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MALTON - ONTARIO

TECHNICAL DEPARTMENT (Aircraft)

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DIGITAL COMPUTER DETERMINATION OF LONGITUDINAL DERIVATIVES FROM OSCILLATORY FLIGHT TESTS

PREPARED BY M.V. Jenkins DATE Oct. 1957.

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INTRODUCTION

This report gives the details of a method for determining the longitudinal aerodynamic derivatives from analysis of longitudinal flight oscillations.

The method may be adopted to a digital computer programme.

It is assumed that the reader is fully converstant with time vector analysis of oscillatory flight which is described in references 2 and 3.

INTRODUCTION TO DIGITAL ANALYSIS OF OSCILLATORY FLIGHT TESTS

From the method of vector analysis described in references 1, and 2, it is known that if the modulus and phase relationships of the incremental variables are known together with the damping and frequency of the oscillation, the main derivatives governing the characteristics of a short period oscillation may be determined.

Until the present time all this information has come from the measurement of the timing and shrinkage of peak values on recorded traces. However this procedure suffers from a number of disadvantages:

1. - Peak values only are utilised

2. - Peak values are obtained by visual interpolation.

3. - An insufficient number of peaks during zero change in equilibrium speed.

4. - Extensive man hours required for conscientious measurement.

These disadvantages have been eliminated by a digital computer best fit curve process which digests every point from which a sinusoidal trace would be formed.

The process is applied to each incremental variable yielding accurately the required relationships.

From the above it may be gathered that the digital computer best fit curve process makes possible a more accurate time vector analysis. However from experience of the method of time vector analysis of Free Flight Model results, this is not considered desirable mainly because of the extensive man-hours required to operate the method and because an alternative method outlined below may be said to have very largely eliminated human error.

In the time vector method the complex relationships of the incremental variables are expressed in polar form; however the alternative is to express these complex relationships in Cartesian co-ordinate form. Hence each equation of motion may be split into a real and an imaginary domain and the solution will yield two 'unknown' values of aerodynamic derivatives. This is analogous to the time vector analysis; however the important point is that the linearised equations of motion although expressed in complex form have been retained in digital notation which may be manipulated by a digital computer.

This makes possible a comprehensive digital computer programme. The input is edited digitalised flight recording on tape and the output is evaluated aerodynamic derivatives.

Visual comprehension of the dynamic motion is the main advantage of the time vector diagrams. This may be retained by drawing out the solutions of the computer programme in time vector diagram form for selected flight cases.

All the main and auxilliary equations of motion solved by the computer programme are given in this report. However all the details of the digital computing programme are given in ref.

References:

1. Dynamic Equations Relative to Body Axes
P/Stab./132 M. V. Jenkins

2. Time Vector Analysis of Free Flight Model Lateral Results
P/Stab./135 M. V. Jenkins

3. Time Vector Analysis of Free Flight Model Longitudual Results
P/Stab./140 M. V. Jenkins

4. Computing Department Report No. A.27
J. Shoosmith

Note on notation

All symbols used in this report are those in current use in Avro for a body axes system unless designated otherwise in the text. The Avro notation which in the main is that accepted by N. A. C. A. is found in reference 1. This reference also contains the derivation of the linear equations of motion which form the basis of this report.

EVALUATION OF LONGITUDINAL DERIVATIVES FROM FLIGHT OSCILLATIONS

Those derivatives which are virtually independent of incremental speed and height changes during short period oscillations are solved by the simultaneous solution of the normal force and pitching equations. This is made possible by the measured interrelationship of the normal acceleration, incidence and pitch variables together with the frequency and damping of the short period oscillation.

The solution will yield $C_{m\alpha}$, $C_{z\alpha}$, $(C_{z\dot{\alpha}} + C_{z_q})$ and $(C_{m\dot{\alpha}} + C_{m_q})$.

However initially the recordings of the α vane will not be accepted until confidence is felt in the corrections to be applied to phase and amplitude of the recordings. Consequently an assumption which is described in the appropriate section concerning the inter-relationship of the incidence and pitch variables is made. This assumption eliminates the experimental determination of $(C_{z\dot{\alpha}} + C_{z_q})$.

In order to find additional longitudinal derivatives with respect to speed an attempt to analyse the phugoid oscillation must be made and incremental speed must be taken into account. Following the principles of the analysis of the short period oscillation, tentatively one would attempt to detect incremental speed through the medium of the longitudinal accelerometer and its relationship to the pitch and normal acceleration variables during the long period oscillation.

However unlike the short period oscillation analysis where several cycles may be examined, the phugoid is unlikely to complete one cycle, since the period in many cases is of the order two minutes, before being subjected to a random disturbance such as a gust or cross-coupling effect due to aileron control. This disturbance will prevent satisfactory determination of the inter-relationship of the longitudinal variables.

In addition to the major consideration above the procurement of a satisfactory longitudinal accelerometer is problematical. Incremental flight path in radians approximately equals incremental normal acceleration. Consequently the maximum value of incremental normal acceleration would be less than 0.1 'g'. The accelerometer would be required not to sense higher frequency accelerations of the same order.

Consequently it appears that the principle of accurately measuring the inter-relationships of the longitudinal variables and substituting these values into the equations of motion to yield the values of aerodynamic derivatives is not possible.

Inclusion of height effects presents a further obstacle in that accurate measurement of heigh variation together with accurate phasing is thought to be unobtainable.

Alternative Method

If the frequency and damping of the phugoid can be satisfactorily measured and the values of the derivatives found from the short period oscillation are substituted into the equations of motion together with the theoretical estimates of lesser important derivatives the inclusion of which is assessed as improving the accuracy of the solution, experimental determination of the speed derivatives is possible.

It is anticipated that measurement of the phugoid damping will be within large tolerances. The consequence of this is that determination of C_x/ M is unreliable. However if the frequency of the phugoid is measured accurately $\partial C_z/\partial M$ and $\partial C_M/\partial M$ values will be reliable.

A series of checks utilising Free Flight Model data yielded the following results. Varying height effects were included.

% error in measuring amplitude ratio over one period	$\partial C_x/\partial M$	$\partial C_z/\partial M$	$\partial C_M/\partial M$
-58.7	-.0274	.0969	.0249
-51.26	-.0106	.0979	.0249
-15.28	+.0195	.0996	.0249
0	+.0250	.0999	.0249
+72.95	+.0382	.1006	.0249
+107.9	+.0412	.1008	.0249

Height Effect In The Phugoid

For accurate determination of the additional derivatives which have a significant bearing on the characteristics of the phugoid oscillation, the effects of incremental height and speed changes during the long period oscillation should be simultaneously introduced. A kinematic relation equation has to be introduced equating incremental $\bar{h}$ to the other variables.

The solution of the equations of motion now involves a stability quintic the roots of which are the short period complex pairs together with a real root which physically will be recognised as a slow convergence or divergence in height relative to equilibrium height.

If the following derivatives with respect to height are calculated and substituted into the equations of motion the resultant determination of the speed derivatives will be more accurate.

$$C_{x_{\bar{h}}} = \frac{\partial C_x}{\partial M} \cdot \frac{\partial M}{\partial \bar{h}} \qquad \frac{\partial \rho}{\partial \bar{h}}$$

$$C_{z_{\bar{h}}} = \frac{\partial C_z}{\partial M} \cdot \frac{\partial M}{\partial \bar{h}} \qquad \frac{\partial T}{\partial \bar{h}}$$

$$C_{m_{\bar{h}}} = \frac{\partial C_m}{\partial M} \cdot \frac{\partial M}{\partial \bar{h}}$$

NOTATION

Let Z be the complex relation between any incremental variable and incremental α.

EXAMPLES

$$q = Z_q = \frac{|q|}{|\alpha|} . |\alpha| \underline{|q} = x_q + iy_q$$

where $\underline{|q}$ is the phase lead of q on α

and $x_q = \frac{|q|}{|\alpha|} . |\alpha| \cos \underline{|q}$ $\qquad y_q = \frac{|q|}{|\alpha|} . |\alpha| \sin \underline{|q}$

$$\dot{q} = \lambda q = Z_{\lambda q} = \frac{|\lambda||q|}{|\alpha|} . |\alpha| \underline{|\lambda q} = x_{\lambda q} + iy_{\lambda q}$$

where $\underline{|\lambda q}$ is the phase lead of $\dot{q}$ on α

and $x_{\lambda q} = \frac{|\lambda||q|}{|\alpha|} |\alpha| \cos \underline{|\lambda q}$ $\qquad y_{\lambda q} = \frac{|\lambda||q|}{|\alpha|} . |\alpha| \sin \underline{|\lambda q}$

$$\lambda = K + i\omega$$

$$\lambda q = Z_{\lambda q} = (K + i\omega)(x_q + iy_q)$$

$$\therefore \quad x_{\lambda q} = \frac{|q||\lambda|}{|\alpha|} . |\alpha| \text{Cos} \underline{|\lambda q} = (Kx_q - \omega y_q) = x_{\dot{q}}$$

$$y_{\lambda q} = \frac{|\lambda||q|}{|\alpha|} . |\alpha| \text{Sin} \underline{|\lambda q} = (Ky_q + \omega x_q) = y_{\dot{q}}$$

$$\dot{\alpha} = \lambda\alpha \qquad \dot{q} = \lambda q$$

SHORT PERIOD OSCILLATION

Instrument phasing errors are such that certain justified assumptions must be made to replace unobtainable accuracy in measurement.

The normal acceleration equation is used for obtaining $\angle q$. Unfortunately it calls for unobtainable accuracy to avoid the assumption that normal acceleration is other than in phase with α.

This assumption having been made, the equations are now framed so that $C_{Z_{\dot{\alpha}}} + C_{Z_q} \triangleq 0$ since $|\alpha| \triangleq |q|$ and the phase lead is small. Should confirmed phase measurement between normal acceleration and α arise this additional fact may be used to determine $(C_{Z_{\dot{\alpha}}} + C_{Z_q})$.

In order to avoid error in the recordings of the pitch functions the following assumptions are made.

$$|\theta| = \frac{1}{3|\lambda|}\left(|\lambda||\theta_t| + |q_t| + \frac{|\dot{q}_t|}{|\lambda|}\right)$$

$$|q| = \frac{1}{3}\left(|\lambda||\theta_t| + |q_t| + \frac{|\dot{q}_t|}{|\lambda|}\right)$$

$$|\dot{q}| = \frac{|\lambda|}{3}\left(|\lambda||\theta_t| + |q_t| + \frac{|\dot{q}_t|}{|\lambda|}\right)$$

Notes on the analysis of the longitudinal short period oscillation are given on the following page.

There are two alternative values of $|\dot{\alpha}|$ which may be accepted viz $|\lambda||\alpha_t|$ or $|q|$. However initially it will be assumed that $|\dot{\alpha}| = |q|$ is more reliable than $|\lambda||\alpha_t|$.

The justification for the assumption $|\dot{\alpha}| = |q|$ is given in P/Stab/140.

For the purposes of finding $\underline{|q}$, Z_q is treated entirely as an unknown.

$$Z_q \cos \alpha_1 = + Z_N + Z_{\dot{\alpha}} \cos \alpha_1 + Z_\alpha q_1 \sin \alpha_1 + Z_\theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1$$

$$Z_q = + Z_N \sec. \alpha_1 + Z_{\dot{\alpha}} + Z_\alpha q_1 \tan \alpha_1 + Z_\theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1 \sec \alpha_1$$

For Z_θ use should be made of the best average result from the following relationships.

$$Z_{\theta_t} = \frac{Z_{q_t}}{\lambda} = \frac{Z_{\dot{q}_t}}{\lambda^2} \qquad Z_\alpha = \frac{Z_{\dot{\alpha}}}{\lambda}$$

$$x_q = + x_N \sec \alpha_1 + x_{\dot{\alpha}} + x_\alpha q_1 \tan \alpha_1 + x_\theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1 \sec \alpha_1$$

$$y_q = + y_N \sec \alpha_1 + y_{\dot{\alpha}} + y_\alpha q_1 \tan \alpha_1 + y_\theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1 \sec \alpha_1$$

$$\underline{|q} = \tan^{-1} \frac{y_q}{x_q} \quad \underline{|\theta} = \underline{|q - \tfrac{\pi}{2} - \varepsilon_D} \quad \underline{|\dot{q}} = \underline{|q + \tfrac{\pi}{2} + \varepsilon_D}$$

Hence Z_α, $Z_{\dot{\alpha}}$, Z_θ, Z_q and $Z_{\dot{q}}$ have now become knowns.

These functions may now be used in the O Z and pitching equations.

LONGITUDINAL SHORT PERIOD OSCILLATION EQUATIONS

The following OZ and Pitching Moment equations are considered to adequately determine the characteristics of the longitudual short period oscillation.

The O Z Equation

$$- \frac{C_{z_\alpha}}{2\tau} \alpha - \dot{\alpha} \frac{C_{z\dot{\alpha}}}{4\mu_1} - q \frac{C_{z_q}}{4\mu_1} + \dot{\alpha} \cos \alpha_1$$

$$- \frac{\sin i}{m V_1} . \frac{\partial T}{\partial \alpha} . \alpha + \alpha q_1 \sin \alpha_1$$

$$- q \cos \alpha_1 + \theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1 = 0$$

Written in complex form the equations becomes:

$$(a + b + c) + (d + e) Z_{\dot{\alpha}} + (f + g) Z_q + h Z_\theta = 0$$

Solve for a and d. All other coefficients are known.

$$a = - \frac{C_{z_\alpha}}{2\tau} \qquad e = + \cos \alpha_1$$

$$b = + q_1 \sin \alpha_1 \qquad f = - \frac{C_{z_q}}{4\mu_1}$$

$$c = - \frac{\sin i}{m V_1} . \frac{\partial T}{\partial \alpha} \qquad g = - \cos \alpha_1$$

$$d = - \frac{C_{z\dot{\alpha}}}{4\mu_1} \qquad h = \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1$$

Check that $C_{z.\alpha} + C_{z_q} \triangleq 0$

Pitching Moment Equation

$$- \alpha \frac{C_{m_\alpha}}{\tau^2 h} - \dot{\alpha} \frac{C_{m_{\dot{\alpha}}} \bar{c}^2}{4 K^2_y \tau} - q \frac{C_{m_q} \bar{c}^2}{4 K^2_y \tau}$$

$$- \frac{e}{I_y} \frac{\partial T}{\partial \alpha} \cdot \alpha + \dot{q} = 0$$

Written in complex form the equation becomes

$$(j + k) + l Z_{\dot{\alpha}} + m Z_q + Z_{\dot{q}} = 0$$

Solve for j and l. All other coefficients are known

$$j = - \frac{C_{m_\alpha}}{\tau^2 h} \qquad l = - \frac{C_{m_{\dot{\alpha}}} \bar{c}^2}{4 K^2_y \tau}$$

$$k = - \frac{e}{I_y} \cdot \frac{\partial T}{\partial \alpha} \qquad m = - \frac{C_{m_q} \bar{c}^2}{4 K^2_y \tau}$$

LINEARISED LONGITUDINAL EQUATIONS TO INCLUDE PHUGOID EFFECTS

The following equations include varying height and speed terms in order to adequately cover the characteristics of the phugoid.

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θ X Equation

$$- \bar{V} \frac{M \frac{\partial C_x}{\partial M}}{2\tau} - \bar{V} \frac{C_{x_1}}{\tau} - \alpha \frac{C_{x_\alpha}}{2\tau} - \dot{\alpha} \frac{C_{x_{\dot{\alpha}}}}{4\mu_1}$$

$$- q \frac{C_{x_q}}{4\mu_1} - \frac{C_{x_{\bar{h}}}}{2\tau} \bar{h} - \frac{C_{x_1}}{\tau 2\rho} \cdot \frac{\partial \rho}{\partial \bar{h}} \cdot \bar{h}$$

$$- \frac{\cos i}{m V_1} \frac{\partial T}{\partial h} \cdot \bar{h} - \frac{\cos i}{m} \cdot \frac{\partial T}{\partial V_1} \cdot \bar{V} - \frac{\cos i}{m V_1} \cdot \frac{\partial T}{\partial \alpha} \cdot \alpha$$

$$+ \dot{\bar{V}} \cos \alpha_1 - \dot{\alpha} \sin \alpha_1 + \bar{V} \cdot \sin \alpha_1 q_1 + \alpha q_1 \cos \alpha_1$$

$$+ q \sin \alpha_1 + \theta \frac{g}{\bar{V}_1} \cos \beta_1 \cos (H) = 0$$

All angles expressed in radians

$$\bar{V} (a + b\lambda) + \alpha (c + d\lambda) + \theta (e + f\lambda) + h (g) = 0$$

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The O Z Equation

$$- \bar{V} \; M \frac{Cz_M}{2\tau} - \bar{V} \frac{Cz_1}{\tau} - \frac{Cz_\alpha}{2\tau} \cdot \alpha - \dot{\alpha} \frac{Cz_{\dot{\alpha}}}{4\mu_1} - q \frac{Cz_q}{4\mu_1}$$

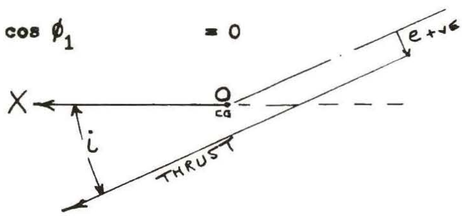
$$- \frac{\bar{h}.Cz_{\bar{h}}}{2\tau} - \frac{Cz_1}{\tau} \frac{\bar{h}}{2\rho} \cdot \frac{\partial \rho}{\partial \bar{h}} - \frac{\sin i}{m V_1} \cdot \frac{\partial T}{\partial \bar{h}} \; \bar{h}$$

$$- \frac{\sin i}{m} \frac{\partial T}{\partial V_1} \cdot \bar{V} - \frac{\sin i}{m V_1} \cdot \frac{\partial T}{\partial \alpha} \cdot \alpha + \dot{\alpha} \cdot \cos \alpha_1$$

$$- \bar{V} q_1 \cos \alpha_1 + \sin \alpha_1 \dot{\bar{V}} + \alpha q_1 \sin \alpha_1$$

$$-q \cos \alpha_1 + \theta \frac{g}{V_1} \sin \textcircled{H} \cos \phi_1 \qquad = 0$$

i is + ve if



All angles expressed in radians.

$$\bar{V} \; (j + \lambda k) + \alpha \, (l + m\lambda) \;\; + \theta \;\; (n + p\lambda) \;\; + \bar{h} \; (q) = 0$$

Pitching Moment Equation

$$- \frac{\bar{V} M \frac{\partial Cm}{\partial M}}{\tau^2 \hbar} - \bar{V} \frac{Cm_1 \; V_1^2}{\mu_1 \; K_Y^2} - \frac{\alpha \, Cm \, \alpha}{\tau^2 \; \hbar} - \dot{\alpha} \frac{C_{m_{\dot{\alpha}}} \; \bar{c}^2}{4 \; K_Y^2 \, \tau}$$

$$- q \frac{C_{m\,q} \; \bar{c}^2}{4 \; K_Y^2 \tau} - \frac{Cm_{\bar{h}} \; \bar{h}}{\hbar \tau^2} - \frac{V_1^{\;2}}{2 \rho} \frac{\partial \rho}{\partial \hbar} \frac{Cm_1 \; \hbar}{\mu_1 K_Y^2}$$

$$- \frac{e}{I_Y} \cdot \frac{\partial T}{\partial V_1} \cdot V_1 \cdot \bar{V} - \frac{e}{I_y} \frac{\partial T}{\partial \hbar} \bar{h} - \frac{e}{I_y} \cdot \frac{\partial T}{\partial \alpha} \cdot \alpha + \dot{q} = 0$$

$e = + 1.06$ for 8" above datum. C.G

All angles expressed in radiaus.

$$\bar{V} \; (r) \; + \alpha \; (s + \lambda t) \; + \Theta \; (\lambda u + \lambda^2) \; + \bar{h} \; (w) = 0$$

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Incremental $\dot{h}$

From P/Stab./132 Sheet 12.5

Linearising the equation of $\dot{h}$ and allowing all products of incremental variables to vanish and putting $\cos\theta = 1$, $\sin\theta = \theta$. Also extracting h_1 $\beta = \phi = 0$ is assumed.

$$+\frac{\bar{V}\ V_1}{\bar{c}}\left\{\cos\alpha_1\sin Ⓗ\cos\beta_1 - \sin\alpha_1\cos Ⓗ\cos\phi_1 - \sin\beta_1\sin\phi_1\cos Ⓗ\right\}$$

$$-\frac{\alpha\ V_1}{\bar{c}}\left\{\sin\alpha_1\sin Ⓗ\cos\beta_1 + \cos\alpha_1\cos Ⓗ\cos\phi_1\right\}$$

$$+\frac{\theta\ V_1}{\bar{c}}\left\{\cos\alpha_1\cos Ⓗ\cos\beta_1 + \sin Ⓗ\sin\alpha_1\cos\phi_1 + \sin\beta_1\sin\phi_1\sin Ⓗ\right\}$$

$$-\ \dot{h}^{-} = 0$$

$$h^{-} = \frac{h}{\bar{c}}\ ;\quad \dot{h}^{-} = \frac{\dot{h}}{\bar{c}}$$

$$\bar{V}x + \alpha y + \theta z + \lambda h = 0$$

CODING OF AERODYNAMIC DERIVATIVES

$a = a_1 + a_2 + a_3 + a_4$

$a_1 = - \dfrac{M \partial Cx / \partial M}{2\tau}$

$a_2 = - Cx_1 / \tau$

$a_3 = - \dfrac{\cos i . \partial T / \partial V_1}{m}$

$a_4 = + \sin \alpha_1 . q_1$

$b = + \cos \alpha_1$

$c = c_1 + c_2 + c_3$

$c_1 = - Cx_\alpha / 2\tau$

$c_2 = - \dfrac{\cos i \; \partial T / \partial \alpha}{m V_1}$

$c_3 = + q_1 \cos \alpha_1$

$d = d_1 + d_2$

$d_1 = - \dfrac{Cx_{\dot{\alpha}}}{4\mu_1}$

$d_2 = - \sin \alpha_1$

$e = + g \cos \beta_1 \cos \textcircled{H} / V_1$

$f = f_1 + f_2$

$f_1 = - Cx_q / 4\mu_1$

$f_2 = + \sin \alpha_1$

$g = g_1 + g_2 + g_3$

$g_1 = - Cx_{\bar{h}} / 2\tau$

$g_2 = - \dfrac{Cx_1 \partial \rho / \partial \bar{h}}{2\tau\rho}$

$g_3 = - \dfrac{\cos i . \partial T / \partial \bar{h}}{m V_1}$

$j = j_1 + j_2 + j_3 + j_4$

$j_1 = - \dfrac{M \partial Cz / \partial M}{2\tau}$

$j_2 = - Cz_1 / \tau$

$j_3 = - \dfrac{\sin i . \partial T / \partial V_1}{m}$

$j_4 = - q_1 \cos \alpha_1$

$k = + \sin \alpha_1$

CODING OF AERODYNAMIC DERIVATIVES

$l = l_1 + l_2 + l_3$

$l_1 = -Cz_\alpha / 2\tau$

$l_2 = -\dfrac{\sin i . \partial T / \partial \alpha}{m V_1}$

$l_3 = + q_1 \sin \alpha_1$

$m = m_1 + m_2$

$m_1 = -Cz_{\dot{\alpha}} / 4\mu_1$

$m_2 = + \cos \alpha_1$

$n = + \dfrac{g. \sin Ⓗ . \cos \phi_1}{V_1}$

$p = p_1 + p_2$

$p_1 = -Cz_q / 4\mu_1$

$p_2 = -\cos \alpha_1$

$q = q_1 + q_2 + q_3$

$q_1 = -Cz_{\bar{h}} / 2\tau$

$q_2 = -\dfrac{Cz_1 \partial \rho / \partial \bar{h}}{2\tau\rho}$

$q_3 = -\dfrac{\sin i . \partial T / \partial \bar{h}}{m V_1}$

$r = r_1 + r_2 + r_3$

$r_1 = -\dfrac{M \partial Cm / \partial M}{\tau^2 \hbar}$

$r_2 = -\dfrac{Cm_1 V_1^2}{\mu_1 K_y^2}$

$r_3 = -\dfrac{V_1 e \partial T / \partial V_1}{Iy}$

$s = s_1 + s_2$

$s_1 = -Cm_\alpha / \tau^2 \hbar$

$s_2 = -\dfrac{e. \partial T / \partial \alpha}{Iy}$

$t = -\dfrac{Cm_{\dot{\alpha}} . \bar{c}^2}{4 K_y^2 \tau}$

$u = -\dfrac{Cm_q . \bar{c}^2}{4 K_y^2 \tau}$

$w = w_1 + w_2 + w_3$

$w_1 = -Cm_{\bar{h}} / \hbar \tau^2$

$w_2 = -\dfrac{V_1^2 . Cm_1 . \partial \rho / \partial \bar{h}}{2\rho \mu_1 K_y^2}$

$w_3 = -\dfrac{e \partial T / \partial \alpha}{Iy}$

$$x = + \frac{V_1}{\bar{c}} \left\{ \cos \alpha_1 \sin Ⓗ \cos \beta_1 - \sin \alpha_1 \cos Ⓗ \cos \phi_1 - \sin \beta_1 \sin \phi_1 \cos Ⓗ \right\}$$

$$y = - \frac{V_1}{\bar{c}} \left\{ \sin \alpha_1 \sin Ⓗ \cos \beta_1 + \cos \alpha_1 \cos Ⓗ \cdot \cos \phi_1 \right\}$$

$$z = + \frac{V_1}{\bar{c}} \left\{ \cos \alpha_1 \cos Ⓗ \cos \beta_1 + \sin Ⓗ \sin \alpha_1 \cos \phi_1 + \sin \beta_1 \sin \phi_1 \sin Ⓗ \right\}$$

$$\bar{\bar{V}}\ (a + b\lambda) + \alpha((c + d\lambda) + \theta\ (e + f\lambda) + \bar{h}\ (g) = 0$$

$$\bar{\bar{V}}\ (j + \lambda k) + \alpha((l + m\lambda) + \theta\ (n + p\lambda) + \bar{h}\ (q) = 0$$

$$\bar{\bar{V}}\ (r) + \alpha\ (s + t\lambda) + \theta\ (\lambda u + \lambda^2) + \bar{h}\ (w) = 0$$

$$\bar{\bar{V}}\ (x) + \alpha\,(y) + \theta\ (z) + \bar{h}\ (\lambda) = 0$$

The determinant of the coefficients in brackets = 0

and the solution of the above is reduced to

$$A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0$$

$A = bm - kd$

$B = ubm + am + bl - pbt - kc - jd - ukd + k.tf$

$= am$

$- jd$

$+ \left[ubm + bl - pbt - kc - ukd + ktf \right]$

$B_1 = m \; ; \; B_2 = -d.$

$\therefore B = aB_1 + jB_2 + B_3$

$$C = +uam + ubl + al - pat - psb - nbt - byq - jc - ujd - ukc + ksf + jtf + kte + kyg + \gamma dp - rmf + xdq - xmg$$

$$C = aum + al - apt - jc - jud + jtf + \gamma dp - rmf + ubl - psb - nbt - byq - ukc + ksf + kte + kyg + xdq - xmg$$

let $(um + l - pt) = C_1$; $(-c - ud + tf) = c_2$; $(dp - mf) = C_3$

let $(+ubl - psb - nbt - byq - ukc + ksf + kte + kyg + xdq - xmg) = C_4$

$$\therefore C = aC_1 + jC_2 + \gamma C_3 + C_4$$

$$
\begin{aligned}
D = & +ual - zwbm + zqbt - nat - nsb - pas \\
& \quad + bywp \\
& - ayq - byqu - ujc + zwkd - zgkt + jsf \\
& \quad + jte \\
& \quad + kse - wfky + gjy + kygu + rcp + rdn - rlf \\
& - rme - xdwp + xcq + xdqu - xlg - xmgu \\
& + xmwf - xtqf + xtgp
\end{aligned}
$$

$$
\begin{aligned}
D = & \, a\,(ul - nt - ps - yq) \text{-------------------------} aD_1 \\
& + j\,(-uc + sf + te + gy) \text{-------------------------} jD_2 \\
& + r\,(+cp + dn - lf - me) \text{-------------------------} rD_3 \\
& (-zwbm + zqbt - nsb + bywp - byqu + zwkd - zgkt \\
& \quad + kse - wfky + kygu - xdwp + xcq + xdqu \\
& \quad - xlg - xmgu + xmwf - xtqf + xtgp \,) \text{---------} D_4
\end{aligned}
$$

$$\therefore D = aD_1 + jD_2 + rD_3 + D_4$$

$$E = -zwam - zwbl + zqat + zqsb - nas + aywp + bywn - ayqu + zwjd + zwkc - zgjt - zgks + jse - weky - wfjy + gujy + rcn - zqrd + zgrm - rle + ryqf - rygp - xcwp - xdwn + xcqu - xlgu + xlwf + xmwe - xsqf - xtqe + xsgp + xtgn$$

$$E = a(-zwm + zqt - ns + ywp - yqu) \text{------------} aE_1$$

$$+ j(+zwd - zgt + se - wfy + guy) \text{------------} jE_2$$

$$+ r(+cn - zqd + zgm - le + yqf - ygp) \text{--} rE_3$$

$$(-zwbl + zqsb + bywn + zwkc - zgks - weky - xcwp - xdwn + xcqu - xlgu + xlwf + xmwe - xsqf - xtqe + xsgp + xtgn) \text{--} E_4$$

$$E = aE_1 + jE_2 + rE_3 + E_4$$

AVRO AIRCRAFT LIMITED
MALTON ONTARIO
TECHNICAL DEPARTMENT

AIRCRAFT

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$$F = -zwal + zqas + aywn + zwjc - zgjs - wejy$$
$$-zqrc + zgrl - ryqe - rygn - xcwn$$
$$+ xlwe - xsqe + xsgn$$

$$F = a(-zwl + zqs + ywn) \text{ ---------- } aF_1$$
$$+ j(+zwc - zgs - wey) \text{ ---------- } jF_2$$
$$+ r(-zqc + zgl + yqe - ygn) \text{ ---------- } rF_3$$
$$(-xcwn + xlwe - xsqe + xsgn) \text{ ---------- } F_4$$

$$F = aF_1 + jF_2 + rF_3 + F_4$$

$$\lambda = k - \omega i$$

$$R_1 = k$$

$$R_2 = (k^2 - \omega^2)$$

$$R_3 = (k^3 - 3\ k\omega^2)$$

$$R_4 = k^4 - 6\ k^2\omega^2 + \omega^4$$

$$R_5 = k^5 - 10\ k^3\omega^2 + 5\ k\omega^4$$

$$I_1 = \omega$$

$$I_2 = 2\ k\omega$$

$$I_3 = 3\ k^2\omega - \omega^3$$

$$I_4 = 4\ (k^3\omega - k\omega^3)$$

$$I_5 = 5\ \omega k^4 - 10\ k^2\omega^3 + \omega^5$$

$$A R_5 + B R_4 + C R_3 + D R_2 + E R_1 + F = 0 \qquad \text{Real}$$

$$A I_5 + B I_4 + C I_3 + D I_2 + E I_1 = 0 \qquad \text{Imaginary}$$

$$\text{Let } \left[A R_5 + B_3 R_4 + C_4 R_3 + D_4 R_2 + E_4 R_1 + F_4\right] = P$$

$$\text{Let } \left[A I_5 + B_3 I_4 + C_4 I_3 + D_4 I_2 + E_4 I_2\right] = Q$$

$$a B_1 R_4 + j B_2 R_4 + a C_1 R_3 + j C_2 R_3 + r C_3 R_3$$

$$+ a D_1 R_2 + j D_2 R_2 + r D_3 R_2 + a E_1 R_1 + j E_2 R_1 + r E_3 R_1$$

$$+ a F_1 + j F_2 + r F_3 + P = 0 \text{ Real}$$

Further simplifying let:

$$B_1 R_4 + C_1 R_3 + D_1 R_2 + E_1 R_1 + F_1 = R$$

$$B_2 R_4 + C_2 R_3 + D_2 R_2 + E_2 R_1 + F_2 = S$$

$$C_3 R_3 + D_3 R_2 + E_3 R_1 + F_3 = T$$

$$\therefore \underline{a R + j S + r T + P = 0} \qquad \underline{\text{Real}}$$

$$a B_1 I_4 + j B_2 I_3 + a C_1 I_3 + j C_2 I_3 + r C_3 I_3$$

$$+ a D_1 I_2 + j D_2 I_2 + r D_3 I_2 + a E_1 I_1 + j E_2 I_1 + r E_3 I_1 + Q = 0 \quad \text{Imaginary}$$

Further Simplifying let:

$$B_1 I_4 + C_1 I_3 + D_1 I_2 + E_1 I_1 = U$$

$$B_2 I_4 + C_2 I_3 + D_2 I_2 + E_2 I_1 = V$$

$$C_3 I_3 + D_2 I_2 + E_3 I_1 = W$$

$$\therefore \underline{a U + j V + r W + Q} = 0 \quad \text{Imaginary}$$

The assumption is made that the damping and frequency of the short period oscillation and the frequency of the phugoid are known accurately; also that the damping of the phugoid is known within large percentage tolerances since the convergence or divergence of the phugoid oscillation is small and difficult to measure.

The equation of the imaginary components of the phugoid is chosen since this minimizes the importance of the damping factor K of the phugoid oscillation when determining $\partial C_z/\partial M$, $\partial C_m/\partial M$

Let primed values be relevant to the roots of the short period oscillation and let the unprimed values be relevant to the oscillatory roots of the phugoid.

$$a R' + j S' + r T' + P' = 0$$

$$a U' + j V' + r W' + Q' = 0$$

$$a U + j V + r . W + Q = 0$$

The above equations yield that:

$$r = \frac{\left(S' \frac{U'}{R'} - V'\right)\left(Q \frac{U'}{U} - Q'\right) - \left(P' \frac{U'}{R'} - Q'\right)\left(V \frac{U'}{U} - V'\right)}{\left(S' \frac{U'}{R'} - V'\right)\left(W \frac{U'}{U} - W'\right) - \left(T' \frac{U'}{R'} - W'\right)\left(V \frac{U'}{U} - V'\right)}$$

$$j = \frac{Q' - P' \frac{U'}{R'} - r\left(T' \frac{U'}{R'} - W'\right)}{S' \frac{U'}{R'} - V'}$$

$$a = -\frac{1}{U}\left(j V + r W + Q\right)$$

$$\frac{\partial C_x}{\partial M} = \frac{2\tau}{M} \left\{ (a_2 + a_3 + a_4) - a \right\}$$

$$\frac{\partial C_z}{\partial M} = \frac{2\tau}{M} \left\{ j_2 + j_3 + j_4 - j \right\}$$

$$\frac{\partial C_m}{\partial M} = \frac{\tau^2 \hbar}{M} \left\{ (r_2 + r_3) - r \right\}$$

Conclusions

1. Cm_α, Cz_α, $(Cm_{\dot{\alpha}} + Cm_q)$ can be determined accurately from flight analysis of short period longitudinal oscillations. If phase relationship between normal acceleration and incidence can be measured very accurately then $(Cz_{\dot{\alpha}} + Cz_q)$ may be determined.

2. $\partial Cz/\partial M$, $\partial Cm/\partial M$ can be determined if the frequency and damping of the phugoid is measured simultaneously with the requisite measurements for the analysis of the short period oscillation. It is assumed that the phugoid period may be measured accurately and the damping can only be measured within large tolerances.

3. Introduction of height derivatives will improve the accuracy of the evaluation of $\partial Cz/\partial M$, $\partial Cm/\partial M$. At transonic and supersonic speeds this is known to be more essential.

4. The discussed digital computer programmes will greatly reduce the time to evaluate derivatives and largely eliminate human error.

5. The discussion in this report is relevant only to the aircraft in the pitch damper disengaged condition.