \left[4 \left(\frac{\pi}{\theta} \right)^2 - 1 \right] \frac{EI}{R^3}$ <p>Proof in Den Hartog Advanced strength of Materials, p. 282.</p>
<p>(6) ARCH FIXED AT ENDS EXTERNAL RADIAL PRESSURE</p> 	$P_{cr} = \left[9 \left(\frac{\pi}{\theta} \right)^2 - 1 \right] \frac{EI}{R^3}$



6.6.2 Numerical Example

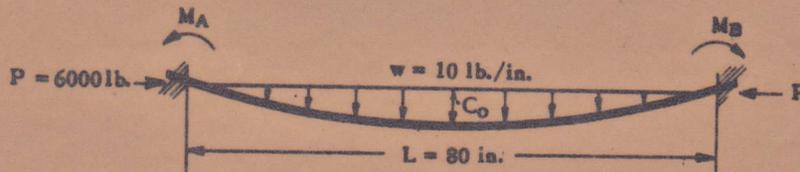


Fig. 6.6.2-1

Slightly bowed column of Fig. 6.6.2-1 having max. bow at centre $c_0 = 2$ in.

To find M_A , M_B and M_C . $EI = 8.68 \times 10^6$.

From tabular summary 6.6.1, Case (3):

$$M_A = -j^2 (w + 2Pk) \left[1 - \frac{U}{2} \cot \frac{U}{2} \right]$$

$$j^2 = \frac{EI}{P} = \frac{8.68 \times 10^6}{6000} = 1448$$

$$M_A = -1448 (10 + 15) [1 - .602]$$

$$= -3.62 \times 10^4 \times .398$$

$$= -14400$$

$$M_C = j^2 (w + 2Pk) \left[\frac{U}{2} \operatorname{cosec} \frac{U}{2} - 1 \right]$$

$$= 3.62 \times 10^4 [1.212 - 1]$$

$$= 3.62 \times 10^4 \times .212$$

$$= \underline{7670}$$

$$k = \frac{4c_0}{L^2} = \frac{4 \times 2}{6400} = 12.5 \times 10^{-4}$$

$$2Pk = 2 \times 6000 \times 12.5 \times 10^{-4} = 1.2 \times 12.5 = 15$$

$$\frac{U}{2} = \frac{L}{2j} = \frac{80}{76.2} = 1.051 \text{ radians} = 60.2 \text{ degrees}$$

$$\cot \frac{U}{2} = .5727$$

$$\frac{U}{2} \cot \frac{U}{2} = 1.051 \times .5727$$

$$= .602$$

$$\operatorname{cosec} \frac{U}{2} = 1.1524$$

$$\frac{U}{2} \operatorname{cosec} \frac{U}{2} = 1.051 \times 1.1524$$

$$= 1.212$$

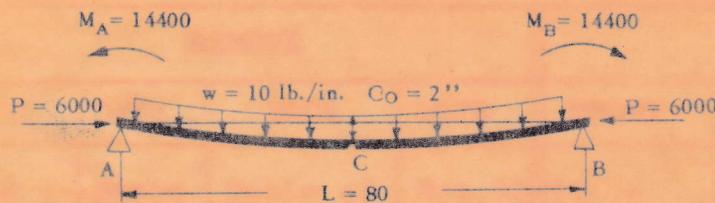
This value checks with that obtained from the Polar Diagram in 6.6.3.

6.6.3 Polar Diagrams

The method of drawing the polar diagram for a column with small initial bow is shown below.

The derivation is in "Aircraft Engineering", Oct. 1952 p. 288 "Solutions of Problems on the Slightly Curved Beam" by Ratzersdorfer.

CONSTANT I: COMPRESSIVE END LOAD. UNIFORMLY DISTRIBUTED LATERAL LOAD
(AGGRAVATING INITIAL BOW) + RELIEVING END MOMENTS



GIVEN DATA: Initial bow $c_0 = 2.0$ in.

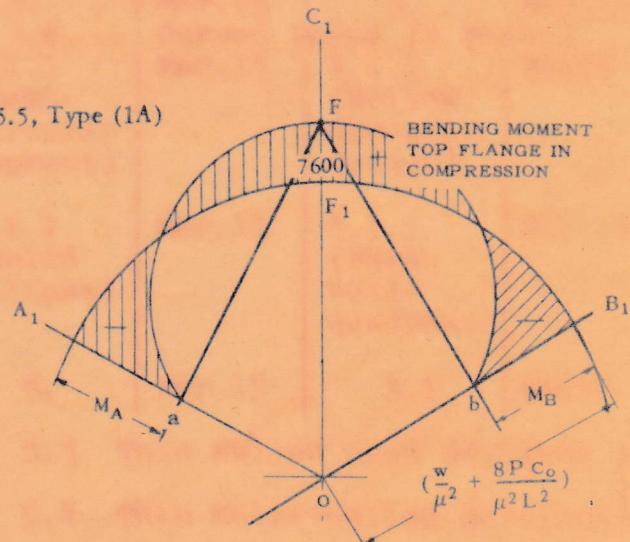
$$\left. \begin{aligned} EI &= 8.68 \times 10^6 \\ \mu^2 &= 6.9 \times 10^{-4} \\ \frac{w}{\mu^2} &= 14500 \\ \mu\alpha &= 60 \text{ degrees} \end{aligned} \right\} \text{From Section 6.5.5, Type (1A)}$$

$$8Pc_0 = 8 \times 6000 \times 2 = 9.6 \times 10^4$$

$$\mu^2 L^2 = 6.9 \times 10^{-4} \times 6400 = 4.42$$

$$\frac{8Pc_0}{\mu^2 L^2} = \frac{9.6 \times 10^4}{4.42} = 2.175 \times 10^4$$

$$\left(\frac{w}{\mu^2} + \frac{8Pc_0}{\mu^2 L^2} \right) = 14500 + 21750 = \underline{36250}$$



Procedure

As for 6.5.5, Type IC except that an arc of radius $\left(\frac{w}{\mu^2} + \frac{8Pc_0}{\mu^2 L^2} \right)$, and not $\frac{w}{\mu^2}$, is struck from O

If w acts so as to relieve the initial bow, strike an arc from O of radius $\left(\frac{w}{\mu^2} - \frac{8Pc_0}{\mu^2 L^2} \right)$

If M_A or M_B is an aggravating moment the position of a and b will be changed accordingly.

B.M. at C is $FF_1 = 7650$ lb. in.

March 15, 1954

AIRCRAFT ENGINEERING MANUAL

VOLUME I (DESIGN)

AMENDMENT LIST S-5

Herewith the following new and revised pages which should be inserted in your copies of Section V, Stress. Pages superseded are to be removed and destroyed.

REMOVE				INSERT			
FRONT	DATE	BACK	DATE	FRONT	DATE	BACK	DATE
Title	Basic	Table of	Oct. 30	Title	Basic	Table of	Mar. 15
Page		Contents		Page		Contents	
3.	Basic	3.0	Basic	3.	Mar. 15	3.	Mar. 15
4.1.1	Basic	4.1.1	Basic	3.6	Mar. 15	Curved Beams (5 Pages)	
(Semi-		(Hollow		4.1.1	Mar. 15	4.1.1	Basic
Circular		Circular		(Semi-		(Hollow	
Segment)		Sector)		Circular		Circular	
				Segment)		Sector)	
4.1.1	Basic	4.1.1	Basic	4.1.1	Mar. 15	4.1.1	Mar. 15
(Solid		(Para-		(Solid		(Para-	
Ellipse)		bolic		Ellipse)		bolic	
		quadrant)				quadrant)	
5.	Basic	5.1	Basic	5.	Mar. 15	5.1	Basic
				5.3		Thin walled open Sections	} 14 Pages
				5.4		Thin multi-walled Sections	

This list should be inserted in the back of the Manual as a record of amendments incorporated.



C.L. Moon,
STANDARDS SUPERVISOR.

December 14, 1953

AIRCRAFT ENGINEERING MANUAL

VOLUME I (DESIGN)

AMENDMENT LIST S-4

Herewith the following new sub-sections for Section V, Stress, comprising 10 pages:

- Torsion
- 5.1 Solid Sections
- 5.2 Hollow Sections

The Table of Contents for Section V will be altered at the next amendment.

Because of the steadily increasing size of the Stress section, those copies which are now in the general Engineering Manual loose-leaf binders are to be transferred to Accopress-type binders which are available on demand from Engineering Services.

This list should be inserted in the back of the Manual as a record of amendments incorporated.



C.L. Moon
STANDARDS SUPERVISOR

October 30, 1953

AIRCRAFT ENGINEERING MANUAL

VOLUME I (DESIGN)

AMENDMENT LIST S-3

Herewith revised Table of Contents for Section V, Stress, dated October 30, 1953, and the following new sub-sections, comprising 34 pages:

- 3.1 Simply Supported Beams
- 3.2 Cantilever Beams
- 3.3 Fixed Beams
- 3.4 Propped Cantilevers
- 3.5 Continuous Beams

This list should be inserted in the back of the Manual as a record of amendments incorporated.



C.L. Moon
STANDARDS SUPERVISOR

March 10, 1953.

AIRCRAFT ENGINEERING MANUAL

VOLUME I (DESIGN)

AMENDMENT LIST NO. S-1

Herewith the following Table of Contents and sub-sections which form the initial issue of Section V, the Stress section of the Engineering Manual.

Table of Contents

Sub-section	2.2.2	Plane Tension Field Beams
	2.2.3	Plane Tension Field Beams with Access Holes.
	2.2.4	Curved Web Field Beams

This amendment list should be inserted in the back of the Manual as a record of amendments incorporated.



C.L. Moon
Standards Supervisor