



A. V. ROE CANADA LIMITED

MALTON - ONTARIO

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT:

REPORT NO. 71/F.A.R./5

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CONTROL SURFACE HINGE MOMENT DERIVATIVES FROM FLIGHT TESTS

PREPARED BY V. Baddeley

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CONTROL SURFACE HINGE MOMENT DERIVATIVES FROM FLIGHT TESTS

INTRODUCTION AND ASSUMPTIONS

Some hinge moment derivatives may be obtained by steady state tests, and provided that the information is accurate enough, further derivatives can be obtained from oscillation tests.

The STEADY STATE tests should be as follows.

Elevator

Horizontal banked circles at steady speed with zero sideslip, or pull ups. The former alternative is the most practical, and the equations in the form presented are equally applicable to spiral dives at constant V.E.A.S. or mach number provided that the effects of longitudinal acceleration are ignored.

The variables are, α , δ_A , q , r , and p , & to obtain the derivatives with respect to all of these parameters and in addition C_H would require at least 6 different normal accelerations. However the effects of p & r will be small and consequently will be assumed zero. q is also likely to be small unless high normal accelerations are used, but its effect should be included if only to improve the accuracy of the other derivatives.

A minimum of 4 different normal accelerations is required for each flight case.

Both sides of the elevator should be analyzed separately so that a mean can be taken to eliminate any stray lateral effects present.

Aileron

1 "S" barrel rolls.

The parameters influencing the hinge moment will be α , δ_A and p . α will be a constant for a given flight case.

It would appear that 4 different rates of roll will be required for each flight case.

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Aileron .. continued..

However by alternately adding and subtracting the contributions of each aileron the number of r.o.r.'s can be reduced to 2. This implies the assumption that $C_{H\delta}$ & $C_{H\alpha}$ are independent of δ_A .

Rudder

Steady Sideslip

The parameters influencing the hinge moment will be β, δ_R and α . To solve for the derivatives with respect to the above variables and $C_{H\delta}$ will require at least 4 different sideslip angles.

It should be noted that the effect of α is really non linear but it is thought that a linear approximation will be of some value.

It is clear from the above that to solve for the steady state derivatives for each control surface at least 10 different manoeuvres are needed for each flight case.

The OSCILLATION should be carried out from steady level flight for simplicity's sake, and will provide approximate values for an additional 2 derivatives per control.

SUMMARY

The following derivatives can be obtained

<u>Control Surface</u>	<u>Steady State Tests</u>	<u>Oscillation Tests</u>
Elevator	$C_{H\delta}, C_{H\alpha}, C_{H\delta_R}, C_{H\alpha_R}$	$C_{H\beta}, C_{H\dot{\alpha}}$
Ailerons	$C_{H\delta}, C_{H\alpha}, C_{H\delta_A}, C_{H\alpha_A}$	$C_{H\beta}, C_{H\dot{\alpha}}$
Rudder	$C_{H\delta}, C_{H\alpha}, C_{H\delta_R}, C_{H\alpha_R}$	$C_{H\beta}, C_{H\dot{\alpha}}$

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NOTATION

In the oscillation section the notation of Ref. 4 is used, but in the steady state section since all velocities and angles are steady state values, the suffix 1 is omitted.

In addition

C_H Non dimensional hinge moment coefficient
hinge moment / $q S c_s$

$C_H (C)$ Hinge moment coefficient applied by the aircraft control system, corrected for inertia and gravity effects

$C_H (A)$ Hinge moment coefficient applied by the aircraft control system.

$C_{H\alpha}$ $\partial C_H / \partial \alpha$

C_{Hq} $\partial C_H / \partial \frac{q c}{2 V_i}$

$C_{H\dot{\alpha}}$ $\partial C_H / \partial \frac{\dot{\alpha} c}{2 V_i}$

$C_{H\delta_s}$ $\partial C_H / \partial \delta_s$

$C_{H\beta}$ $\partial C_H / \partial \beta$

C_{Hp} $\partial C_H / \partial \frac{p b}{2 V_i}$

$C_{H\dot{p}}$ $\partial C_H / \partial \frac{\dot{p} b}{2 V_i}$

C_{Hr} $\partial C_H / \partial \frac{r b}{2 V_i}$

$C_{H\dot{\beta}}$ $\partial C_H / \partial \frac{\dot{\beta} b}{2 V_i}$

C_{H_0} Hinge moment coefficient for all angles, velocities (except forward speed) and accelerations zero.

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NOTATION

S_S	control surface area
c_S	control surface mean chord
$\bar{c}$	aircraft mean aerodynamic chord
n_n	normal acceleration
n_l	lateral acceleration
$\bar{q}$	$\frac{\rho V_1^2}{2}$
Γ	wing anhedral angle
χ_S	sweepback of control surface hinge line
δ_S	control surface deflection
T	$m_S / \rho S_S V_1$
ρ	air density
$\dot{\psi}$	rate of yaw in horizontal plane
l_S	distance of control surface cg to hinge line
k_{EY}^2, k_{EX}^2	defined in Ref. 1
k_{AY}^2, k_{AX}^2	
k_{RZ}^2, k_{RX}^2	
x_{OR}	
x_{OA}, x_{OE}	
e_{CA}, e_{CE}	

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Notation ... continued...

- ξ general disturbance angle, velocity, or acceleration of the oscillation.
 ξ_0 amplitude of the envelope of the oscillation at some mean time.
 ξ phase lead of ξ , relative to α in the longitudinal oscillation, β in the lateral oscillation.

Other parameters are defined in the section to which they are appropriate.

Note that the foot, slug second, radian system should be used throughout.

SUFFICES

A	Aileron
E	Elevator
R	Rudder
S	Control Surface
PT	Port
ST	Starboard

SIGN CONVENTION

The sign convention for control surface deflections and hinge moments are defined in Ref. 1.

REFERENCES

- 71/F.A.R./4 Control surface mass contribution to hinge moment - V. Baddeley
- 71/STAB/5 Digital computer determination of lateral derivatives from oscillatory flight tests - M. Jenkins
- 71/STAB/6 Digital computer determination of longitudinal derivatives from oscillatory flight test - M. Jenkins
- P/STAB/132 Dynamic equations relative to body axes - M. Jenkins

STEADY STATE TESTS

THE HINGE MOMENT, CORRECTED FOR INERTIA & GRAVITY EFFECTS BY THE EQUATIONS OF REF. 1 SHOULD FIRST BE OBTAINED. IN THIS SECTION SINCE ALL VELOCITIES & ANGLES ARE STEADY STATE, NO SUFFIXES WILL BE USED TO DENOTE THIS.

ELEVATOR

$$0 = C_H(C) + C_{H_0} + \alpha C_{H_{\alpha}} + \delta_E C_{H_{\delta_E}} + \frac{qE}{2V} C_{H_q}$$

$$\text{NOW } z' = \frac{\pi \rho g}{V} \sin \phi$$

$$q = z' \cos \Theta \sin \phi = \frac{\pi \rho g}{V} \cos \Theta \sin^2 \phi = Q \frac{2V}{E}$$

$$\therefore -C_H(C) = C_{H_0} + \alpha C_{H_{\alpha}} + \delta_E C_{H_{\delta_E}} + Q C_{H_q}$$

4 DIFFERENT CASES SUFFIXES 1 TO 4 :

$$C_{H_0} = \frac{\begin{vmatrix} -C_H(C)_1 & \alpha_1 & \delta_{E1} & Q_1 \\ -C_H(C)_4 & \alpha_4 & \delta_{E4} & Q_4 \end{vmatrix}}{\Delta} = \nabla(C_{H_0})/\Delta$$

SIMILARLY

$$C_{H_{\alpha}} = \nabla(C_{H_{\alpha}})/\Delta ; C_{H_{\delta_E}} = \nabla(C_{H_{\delta_E}})/\Delta ; C_{H_q} = \nabla(C_{H_q})/\Delta$$

WHERE

$$\begin{aligned} \Delta &= \alpha_2(\delta_{E3} Q_4 - \delta_{E4} Q_3) - \alpha_3(\delta_{E2} Q_4 - \delta_{E4} Q_2) + \alpha_4(\delta_{E2} Q_3 - \delta_{E3} Q_2) \\ &\quad - \alpha_1(\delta_{E3} Q_4 - \delta_{E4} Q_3) + \alpha_3(\delta_{E1} Q_4 - \delta_{E4} Q_1) - \alpha_4(\delta_{E1} Q_3 - \delta_{E3} Q_1) \\ &\quad + \alpha_1(\delta_{E2} Q_4 - \delta_{E4} Q_2) - \alpha_2(\delta_{E1} Q_4 - \delta_{E4} Q_1) + \alpha_4(\delta_{E1} Q_2 - \delta_{E2} Q_1) \\ &\quad - \alpha_1(\delta_{E2} Q_3 - \delta_{E3} Q_2) + \alpha_2(\delta_{E1} Q_3 - \delta_{E3} Q_1) - \alpha_3(\delta_{E1} Q_2 - \delta_{E2} Q_1) \\ \nabla(C_{H_0}) &= -C_H(C)_1 \{ \alpha_1(\delta_{E3} Q_4 - \delta_{E4} Q_3) - \alpha_3(\delta_{E2} Q_4 - \delta_{E4} Q_2) + \alpha_4(\delta_{E2} Q_3 - \delta_{E3} Q_2) \} \\ &\quad + C_H(C)_2 \{ \alpha_1(\delta_{E3} Q_4 - \delta_{E4} Q_3) - \alpha_3(\delta_{E1} Q_4 - \delta_{E4} Q_1) + \alpha_4(\delta_{E1} Q_3 - \delta_{E3} Q_1) \} \\ &\quad - C_H(C)_3 \{ \alpha_1(\delta_{E2} Q_4 - \delta_{E4} Q_2) - \alpha_2(\delta_{E1} Q_4 - \delta_{E4} Q_1) + \alpha_4(\delta_{E1} Q_2 - \delta_{E2} Q_1) \} \\ &\quad + C_H(C)_4 \{ \alpha_1(\delta_{E2} Q_3 - \delta_{E3} Q_2) - \alpha_2(\delta_{E1} Q_3 - \delta_{E3} Q_1) + \alpha_3(\delta_{E1} Q_2 - \delta_{E2} Q_1) \} \end{aligned}$$

$$\begin{aligned} \nabla(C_{H_x}) = & -C_{H_2}(C_2)(\delta_{E_3}Q_4 - \delta_{E_4}Q_3) + C_{H_3}(C_3)(\delta_{E_2}Q_4 - \delta_{E_4}Q_2) - C_{H_4}(C_4)(\delta_{E_2}Q_3 - \delta_{E_3}Q_2) \\ & + C_{H_1}(C_1)(\delta_{E_3}Q_4 - \delta_{E_4}Q_3) - C_{H_3}(C_3)(\delta_{E_1}Q_4 - \delta_{E_4}Q_1) + C_{H_4}(C_4)(\delta_{E_1}Q_3 - \delta_{E_3}Q_1) \\ & - C_{H_1}(C_1)(\delta_{E_2}Q_4 - \delta_{E_4}Q_2) + C_{H_2}(C_2)(\delta_{E_1}Q_4 - \delta_{E_4}Q_1) - C_{H_4}(C_4)(\delta_{E_1}Q_2 - \delta_{E_2}Q_1) \\ & + C_{H_1}(C_1)(\delta_{E_2}Q_3 - \delta_{E_3}Q_2) - C_{H_2}(C_2)(\delta_{E_1}Q_3 - \delta_{E_3}Q_1) + C_{H_3}(C_3)(\delta_{E_1}Q_2 - \delta_{E_2}Q_1) \end{aligned}$$

$$\begin{aligned} \nabla(C_{H_y}) = & \alpha_2(C_{H_4}Q_3 - C_{H_3}Q_4) - \alpha_3(C_{H_4}Q_2 - C_{H_2}Q_4) + \alpha_4(C_{H_3}Q_2 - C_{H_2}Q_3) \\ & - \alpha_1(C_{H_4}Q_3 - C_{H_3}Q_4) + \alpha_3(C_{H_4}Q_1 - C_{H_1}Q_4) + \alpha_4(C_{H_3}Q_1 - C_{H_1}Q_3) \\ & + \alpha_1(C_{H_4}Q_2 - C_{H_2}Q_4) - \alpha_2(C_{H_4}Q_1 - C_{H_1}Q_4) + \alpha_4(C_{H_3}Q_1 - C_{H_1}Q_2) \\ & - \alpha_1(C_{H_3}Q_2 - C_{H_2}Q_3) + \alpha_2(C_{H_3}Q_1 - C_{H_1}Q_3) - \alpha_3(C_{H_2}Q_1 - C_{H_1}Q_2) \end{aligned}$$

$$\begin{aligned} \nabla(C_{H_q}) = & \alpha_2(\delta_{E_4}C_H(C_3) - \delta_{E_3}C_H(C_4)) - \alpha_3(\delta_{E_4}C_H(C_2) - \delta_{E_2}C_H(C_4)) + \alpha_4(\delta_{E_3}C_H(C_2) - \delta_{E_2}C_H(C_3)) \\ & - \alpha_1(\delta_{E_4}C_H(C_3) - \delta_{E_3}C_H(C_4)) + \alpha_3(\delta_{E_4}C_H(C_1) - \delta_{E_1}C_H(C_4)) - \alpha_4(\delta_{E_3}C_H(C_1) - \delta_{E_1}C_H(C_3)) \\ & + \alpha_1(\delta_{E_4}C_H(C_2) - \delta_{E_2}C_H(C_4)) - \alpha_2(\delta_{E_4}C_H(C_1) - \delta_{E_1}C_H(C_4)) + \alpha_4(\delta_{E_3}C_H(C_1) - \delta_{E_1}C_H(C_2)) \\ & - \alpha_1(\delta_{E_3}C_H(C_2) - \delta_{E_2}C_H(C_3)) + \alpha_2(\delta_{E_3}C_H(C_1) - \delta_{E_1}C_H(C_2)) - \alpha_3(\delta_{E_2}C_H(C_1) - \delta_{E_1}C_H(C_2)) \end{aligned}$$

RUDDER

$$O = C_H(C) + C_{H_0} + \alpha C_H + \delta_R C_{H\delta_R} + \beta C_{H\beta}$$

HAVE A DIFFERENT CASES

IT WILL BE NOTICED THAT THE ABOVE EXPRESSION IS ALMOST IDENTICAL WITH THAT FOR THE ELEVATORS & IF

$$\begin{aligned} \delta_R & \text{ IS READ FOR } \delta_E \\ \beta & = 1 \quad \text{ " } Q \\ C_{H\beta} & = 1 \quad \text{ " } C_{Hq} \end{aligned}$$

THE EQUATIONS FOR THE ELEVATOR DERIVATIVES CAN BE USED FOR THE RUDDER

AILERONS

$$0 = C_H(C) + C_{H_0} + \alpha C_{H_\alpha} + \delta_R C_{H_{\delta_R}} + \frac{p b}{2V} C_{H_P}$$

IF PORT & STBD HINGE MOMENTS ARE SUMMED

$$0 = C_H(C)_{PT} + C_H(C)_{ST} + C_{H_{\delta_R}}(\delta_{RPT} + \delta_{RST}) + \frac{p b}{V} C_{H_P}$$

PROVIDED THAT THE AILERONS ARE A MIRROR IMAGE OF EACH OTHER IN THE PLANE OF SYMMETRY

NOW SUBTRACTING THE HINGE MOMENTS

$$0 = C_H(C)_{PT} - C_H(C)_{ST} + 2(C_{H_0} + \alpha C_{H_\alpha}) + C_{H_{\delta_R}}(\delta_{RPT} - \delta_{RST})$$

2 DIFFERENT RATES OF ROLL, SUFFICES 1 & 2

$$C_{H_{\delta_R}} = \frac{[C_H(C)_{PT} - C_H(C)_{ST}]_2 P_1 - [C_H(C)_{PT} + C_H(C)_{ST}]_1 P_2}{[(\delta_{RPT} + \delta_{RST})_1 P_2 - (\delta_{RPT} + \delta_{RST})_2 P_1]}$$

$$\frac{1}{V} C_{H_P} = \frac{[(\delta_{RPT} + \delta_{RST})_2 (C_H(C)_{PT} + C_H(C)_{ST})_1 - (\delta_{RPT} + \delta_{RST})_1 (C_H(C)_{PT} + C_H(C)_{ST})_2]}{[(\delta_{RPT} + \delta_{RST})_1 P_2 - (\delta_{RPT} + \delta_{RST})_2 P_1]}$$

THE VALUE OF $C_{H_{\delta_R}}$ CALCULATED ABOVE WILL NOW BE USED IN THE ESTIMATION OF C_{H_0} & C_{H_α}

$$0 = \frac{C_H(C)_{PT} - C_H(C)_{ST} + C_{H_{\delta_R}}(\delta_{RPT} - \delta_{RST})}{2} + C_{H_0} + \alpha C_{H_\alpha}$$

$$= R + C_{H_0} + \alpha C_{H_\alpha}$$

$$C_{H_0} = (R_2 \alpha_1 - R_1 \alpha_2) / (\alpha_2 - \alpha_1)$$

$$C_{H_\alpha} = (R_1 - R_2) / (\alpha_2 - \alpha_1)$$

IT SHOULD BE NOTED THAT THE VALUES OF C_{H_0} & C_{H_α} APPLY TO THE PORT AILERON. THE STBD AILERON VALUES ARE NUMERICALLY THE SAME BUT THE SIGN MUST BE REVERSED.

OSCILLATION TESTS

ALL ANGLES, VELOCITIES, ACCELERATIONS, & HINGE MOMENTS ARE DISTURBANCE VALUES FROM THE STEADY STATE UNLESS A SUFFIX 1 IS ATTACHED, IN WHICH CASE THEY ARE STEADY STATE VALUES.

THE TESTS SHOULD BE MADE BY DISTURBANCE FROM STEADY LEVEL FLIGHT, OTHERWISE THE FOLLOWING EQUATIONS DO NOT HOLD

ELEVATOR

SINCE THE ESTIMATE OF $C_{H\dot{\alpha}}$ FROM THE STEADY STATE IS NOT EXPECTED TO BE RELIABLE IT WILL BE RE-ESTIMATED HERE.

$$0 = C_H(C) + \alpha C_{H\alpha} + \frac{g \bar{c}}{2V_1} C_{H\dot{q}} + \frac{\dot{\alpha} \bar{c}}{2V_1} C_{H\ddot{\alpha}}$$

$$\dot{\alpha} - \dot{q} = -(\eta_n + \theta \sin \Theta) \frac{g}{V_1}$$

$$\therefore 0 = C_H(C) + \alpha C_{H\alpha} + \frac{g \bar{c}}{2V_1} (C_{H\dot{q}} + C_{H\ddot{\alpha}}) - \eta_n \frac{\bar{c} g}{2V_1^2} C_{H\alpha} - \theta (\sin \Theta) \frac{g \bar{c}}{2V_1^2} C_{H\ddot{\alpha}}$$

NOW FROM REF 1

$$C_H(C) = C_H(A) + \frac{m_E V_1}{\bar{q} S_E C_E} \left(I_1 \frac{g}{V_1} (\eta_n + \theta \sin \Theta) - I_2 \alpha \frac{g}{V_1} + I_3 \frac{\dot{q}}{V_1} - \theta W_1 \frac{g}{V_1} \right)$$

THE RESULTING EQN. FOR THE HINGE MOMENT DERIVATIVES CAN BE EXPRESSED THUS:

$$0 = C_H(A) + \alpha C_{H\alpha} + \eta_n k_1 (k_2 - C_{H\ddot{\alpha}}) + \dot{q} k_3 (k_4 + C_{H\dot{q}} + C_{H\ddot{\alpha}}) + \dot{q} k_5 k_6 - \theta k_1 (k_7 + \sin \Theta (C_{H\ddot{\alpha}} - k_2))$$

THE AMPLITUDE & PHASING OF THE OSCILLATION SHOULD BE OBTAINED BY THE METHOD INDICATED IN REF. 3. THE PHASING IS MEASURED W.R.T. α . AFTER EQUATING REAL & IMAGINARY PARTS OF THE ABOVE & SOLVING THE RESULTING EQNS.

$$C_{H\dot{\alpha}} = (c_1 b_2 - c_2 b_1) / (a_1 b_2 - a_2 b_1)$$

$$C_{H\ddot{\alpha}} = (a_1 c_2 - a_2 c_1) / (a_1 b_2 - a_2 b_1)$$

WHERE

$$\begin{aligned} -c_1 &= |C_H(A)_0| \sin |C_H(A)| + k_1 k_2 |\eta_n| \sin |\eta_n| + k_3 k_4 |\dot{q}_0| \sin |\dot{q}| + k_5 k_6 |\dot{q}_0| \sin |\dot{q}| \\ &\quad - k_1 (k_7 - k_2 \sin \Theta) |\Theta_0| \sin |\Theta| \end{aligned}$$

$$-C_2 = |C_H(A_0)| \cos(C_H(F)) + C_{H_2} |\alpha_0| + k_1 k_2 |n_{n_0}| \cos(r_T) + k_3 k_4 |q_0| \cos(q_-) \\ + k_5 k_6 |q_0| \cos(q_+) - k_1 (k_2 - k_2 \sin \Theta) |B_0| \cos \Theta$$

$$C_1 = k_3 |q_0| \sin(q_-) - k_1 (|n_{n_0}| \sin(r_T) + |B_0| \sin \Theta \sin \Theta)$$

$$a_2 = k_3 |q_0| \cos(q_+) \quad k_1 |n_{n_0}| \cos(r_T) + |B_0| \cos \Theta \sin \Theta$$

$$b_1 = k_3 |q_0| \sin(q_-)$$

$$b_2 = k_3 |q_0| \cos(q_+)$$

$$k_1 = \frac{q_0}{2V_1^2} ; k_2 = \frac{4\pi E I_1}{\rho \delta_E c_E L} ; k_3 = \frac{c}{2V_1} ; k_4 = -\frac{4m_E I_2 \alpha_1}{\rho \delta_E c_E L} ; k_5 = \frac{c T}{2V_1}$$

$$k_6 = \frac{4I_3}{c_E L} ; k_7 = \frac{4m_E W_1}{\rho \delta_E c_E L} ; T = \frac{m_E}{\rho \delta_E V_1}$$

$$I_1 = (\cos \Gamma \cos \delta_E + \sin \chi_E \sin \Gamma \sin \delta_E) \ell_E$$

$$I_2 = \cos \chi_E \sin \delta_E \ell_E$$

$$I_3 = -\{ I_1 x_{0E} + I_2 z_{0E} + \cos \chi_E \cos \Gamma \cdot k_{EY}^2 \\ + (\sin \chi_E \cos \Gamma \cos \delta_E + \sin \Gamma \sin \delta_E) k_{EXY} \}$$

$$W_1 = \{ (\sin \chi_E \sin \Gamma \sin \Theta + \cos \chi_E \cos \Theta) \sin \delta_E + \cos \Gamma \sin \Theta \cos \delta_E \} \ell_E$$

ALL THE ABOVE EQNS. APPLY EQUALLY WELL TO BOTH SIDES OF THE ELEVATOR

AILERONS

$$0 = C_H(C) + \beta C_{H_P} + \frac{\rho b}{2V_1} C_{H_P} + \frac{\rho b \tau}{2V_1} C_{H_P}$$

STBY AILERON

$$C_H(C) = C_H(A) + \frac{m_A}{\rho S_A c_A} (-g(n_e + \phi \cos \Theta) I_1 + \rho I_2 + i I_3 + \rho g W_1)$$

&

$$0 = C_H(A) + \beta C_{H_P} + n_e k_1 k_2 + \rho k_3 C_{H_P} + \rho k_4 (k_5 + C_{H_P}) + \phi k_1 (k_6 + k_7) + i k_4 k_8$$

THE AMPLITUDE & PHASING OF THE OSCILLATION SHOULD BE OBTAINED BY THE METHOD OF REF 2. THE PHASING IS MEASURED W.R.T β . AFTER EQUATING REAL & IMAGINARY PARTS OF THE ABOVE & SOLVING THE RESULTING EQNS,

$$C_{H_P} = a_1 / b_1$$

$$C_{H_P} = c_2 / a_2 - b_2 c_1 / a_2 b_1$$

WHERE

$$-c_1 = |C_H(A)| \sin |C_H(A)| + k_1 k_2 |n_e| \sin |n_e| + k_3 C_{H_P} |\rho| \sin |P| + k_4 k_5 |\dot{\rho}| \sin |P| \\ + k_1 (k_6 + k_7) |\dot{\phi}| \sin |\phi| + k_4 k_8 |\dot{i}| \sin |i|$$

$$-c_2 = |C_H(A)| \cos |C_H(A)| + k_1 k_2 |n_e| \cos |n_e| + k_3 C_{H_P} |\rho| \cos |P| + k_4 k_5 |\dot{\rho}| \cos |P| \\ + k_1 (k_6 + k_7) |\dot{\phi}| \cos |\phi| + k_4 k_8 |\dot{i}| \cos |i|$$

$$a_1 = 0 \quad ; \quad b_1 = k_4 |\dot{\rho}| \sin |P|$$

$$a_2 = |\dot{\rho}| \quad ; \quad b_2 = k_4 |\dot{\rho}| \cos |P|$$

$$k_1 = \frac{g b}{2 V_1^2} \quad ; \quad k_2 = \frac{-4 m_A I_1}{\rho S_A c_A b} \quad ; \quad k_3 = \frac{b}{2 V_1} \quad ; \quad k_4 = \frac{b \tau}{2 V_1} \quad ; \quad k_5 = \frac{4 I_2}{c_A b} \quad ; \quad k_6 = \frac{4 m_A W_1}{\rho S_A c_A b}$$

$$k_7 = \frac{-4 m_A \cos \Theta I_1}{\rho S_A c_A b} \quad ; \quad k_8 = \frac{4 I_3}{c_A b} \quad ; \quad \tau = \frac{m_A}{\rho S_A V_1}$$

$$I_1 = (\sin \chi_A \cos \Gamma \sin \delta_A - \sin \Gamma \cos \delta_A) e_A$$

$$I_2 = (Z_{O_A} I_1 + \sin \chi_A k_{A Y}^2 - \cos \chi_A \cos \delta_A k_{A X Y})$$

$$I_3 = (X_{O_A} I_1 - \cos \chi_A \sin \Gamma k_{A Y}^2 + (\cos \Gamma \sin \delta_A - \sin \chi_A \sin \Gamma \cos \delta_A) k_{A X Y})$$

$$W_1 = (\sin \chi_R \cos \Gamma \cos \Theta \sin \delta_R + \sin \Gamma \cos \Theta \cos \delta_R) \ell_R$$

PORT AILERON

WITH THE FOLLOWING VALUES FOR I & W THE EQNS. FOR $C_{H\dot{\beta}}$ & $C_{H\beta}$ ARE THE SAME AS FOR THE STARBOARD AILERON.

$$I_1 = -(\sin \chi_R \cos \Gamma \sin \delta_R + \sin \Gamma \cos \delta_R) \ell_R$$

$$I_2 = z_{0R} I_1 + \sin \chi_R k_{Ry}^2 - \cos \chi_R \cos \delta_R k_{Rxy}$$

$$I_3 = x_{0R} I_1 - \cos \chi_R \sin \Gamma k_{Ry}^2 - (\cos \Gamma \sin \delta_R + \sin \chi_R \sin \Gamma \cos \delta_R) k_{Rxy}$$

$$W_1 = -(\sin \chi_R \cos \Gamma \cos \Theta \sin \delta_R + \sin \Gamma \cos \Theta \cos \delta_R) \ell_R$$

RUDDER

$$0 = C_H(C) + \beta C_{H\beta} + \frac{3}{2V_1}(C_{H\alpha} - C_{H\beta}) + \left(\frac{3}{V_1}(\eta_e + \phi \cos \Theta) + \rho \alpha\right) \frac{b}{2V_1} C_{H\beta}$$

$$C_H(C) = C_H(A) + \frac{m_k}{2S_R C_R} (g(\eta_e + \phi \cos \Theta) I_1 - i I_2 - \rho I_3 - g \phi W_1)$$

$$\begin{aligned} 0 = & C_H(A) + \beta C_{H\beta} + \eta_e k_1 (k_2 + C_{H\beta}) + \rho k_3 \alpha_1 C_{H\beta} + \dot{\rho} k_4 k_5 + k_3 (C_{H\alpha} - C_{H\beta}) + i k_4 k_6 \\ & + \phi k_1 (k_2 \cos \Theta + k_7 + C_{H\beta} \cos \Theta) \end{aligned}$$

THE AMPLITUDE & PHASING SHOULD BE OBTAINED BY THE METHOD OF REF. 2. THE PHASING IS MEASURED W.R.T. β . AFTER EQUATING IMAGINARY & REAL PARTS OF THE ABOVE & SOLVING THE RESULTING EQUATIONS:

$$C_{H\beta} = (c_1 \underline{U}_2 - c_2 \underline{U}_1) / (a_1 \underline{U}_2 - a_2 \underline{U}_1)$$

$$C_{H\alpha} = (a_1 c_2 - a_2 c_1) / (a_1 \underline{U}_2 - a_2 \underline{U}_1)$$

WHERE

$$\begin{aligned} -c_1 = & |C_H(A)_0| \sin |C_H(A)| + k_1 k_2 |\tau_e| \sin |\underline{m}_e| + k_4 k_5 |\dot{\rho}_0| \sin |\dot{\rho}| + k_4 k_6 |\dot{\tau}_0| \sin |\dot{\tau}| \\ & + k_1 (k_2 \cos \Theta + k_7) |\phi_0| \sin |\phi| \end{aligned}$$

$$\begin{aligned} -c_2 = & |C_H(A)_0| \cos |C_H(A)| + C_{H\beta} |\beta_0| + k_1 k_2 |\tau_e| \cos |\underline{m}_e| + k_4 k_5 |\dot{\rho}_0| \cos |\dot{\rho}| \\ & + k_4 k_6 |\dot{\tau}_0| \cos |\dot{\tau}| + k_1 (k_2 \cos \Theta + k_7) |\phi_0| \cos |\phi| \end{aligned}$$

$$a_1 = k_1 |\eta_e| \sin |\underline{m}_e| + k_3 \alpha_1 |\rho_0| \sin |\rho| - k_3 |\tau_0| \sin |\underline{\tau}| + k_1 \cos \Theta |\phi_0| \sin |\phi|$$

$$a_2 = k_1 |\eta_e| \cos |\underline{m}_e| + k_3 \alpha_1 |\rho_0| \cos |\rho| - k_3 |\tau_0| \cos |\underline{\tau}| + k_1 \cos \Theta |\phi_0| \cos |\phi|$$

$$\underline{U}_1 = k_3 |\tau_0| \sin |\underline{\tau}| ; \underline{U}_2 = k_3 |\tau_0| \cos |\underline{\tau}|$$

$$\tau_1 = \frac{b}{2V_1} V_1^2 ; k_2 = 4 I_1 m_k / \rho S_R C_R b ; k_3 = \frac{b}{2V_1} ; k_4 = \frac{b}{2V_1}$$

$$k_5 = -4 I_3 / C_R b ; k_6 = -4 I_2 / C_R b ; k_7 = -4 m_k W_1 / \rho S_R C_R b$$

$$I_1 = \cos \delta_R \cdot \ell_R$$

$$I_2 = \cos \chi_R k_{Rz}^2 + \cos \delta_R (x_{OR} \ell_R + \sin \chi_R k_{Rxz})$$

$$I_3 = \sin \chi_R k_{Rz}^2 - \cos \delta_R \cos \chi_R k_{Rxz}$$

$$W_1 = \cos \Theta \cdot \cos \delta_R \cdot \ell_R$$

NOTE THAT IN USING p, i, p & μ , THE CORRECTED VALUES NOT THE TRACE VALUES SHOULD BE USED. THE CORRECTED VALUES CAN BE OBTAINED BY THE METHOD INDICATED IN REF. 2.