A. V ROE CANADA LIMITED
MALTON - ONTARIO

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CONTROL MASS CONTRIBUTION TO HINGE MOMENT

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AVRO AIRCRAFT LIMITED MALTON - ONTARIO TECHNICAL DEPARTMENT		REPORT NO. 71/F.A.R./4 SHEET NO. 1	
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CONTROL MASS CONTRIBUTION TO HINGE MOMENT

INTRODUCTION

The contribution of the control mass to the hinge moment is in three parts.

1. due to the aircraft velocitise and accelerations.
2. due to the control movement relative to the aircraft and
3. due to gravity.

The contributions due to 1. and 2. can be conveniently expressed together as the inertia contribution. The report is divided into two sections:-

Section 1. deals with the special case of steady manoeuvres thus:
Horizontal circle with zero sideslip.
Steady sideslip
1-"g" barrel roll.

Section 2. deals with the general case of accelerations and velocities along and about all aircraft axes and the aircraft with any orientation in space.

To the hinge moment applied must be added the hinge moment due to gravity, and subtracted the hinge moment due to inertia. The resultant hinge moment is minus the hinge moment due to aerodynamic forces i.e.

H (applied) + H (gravity) - H (inertia) + H (aerodynamic) = 0

CONTROL MASS CONTRIBUTING TO HINGE MOMENT

NOTATION

Unless stated to the contrary the notation is the same as that in P/Stability/132 and in addition

δ Control deflection, positive to port for the rudder, down for the elevator, down for the starboard aileron and up for the port aileron.

k_{AY}; k_{EY} ; k_{RZ} Radii of gyration of the aileron, elevator and rudder about their respective hinge lines.

$k_{A_{XY}}$; $k_{E_{XY}}$ Products of inertia of the aileron and elevator respectively with zero deflection divided by their mass. The products are measured about the hinge line, and the normal to the hinge line in the plane of the wing, intersecting the hinge line in the plane of symmetry. The positive direction of the hinge line is away from the plane of symmetry, and of the normal aft.

$k_{R_{XZ}}$ Product of inertia of the rudder with zero deflection divided by its mass. The product is measured about the hinge line, and the normal to the hinge line in the plane of the fin, intersecting the hinge line in the aircraft OXY plane.

CONTROL MASS CONTRIBUTING TO

HINGE MOMENT

The positive direction of the hinge line is away from the aircraft OXY plane, and of the normal aft.

ℓ	Distance from hinge line to control cg.
x_{O_R}	Coordinate of the intersection of the rudder hinge line with the aircraft OX axis, w.r.t. the -OX aircraft axis.
x_{O_A} ; z_{O_A} x_{O_E} ; z_{O_E}	Coordinates of the intersection of the hinge line with the plane of symmetry w.r.t. aircraft axes with the OX axis reversed.
H	Hinge moment, positive nose up for the elevator clockwise from above for the rudder, nose up on the starboard aileron and nose down on the port aileron.
Γ	Wing anhedral angle
χ	Angle of sweepback of hinge line
n_n	Normal acceleration

SUFFICES

A	Aileron
E	Elevator
R	Rudder

Note that the foot, slug, second, radian system should be used throughout.

SECTION 1

(i) HORIZONTAL CIRCLE - ELEVATOR HINGE MOMENT

$\dot{V}_1 = \dot{\alpha}_1 = \beta_1 = \beta_1 = p_1 = \dot{q}_1 = \dot{r}_1 = 0$; $p_1 = -\frac{n_n g}{V_1} \sin\Theta . \sin\phi_1$; $q_1 = \frac{n_n g}{V_1} . \cos\Theta . \sin^2\phi_1$; $r_1 = \frac{n_n g}{V_1} . \cos\Theta \cos\phi_1 . \sin\phi_1$

STBD' ELEVATOR

$$\frac{H(\text{INERTIA})}{m_E} = -n_n g . \cos\Theta . \sin^2\phi_1 \{ \cos\alpha_1 (\cos\Gamma . \cos\delta_E + \sin\chi_E \sin\Gamma . \sin\delta_E) - \sin\alpha_1 . \cos\chi_E . \sin\delta_E \} \ell_E$$

$$- \frac{n_n^2 g^2}{V_1^2} \cos^2\Theta . \cos\phi_1 . \sin^3\phi_1 \{ - z_{o_E} (\sin\chi_E \cos\Gamma \sin\delta_E - \sin\Gamma . \cos\delta_E) \ell_E$$

$$+ (\sin\chi_E . \cos 2\Gamma . \cos 2\delta_E + [1 + \sin^2\chi_A] \frac{\sin 2\Gamma}{2} \sin 2\delta_A) k_{E_Y}^2$$

$$- (\cos\chi_E . \cos 2\Gamma . \cos\delta_E + \sin 2\dot{\chi}_E . \frac{\sin 2\Gamma}{2} \sin\delta_E) k_{E_{XY}} \}$$

$$+ \frac{n_n^2 g^2}{V_1^2} . \cos\Theta . \sin\Theta . \sin^3\phi_1 \{ x_{o_E} (\sin\chi_E . \cos\Gamma \sin\delta_E - \sin\Gamma . \cos\delta_E) \ell_E$$

$$+ (\frac{\sin 2\chi_E}{2} . \cos\Gamma . \sin 2\delta_E - \cos\chi_E . \sin\Gamma . \cos 2\delta_E) k_{E_Y}^2 - (\cos 2\chi_E . \cos\Gamma . \sin\delta_E + \sin\chi_E . \sin\Gamma . \cos\delta_E) k_{E_{XY}} \}$$

$$+ \frac{n_n^2 g^2}{V_1^2} \sin\Theta . \cos\Theta . \cos\phi_1 . \sin^2\phi_1 \{ [x_{o_E} (\cos\Gamma . \cos\delta_E + \sin\chi_E . \sin\Gamma . \sin\delta_E) - z_{o_E} \cos\chi_E . \sin\delta_E] \ell_E$$

$$+ (\cos\chi_E . \cos\Gamma . \cos 2\delta_E + \frac{\sin 2\chi_E}{2} . \sin\Gamma . \sin 2\delta_E) k_{E_Y}^2$$

$$+ (\sin\chi_E . \cos\Gamma . \cos\delta_E - \cos 2\chi_E . \sin\Gamma . \sin\delta_E) k_{E_{XY}} \}$$

$$- \frac{n_n^2 g^2}{V_1^2} \sin^2\Theta . \sin^2\phi_1 \{ z_{o_E} (\cos\Gamma \cos\delta_E + \sin\chi_E \sin\Gamma . \sin\delta_E) \ell_E$$

$$+ (-\sin\chi_E \frac{\sin 2\Gamma}{2} + \cos^2\chi_E . \frac{\sin 2\delta_E}{2}) k_{E_Y}^2 + \frac{\sin 2\chi_E}{2} . \sin\delta_E . k_{E_{XY}} \}$$

$$- \frac{n_n^2 g^2}{V_1^2} \cos^2\Theta \cos^2\phi_1 . \sin^2\phi_1 \{ - x_{o_E} \cos\chi_E . \sin\delta_E . \ell_E$$

$$+ ([\sin^2\chi_E . \sin^2\Gamma - \cos^2\Gamma] \frac{\sin 2\delta_E}{2} - \sin\chi_E . \frac{\sin 2\Gamma}{2} . \cos 2\delta_E) k_{E_Y}^2$$

$$- (\frac{\sin 2\chi_E}{2} . \sin^2\Gamma . \sin\delta_E + \cos\chi_E . \frac{\sin 2\Gamma}{2} . \cos\delta_E) k_{E_{XY}} \}$$

$$- \frac{n_n^2 g^2}{V_1^2} \cos^2\Theta . \sin^4\phi_1 \{ (z_{o_E} [\cos\Gamma . \cos\delta_E + \sin\chi_E . \sin\Gamma . \sin\delta_E] - x_{o_E} \cos\chi_E . \sin\delta_E) \ell_E$$

$$- (\sin\chi_E . \frac{\sin 2\Gamma}{2} . \cos 2\delta_E + [\sin^2\Gamma - \sin^2\chi_E \cos^2\Gamma] \frac{\sin 2\delta_E}{2}) k_{E_Y}^2$$

$$+ (\cos\chi_E \frac{\sin 2\Gamma}{2} . \cos\delta_E - \frac{\sin 2\chi_E}{2} . \cos^2\Gamma . \sin\delta_E) k_{E_{XY}} \}$$

$$+ n_n g \sin\phi_1 (\sin\Theta . \sin\alpha_1 + \cos\Theta . \cos\phi_1 \cos\alpha_1) (\sin\chi_E . \cos\Gamma . \sin\delta_E - \sin\Gamma . \cos\delta_E) \ell_E$$

THE ABOVE EXPRESSION CAN BE APPROXIMATED TO

$$\frac{H(\text{INERTIA})}{m_E} \doteq -n_n g \sin\phi_1 \{ \cos\Theta . \sin\phi_1 [\cos\alpha_1 (\cos\Gamma \cos\delta_E + \sin\chi_E . \sin\Gamma . \sin\delta_E) - \sin\alpha_1 \cos\chi_E \sin\delta_E]$$

$$- (\sin\Theta . \sin\alpha_1 + \cos\Theta . \cos\phi_1 . \cos\alpha_1)(\sin\chi_E . \cos\Gamma . \sin\delta_E - \sin\Gamma . \cos\delta_E) \} \ell_E$$

& SHOULD BE ACCURATE TO WITHIN 1%

$$\frac{H(\text{GRAVITY})}{m_E} = +g\{(\sin\overline{\phi_1+\Gamma} . \cos\Theta . \sin\chi_E - \sin\Theta . \cos . \chi_E)\sin\delta_E + \cos\Theta . \cos\overline{\phi_1+\Gamma} . \cos\delta_E\}\ell_E$$

PORT ELEVATOR

$$\frac{H(\text{INERTIA})}{m_E} \text{ APPROXIMATE VALUE } = -n_n g \sin\phi_1 \{ \cos\Theta . \sin\phi_1 [\cos\alpha_1 (\cos\Gamma . \cos\delta_E + \sin\chi_E . \sin\Gamma \sin\delta_E) - \sin\alpha_1 \cos\chi_E . \sin\delta_E]$$

$$+ (\sin\Theta \sin\alpha_1 + \cos\Theta . \cos\phi_1 . \cos\alpha_1)(\sin\chi_E . \cos\Gamma . \sin\delta_E - \sin\Gamma . \cos\delta_E)\} \ell_E$$

$$\frac{H(\text{GRAVITY})}{m_E} = -g\{(\sin\overline{\phi_1-\Gamma} . \cos\Theta . \sin\chi_E + \sin\Theta . \cos\chi_E)\sin\delta_E - \cos\overline{\phi_1-\Gamma} . \cos\Theta . \cos\delta_E\}$$

NOTE THAT IF THE PORT & STBD' HINGE MOMENTS ARE ADDED THE INERTIA CONTRIBUTION BECOMES EVEN SIMPLER

(ii) STEADY SIDESLIP - RUDDER HINGE MOMENT

$\dot{V}_1 = \alpha_1 = \dot{\beta}_1 = p_1 = q_1 = r_1 = \dot{p}_1 = \dot{q}_1 = \dot{r}_1 = 0$

THERE IS NO INERTIA CONTRIBUTION TO THE HINGE MOMENT

$$\frac{H(\text{GRAVITY})}{m_R} = -g\left\{(\cos\Theta.\cos\phi_1 \sin\chi_R + \sin\Theta.\cos\chi_R)\sin\delta_R + \cos\Theta\sin\phi_1.\cos\delta_R\right\}$$

(iii) 1"g" BARREL ROLL - AILERON HINGE MOMENT

$V_1 = \alpha_1 = \beta_1 = \dot{\beta}_1 = q_1 = r_1 = \dot{p}_1 = \dot{q}_1 = \dot{r}_1 = 0$

STBD' AILERON

$$\frac{H(\text{INERTIA})}{m_A} = -p_1^2\left\{z_{0_A}(\cos\Gamma.\cos\delta_A + \sin\chi_A.\sin\Gamma\sin\delta_A)\ell_A\right.$$
$$\left.+\left(-\sin\chi_A.\frac{\sin 2\Gamma}{2} + \cos^2\chi_A\frac{\sin 2\delta_A}{2}\right)k_{A_y}^2 + \frac{\sin 2\chi_A}{2}.\sin\delta_A k_{A_{xy}}\right\}$$

$$\frac{H(\text{GRAVITY})}{m_A} = g\left\{(\sin\overline{\phi_1+\Gamma}.\cos\Theta\sin\chi_A - \sin\Theta\cos\chi_A)\sin\delta_A + \cos\Theta.\cos\overline{\phi_1+\Gamma}.\cos\delta_A\right\}\ell_A$$

PORT AILERON

$$\frac{H(\text{INERTIA})}{m_A} = p_1^2\left\{z_{0_A}(\cos\Gamma.\cos\delta_A - \sin\chi_A.\sin\Gamma\sin\delta_A)\ell_A\right.$$
$$\left.-\left(\frac{\sin 2\Gamma}{2}.\sin\chi_A + \cos^2\chi_A.\frac{\sin 2\delta_A}{2}\right)k_{A_y}^2 - \frac{\sin 2\chi_A}{2}\sin\delta_A.k_{A_{xy}}\right\}$$

$$\frac{H(\text{GRAVITY})}{m_A} = -g\left\{(\sin\overline{\phi_1-\Gamma}\cos\Theta.\sin\chi_A + \sin\Theta.\cos\chi_A)\sin\delta_A + \cos\overline{\phi_1-\Gamma}.\cos\Theta.\cos\delta_A\right\}\ell_A$$

NOTE THAT IF $\delta_{A.PORT} = \delta_{A.STBD'}$ OR BOTH ARE SMALL THEN THE SUM OF THE INERTIA TERMS INVOLVES ONLY TERMS IN $\sin\delta_E$.