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DIGITAL COMPUTER DETERMINATION OF LATERAL DERIVATIVES FROM OSCILLATORY FLIGHT TESTS

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INTRODUCTION

This report gives the details of a method for determining the main lateral aerodynamic derivatives from analysis of lateral flight oscillations.

The method may be adapted to a digital computer programme.

It is assumed that the reader is fully conversant with time vector analysis of oscillatory flight which is described in references 2 and 3.

INTRODUCTION TO DIGITAL ANALYSIS OF OSCILLATORY FLIGHT TESTS

From the method of vector analysis described in references 1, and 2, it is known that if the modulus and phase relationships of the incremental variables are known together with the damping and frequency of the oscillation, the main derivatives governing the characteristics of a short period oscillation may be determined.

Until the present time all this information has come from the measurement of the timing and shrinkage of peak values on recorded traces. However this procedure suffers from a number of disadvantages:

1. - Peak values only are utilised

2. - Peak values are obtained by visual interpolation.

3. - An insufficient number of peaks during zero change in equilibrium speed.

4. - Extensive man hours required for conscientious measurement.

These disadvantages have been eliminated by a digital computer best fit curve process which digests every point from which a sinusoidal trace would be formed.

The process is applied to each incremental variable yielding accurately the required relationships.

From the above it may be gathered that the digital computer best fit curve process makes possible a more accurate time vector analysis. However from experience of the method of time vector analysis of Free Flight Model results, this is not considered desirable mainly because of the extensive man-hours required to operate the method and because an alternative method outlined below may be said to have very largely eliminated human error.

In the time vector method the complex relationships of the incremental variables are expressed in polar form; however the alternative is to express these complex relationships in Cartesian co-ordinate form. Hence each equation of motion may be split into a real and an imaginary domain and the solution will yield the 'unknown' values of aerodynamic derivatives. This is analogous to the time vector analysis; however the important point is that the linearised equations of motion although expressed in complex form have been retained in digital notation which may be manipulated by a digital computer.

This makes possible a comprehensive digital computer programme. The input is edited digitalised flight recording on tape and the output is evaluated aerodynamic derivatives.

Visual comprehension of the dynamic motion is the main advantage of the time vector diagrams. This may be retained by drawing out the solutions of the computer programme in time vector diagram form for selected flight cases.

All the main and auxilliary equations of motion solved by the computer programme are given in this report. However all the details of the digital computing programme are given in ref.

References:

1. Dynamic Equations Relative to Body Axes
P/Stab./132 M. V. Jenkins

2. Time Vector Analysis of Free Flight Model Lateral Results
P/Stab./135 M. V. Jenkins

3. Time Vector Analysis of Free Flight Model Longitudual Results
P/Stab./140 M. V. Jenkins

4. Computing Department Report No. A.27
J. Shoosmith

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Note on notation

All symbols used in this report are those in current use in Avro for a body axes system unless designated otherwise in the text. The Avro notation which in the main is that accepted by N. A. C. A. is found in reference 1. This reference also contains the derivation of the linear equations of motion which form the basis of this report.

Prior to the solution of the lateral equations of motion certain relationships must be determined from the following measured values.

β	Angle of sideslip
ϕ	Angle of Bank
p_t	Rate of roll
$\dot{p}_t$	Roll acceleration
r_t	yaw rate
$\dot{r}_t$	yaw acceleration
n_y	transversal acceleration
q	rate of pitch
α	Angle of incidence
Θ	Angle of pitch

In the linearised equations all the incremental variables are assumed of the form

$$y = A_1 e^{K_1 t} + A_2 e^{K_2 t} \cos (wt + \alpha')$$

A_1 is the steady state value of the variable and $e^{K_1 t}$ is the decay factor of the steady state value

A_2 is the oscillatory increment modulus of which $e^{K_2 t}$ is the decay factor

ω is the frequency of the oscillation in radians per sec.

α' phase displacement angle

NOTATION

Let Z be the complex relation between any incremental variable and incremental β

EXAMPLES

$$Z_\phi = \frac{|\phi|}{|\beta|}\angle\phi = x_\phi + j y_\phi \quad , \quad Z_\phi |\beta| = \phi$$

WHERE $\angle\phi$ IS THE PHASE LEAD OF ϕ ON β

AND $x_\phi = \frac{|\phi|}{|\beta|}\cos\angle\phi \qquad y_\phi = \frac{|\phi|}{|\beta|}\sin\angle\phi$

$$Z_{\lambda\phi} = \frac{|\lambda||\phi|}{|\beta|}\angle\lambda\phi = x_{\lambda\phi} + j y_{\lambda\phi} \, , \, Z_{\lambda\phi}|\beta| = p$$

WHERE $\angle\lambda\phi$ IS THE PHASE LEAD OF p ON β

AND $x_{\lambda\phi} = \frac{|\lambda||\phi|}{|\beta|} . \cos\angle\lambda\phi \qquad y_{\lambda\phi} = \frac{|\lambda||\phi|}{|\beta|}\sin\angle\lambda\phi$

$$\lambda = K + j\omega$$

$$Z_{\lambda\phi} = (K + j\omega)(x_\phi + j y_\phi)$$

$$\therefore \; x_{\lambda\phi} = \frac{|\lambda||\phi|}{|\beta|}\cos\angle\lambda\phi = K . x_\phi - \omega . y_\phi = x_p$$

$$y_{\lambda\phi} = \frac{|\lambda||\phi|}{|\beta|}\sin\angle\lambda\phi = \omega . x_\phi + K y_\phi = y_p$$

$$Z_p = Z_{\lambda\phi} = x_{\lambda\phi} + j y_{\lambda\phi}$$

The following relationships are now determined from the computer curve fitting process.

For convenience $\frac{g}{V} n_Y$ is denoted A

$$A_{2\beta} = |\beta|$$

$$\frac{A_2\,\phi}{A_{2\beta}} = \frac{|\phi|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'_{\phi} = \angle\phi \longrightarrow Z_{\phi}$$

$$\frac{A_2\,p_t}{A_{2\beta}} = \frac{|p_t|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'\,p_t = \angle p_t \longrightarrow Z_{p_t}$$

$$\frac{A_2\,\dot{p}_t}{A_{2\beta}} = \frac{|\dot{p}_t|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'\,\dot{p}_t = \angle\dot{p}_t \longrightarrow Z^{\circ}_{\ddot{p}_t}$$

$$\frac{A_2\,r_t}{A_{2\beta}} = \frac{|r_t|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'\,r_t = \angle r_t \longrightarrow Z_{r_t}$$

$$\frac{A_2\,\dot{r}_t}{A_{2\beta}} = \frac{|\dot{r}_t|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'_{\dot{r}_t} = \angle\dot{r}_t \longrightarrow \mathring{Z}_{\ddot{r}_t}$$

$$\frac{A_2\,A}{A_{2\beta}} = \frac{|A|}{|\beta|} \qquad \alpha'_{\beta} - \alpha'\,A = \angle A \longrightarrow Z_A$$

$$q_{1,} = A_1\,q$$

$$\alpha_{1,} = A_1\,\alpha$$

$$\beta_{1,} = A_1\,\beta$$

DETERMINATION OF AVERAGE λ

$\lambda = k + \omega i$ complex root relevant to short period oscillation

For greater accuracy an average λ is used in the solution of the equations.

$$\lambda = \frac{1}{7}\left\{\lambda_{\beta} + \lambda_{\phi} + \lambda_{pt} + \lambda_{\dot{p}t} + \lambda_{rt} + \lambda_{\dot{r}t} + \lambda_{A}\right\}$$

SOLUTION OF THE LATERAL EQUATIONS

x_r

Let transverse acceleration/V be denoted A

$$A = \dot{\beta} - p\,\alpha_1 + r\cos\alpha_1 - \phi\,\frac{g}{V}\,\cos\phi_1\,\cos\beta_1\cos\text{Ⓗ}$$

$$r\cos\alpha_1 = -\dot{\beta} + p\,\alpha_1 + \phi\,.\,\frac{g}{V}\,\cos\phi_1\cos\beta_1\cos\text{Ⓗ} + A \;\ldots\ldots\ldots\; (1)$$

Splitting the equation into complex form.

$$x_r\ \cos\alpha_1 = -x_{\dot{\beta}} + x_p\alpha_1 + x_\phi\,.\,\frac{g}{V}\,.\cos\phi_1\cos\beta_1\cos\text{Ⓗ} + x_A\ldots(2)$$

A check on this equation is that $x_r \triangleq -|\lambda||\beta|$

$$y_r\ \cos\alpha_1 = -y_{\dot{\beta}} + y_p\,\alpha_1 + y_\phi\,\frac{g}{V}\,.\cos\phi_1\cos\beta_1\cos\text{Ⓗ} + y_A\ldots(3)$$

However the value of y_r may be obtained more accurately in the next stage.

<u>This stage is only for the obtaining of the phase of r accurately and hence x_r is extracted.</u>

<u>y_r</u>

The yaw accelerometer measures $\dot{r}_t$

$$\dot{r} = \dot{p}\,\frac{K_{xz}}{K^2_z} - q_1 \cdot \left|\frac{K^2_y - K^2_x}{K^2_z}\right| p - \frac{K_{xz}}{K^2_z} \cdot q_1 \cdot r + \dot{r}_t \text{ ----------(4)}$$

Expressing the equation in complex form.

$$Z_{\dot{r}} = Z_{\dot{p}}\,\frac{K_{xz}}{K^2_z} - q_1 \left|\frac{K^2_y - K^2_x}{K^2_z}\right| Z_p \;\frac{-K_{xz}}{K^2_z} \cdot q_1 \cdot Z_r + Z_{\dot{r}_t}$$

Negligible accuracy is lost by the substitution

of $Z_{\dot{p}} = Z_{\dot{p}_t}$; $Z_p = Z_{p_t}$; $Z_r = Z_{r_t}$

$$x_{\dot{r}} = x_{\dot{p}}\,\frac{K_{xz}}{K^2_z} - q_1 \left|\frac{K^2_y - K^2_x}{K^2_z}\right| x_p \;\frac{-K_{xz}}{K^2_z} \cdot q_1 \cdot x_r + x_{\dot{r}_t} \text{ ---------(5)}$$

$$y_{\dot{r}} = y_{\dot{p}}\,\frac{K_{xz}}{K^2_z} - q_1 \left|\cdot\frac{K^2_y - K^2_x}{K^2_z}\right| \cdot y_p \;\frac{-K_{xz}}{K^2_z} \cdot q_1 \cdot y_r + y_{\dot{r}_t} \text{ --------(6)}$$

$$|r| = \frac{\sqrt{(x_{\dot{r}})^2 \quad (y_{\dot{r}})^2}}{|\lambda|}$$

$y_r = -\sqrt{|r|^2 - (x_r)^2}$, x_r from page 10

x_r and y_r are now established

Hence $Z_{\dot{r}} = \lambda Z_r = \lambda^2 Z_{\psi'}$

where $\psi' = \int_{t_1}^{t_2} r \, dt$

DETERMINATION OF Z_ϕ , Z_p , $Z_{\dot{p}}$

Substituting Z_r , $Z_{\dot{r}}$ into equation (4)

$$Z_{r_t} = Z_{\dot{r}} - \frac{K\,x\,z}{K^2 z} Z_{\dot{p}} + q_1 \frac{K^2 y - K^2 x}{K^2 z} Z_p + q_1 \frac{K\,x\,z.r}{K^2 z}$$

Negligible accuracy is lost by substituting into this equation

$Z_{\dot{p}} = Z_{\dot{p}_t}$ and $Z_p = Z_{p_t}$

$$x_{\dot{r}_t} = x_{\lambda r} - \frac{K\,x\,z}{K^2 z} x_{\dot{p}_t} + q_1 \frac{K^2 y - K^2 x}{K^2 z} x_{p_t} + \frac{K\,x\,z}{K^2 z} q_1\; x_r \;\ldots\ldots \quad (7)$$

$$y_{\dot{r}_t} = y_{\lambda r} - \frac{K\,x\,z}{K^2 z} y_{\dot{p}_t} + q_1 \frac{K^2 y - K^2 x}{K^2 z} y_{p_t} + \frac{K\,x\,z}{K^2 z} q_1\; y_r \;\ldots\ldots \quad (8)$$

$$\angle \dot{r}_t = \tan^{-1} \frac{y_{\dot{r}_t}}{x_{\dot{r}_t}}$$

Hence $\dot{r}_t$, and $\angle \dot{r}_t - 90 - \varepsilon_D = \angle r_t$ are now correctly known

where $(90 + \varepsilon_D) = 90 + \tan^{-1}\left\{\frac{-k}{\omega}\right\}$

Let $\angle \dot{P}_t$ be the averaged value

$$\angle \dot{P}_t = 1/3\left\{\left(\angle \dot{r}_t + \angle \dot{P}_t - \dot{r}_t\right) + \left(\angle \dot{r}_t + \angle P_t - r_t\right) + \angle \dot{P}_t\right\}$$

where $\angle \dot{P}_t - \dot{r}_t$, $\angle P_t - r_t$ are measured values.

Accepting the measured value of $\frac{|\dot{P}_t|}{|\beta|}$

$Z_{\dot{p}_t} = \frac{|\dot{P}_t|}{|\beta|} . \angle \dot{P}_t$ now becomes a known.

DETERMINATION OF Z_{ϕ}, Z_p, $Z_{\dot p}$ (Continued)

Roll accelerometer measures $\dot{P}_t$

$$-\dot{p}_t = -\dot{p} + q_1 \frac{K_{xz}}{K^2_x} . p - r q_1 \frac{K^2_z - K^2_y}{K^2_x} + \frac{K_{xz}}{K^2_x} . \dot{r}$$

$$Z_{\dot p} = Z_{\dot p_t} + q_1 \frac{K_{xz}}{K^2_x} Z_{\lambda\phi_t} - r q_1 \frac{K^2_z - K^2_y}{K^2_x} + \frac{K_{xz}}{K^2_x} . \dot{r}$$

Splitting this into real and imaginary equations.

$$x_{\dot p} = x_{\dot p_t} + q_1 \frac{K_{xz}}{K^2_x} x_{\lambda\phi_t} - q_1 \frac{K^2_z - K^2_y}{K^2_x} . x_r - \frac{K_{xz}}{K^2_x} . x_{\dot r}$$

$$y_{\dot p} = y_{\dot p_t} + q_1 \frac{K_{xz}}{K^2_x} . y_{\lambda\phi_t} - q_1 \frac{K^2_z - K^2_y}{K^2_x} . y_r + \frac{K_{xz}}{K^2_x} . y_{\dot r}$$

Hence $Z_{\dot p}$ is now known

And using the relationships $\lambda^2 . Z_{\phi} = Z_{\dot p}$

$$\lambda . Z_{\phi} = Z_p$$

Z_{ϕ}, Z_p, $Z_{\dot p}$ are now known.

All the required lateral variables have now been expressed in terms of β and may be substituted into the equations of motion.

NOTATION

$a = -\dfrac{C_{Y_\beta}}{2\tau}$ $b = -\dfrac{C_{Y_p}}{4\mu_2}$

$c = -\dfrac{g}{V} \cos \phi_1 \cos \beta_1 \cos \textcircled{H}$ $d = \dfrac{-C_{Y_r}}{4\mu_2}$

$e = -\dfrac{C_{l_\beta}}{i_x}$ $f = \dfrac{-C_{l_p}}{i_x} \cdot \dfrac{b}{2V}$

$g = -\dfrac{K_{xz}}{K^2x}$ $h = \dfrac{-C_{l_r}}{i_x} \cdot \dfrac{b}{2V}$

$j = \dfrac{K^2z - K^2_Y}{K^2x}$ $k' = \dfrac{-C_{n_\beta}}{i_z}$

$l = \dfrac{K^2_Y - K^2x}{K^2z}$ $m = -\dfrac{K\,x\,z}{K^2z}$

$n = \dfrac{-C_{n_p}}{i_z} \cdot \dfrac{b}{2V}$ $s = \dfrac{-C_{n_r}}{i_z} \cdot \dfrac{b}{2V}$

LATERAL EQUATIONS OF MOTION

$$(a + \lambda) + (b - \alpha_1) Z_p + C Z_\phi + (\cos \alpha_1 + d) Z_r = 0$$

$$e + (f + q_1 g) Z_p + \dot{Z}_p + (h + q_1 j) Z_r + g \dot{Z}_r = 0$$

$$k' + (q_1 l + n) Z_p + m \dot{Z}_p + (s - q_1 m) Z_r + \dot{Z}_r = 0$$

Unless specified otherwise assume b = 0

Find a d e f n k'
(all other coefficients are known)

Convert $\dfrac{-C_{y_\beta}}{2\tau}$ $\dfrac{-C_{y_r}}{4\mu_2}$ $\dfrac{-C_{l_\beta}}{i_x}$ $\dfrac{-C_{l_p}}{i_x} \cdot \dfrac{b}{2V}$ $\dfrac{-C_{n_p}}{i_z} \dfrac{b}{2V}$ $\dfrac{-C_{n_\beta}}{i_z}$

By notation to C_{y_β} C_{Y_r} C_{l_β} C_{l_p} C_{n_p} C_{n_β}

CHECKING PROGRAMME

Solve $A'\lambda^4 + B'\lambda^3 + C'\lambda^2 + D'\lambda + E' = 0$

$A' = 1 - gm$

$B' = -mag - mh - q_1 mj - q_1 lg - gn + a + s - q_1 m + f + gq_1$

$C' = as - q_1 am + af + q_1 ag - ahm - q_1 ajm$
$- q_1 agl - agn + me\cos\alpha_1 + med + e\alpha_1$
$- eb - \alpha_1 k'g + bk'g - \cos\alpha_1 k' - dk'$
$- q_1 lh - q_1^2 jl - nh - q_1 jn + fs - q_1 fm$
$+ q_1 gs - gq_1^2 m$

$D' = -k'\alpha_1 h - k'q_1\alpha_1 j + k'bh + jk'bq_1 + k'cg - k'\cos\alpha_1 f$
$- k'q_1 g\cos\alpha_1 - k'df - q_1 k'gd + q_1 el\cos\alpha_1$
$+ en\cos\alpha_1 + q_1 edl + edn - ec - e\alpha_1 q_1 m$
$+ eq_1 bm + e\alpha_1 s - esb - q_1 alh - ljaq_1^2 - anh$
$- q_1 ajn + asf + q_1 ags - q_1^2 agm - q_1 amf$

$E' = ck'h + q_1 ck'j - ces + q_1 cme$

Conclusions

1. Values of C_{y_β}, C_{l_β} C_{n_β}, C_{l_p} C_{n_p} C_{y_r} are determined by the method.

2. Implicit in the solution is that the values of C_{y_p} C_{l_r} C_{n_r} are known. The terms containing C_{y_p} and C_{l_r} are of negligible importance.

3. Evaluation of C_{n_p} determines the resultant component of $(r\,C_{n_r} + p\,C_{n_p})$. The isolated values of C_{n_p} and C_{n_r} cannot be obtained experimentally accurately as sufficient accuracy of measurement of the phase relationship between p and r is unobtainable. Any method of analysis of the Dutch Roll oscillation suffers from the same practical snag. However, if the determined value of C_{n_p} and the 'known' value of C_{n_r} are substituted into the equations of motion the consequent evaluation of response of the aircraft will be accurate because the resultant component $(r\,C_{n_r} + p\,C_{n_p})$ within practical limits remains constant regardless of the isolated values of C_{n_r} and C_{n_p}. C_{n_β} is slightly affected by the breakdown and this is seen by reference to the time vector yawing diagram in reference 2. However, probable values of C_{n_β} have been established by wind tunnel and free flight models, forming a boundary condition or check on the evaluation of the components of the yawing equation of motion.

Conclusions (continued)

4. It will be realized that C_{n_r} as treated in the method absorbs the $Cn_{\dot{\beta}}$ effect. This simplification is considered justified since r and $\dot{\beta}$ are almost in counterphase and isolation of the separate contributions is impossible. Evaluation of the response of the aircraft remains unchanged by the substitution of the resultant component into the equations of motion.
5. The discussed digital computer programmes will greatly reduce the time to evaluate derivatives and largely eliminate human error.
6. The discussion in this report is relevant only to the aircraft in the lateral damper disengaged condition.