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DIGITAL COMPUTATION OF RESPONSE USING AN APPROXIMATION TO LATERAL DAMPER SYSTEM

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NOTATION

Symbols are in current Avro notation unless otherwise stated in the test.

INTRODUCTION AND SUMMARY

This report includes a method of calculating aircraft response by a step by step time solution. The 704 digital computer facilitates the determination of incremental response during extremely small time increments and hence the usual inaccuracy due to employing relatively large time increments, is eliminated.

The report also includes a method of determining , damper engaged condition, effective aircraft derivatives which may alternatively be described as synthetic aerodynamic derivatives, since damper system aerodynamic outputs are classified according to which particular aircraft response promoted the input and then summed with the natural aircraft derivatives. A typical but simplified example of this, is an aircraft response p which promotes a proportional signal to the differential servo producing a proportional aileron pure rolling moment output in phase with p. A ΔCl_p has been created and this may be added to the natural aircraft Cl_p to form an effective or synthetic aerodynamic $\overline{Cl}_p$.

The method also includes determination of the control derivatives and this yields values of the natural aircraft derivatives subsequent to the evaluation of the effective derivatives.

It is thought that the framing of the damper engaged condition equations of motion into a notation analogous to the free aircraft equations of motion expressed in familiar aerodynamic notation facilitates a quick clear physical interpretation of the functions of the damper system as regards aircraft response.

The parallel servo output is proportional to the integration with respect to time of the error between command and response. As command is arbitrary a numerical step by step time solution was preffered to a Laplace transformation, utilisation of which, would necessitate in many instances an approximate transform of the arbitrary command, the approach to which would vary according to the circumstances.

DEVELOPMENT OF THE SCOPE OF THE DIGITAL TIME RESPONSE SOLUTION

Having embarked on analysis of time solutions of the response of the aircraft rather than restricting analysis to linear equations of motion with zero flying control forcing functions, it is no longer necessary to restrict the analysis to small motions, nor is it necessary for the longitudinal and lateral motions to be uncoupled. Caution must be applied if it is suspected that aircraft response frequency approaches that of the servo-mechanism system.

ADVANTAGES OF DIGITAL SOLUTION

Once the response programmes have been proven chances of any error in the analysis are remote and time to solution using exact aerodynamic derivatives rather than "simulator" aerodynamic derivatives may well compare favourably with the analogue computer facility. In any event a very positive check is possible. Changes in servo-mechanism gains are also easily incorporated in the digital programme. Extropolation to other flight conditions is an easy matter.

δa output from parallel servo due to P,p is assumed of the form $\delta a = \int (K_4' P - K_4 p)\, dt$

δa output from differential servo due to P,p is assumed of the form $\delta a = K_5' P - K_5 p$

Therefore total δa from P,p is $\int (K_4' P - K_4 p)\, dt + K_5' P - K_5 p$

where K_4, K_4', K_5, K_5' are constants for any given flight case and negative in value.

Forcing side force along OY axis due to P,p

$$\text{is} \left\{ \frac{\Delta_1 Cy_p\, p}{4\mu_2} + \frac{Cy_{\delta a}}{2\tau} \left[\int (K_4' P - K_4 p)\, dt + K_5' P \right] \right\} Vm$$

where $\Delta_1 Cy_p\, p = -K_5\, Cy_{\delta a} \cdot \frac{2V}{b}$

Forcing rolling moment due to P,p

$$\text{is} \left\{ \frac{\Delta_1 Cl_p\, b \cdot p}{2Vi_x} + \frac{Cl_{\delta a}}{i_x} \left[\int (K_4' P - K_4 p)\, dt + K_5' P \right] \right\} mK^2_x$$

where $\Delta_1 Cl_p = -K_5\, Cl_{\delta a} \cdot \frac{2V}{b}$

FORCING YAWING MOMENT DUE TO P, p

$$\text{IS} \left\{ p\,\frac{\Delta_1 C_{n_p} b}{2V i_z} + \frac{C_{n_{\delta a}}}{i_z}\left[\int (K'_4 P - K_4 p)\,dt + K'_5 P\right]\right\} m K_z^2$$

$$\text{WHERE} \quad \Delta_1 C_{n_p} = -K_5 C_{n_{\delta a}} \frac{2V}{b}$$

δ_T OUTPUT FROM DIFFERENTIAL SERVO DUE TO δ_a, $(r - .061p)$, a_y IS ASSUMED OF THE FORM

$$\delta_T = K_6 \delta a + K_7 (r - .061p) + K_8 a_y$$

WHERE a_y IS THE ACCELERATION MEASURED BY THE TRANSVERSE ACCELEROMETER AND $(r - .061p)$ IS THE YAW RATE MEASURED BY THE TILTED GYRO

K_8 IS A CONSTANT FOR ANY GIVEN FLIGHT CASE AND POSITIVE IN VALUE

$$\delta_{T_6} = K_6 \delta_a$$

$$K_6 = K'_6 + K''_6 \;:\; K'_6 = K_{6(t=0+)} \;:\; K''_6 = -\frac{.5}{\delta_a}\int \delta_{T_6}\,dt$$

$$\delta_{T_7} = K_7 (r - .061p)$$

$$K_7 = K'_7 + K''_7 \;:\; K'_7 = K_{7(t=0+)} \;:\; K''_7 = -\frac{.5}{r - .061p}\int \delta_{T_7}\,dt$$

Forcing side force along OY axis due to δa, $(r - .061p)$, a_y

is

$$\left\{ \frac{\Delta Cy_{\delta a}}{2\tau} \left[\int (K'_4 P - K_4 p)\, dt + K'_5 P \right] + \frac{\Delta Cy_r \cdot r}{4\mu_2} + \frac{\Delta_2 Cy_p \cdot p}{4\mu_2} + \frac{Cy_{a_y}}{2\tau} . a_y \right\} Vm$$

where $\Delta Cy_{\delta a} = K_6\, Cy_{\delta r}$; $\Delta Cy_r = \frac{2V}{b} \cdot Cy_{\delta r}\, K_7$

$$\Delta_2 Cy_p = -(K_6 K_5 + .061 K_7)\, \frac{2V}{b} \cdot Cy_{\delta r} \;;\; Cy_{a_y} = K_8\, Cy_{\delta r}$$

Forcing rolling moment due to δa, $(r - .061p)m$ a_y is

$$\left\{ \frac{\Delta Cl_{\delta a}}{i_x} \left[\int (K'_4 P - K_4 p)\, dt + K'_5 P \right] + \frac{\Delta Cl_r}{i_x} \frac{.b}{2V} \cdot r + \frac{\Delta_2 Cl_p}{i_x} \frac{.b}{2V} \cdot p + \frac{Cl_{a_y}}{i_x} \cdot a_y \right\} mK^2_x$$

where $\Delta Cl_{\delta a} = K_6\, Cl_{\delta r}$; $\Delta Cl_r = K_7\, Cl_{\delta r} \cdot \frac{2V}{b}$

$$\Delta_2 Cl_p = -(K_6 K_5 + .061 K_7) . \frac{2V}{b} . Cl_{\delta r} \;;\; Cl_{a_y} = K_8\, Cl_{\delta r}$$

Forcing yawing moment due to δa, $(r - .061p)$, a_y is

$$\left\{ \frac{\Delta Cn_{\delta a}}{i_z}\left[\int\left(K'_4 P - K_4 p\right) dt + K'_5 P\right] + \frac{\Delta Cn_r}{i_z}\,\frac{\cdot b}{2V}\cdot r + \frac{\Delta_2 Cn_p}{i_z}\,\frac{\cdot b}{2V}\cdot p + \frac{Cn_{a_y}}{i_z}\cdot a_y \right\}$$

where $\Delta Cn_{\delta a} = K_6\, Cn_{\delta r}$; $\Delta Cn_r = K_7\, \frac{.2V}{b}\, .Cn_{\delta r}$

$$\Delta_2 Cn_p = -(K_6\, K_5 + .061K_7)\,\frac{2V}{b}\cdot Cn_{\delta r}$$

$$Cn_{a_y} = K_8\, Cn_{\delta r}$$

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δ_T OUTPUT FROM DIFFERENTIAL SERVO DUE TO $(\delta_a q_1)$ IS ASSUMED OF THE FORM $K_q \delta_a q_1$. AS THE RESPONSE SOLUTION IS RESTRICTED TO 3 DEGREES OF FREEDOM THE ASSUMPTION IS MADE THAT $(\delta a_1 + \delta a)(q_1 + q) - \delta a_1 q_1 \doteqdot \delta_a q_1$

FORCING SIDE FORCE ALONG OY AXIS DUE TO $(\delta a . q_1)$

IS $$\frac{C_{Y_{\delta T}}}{2\tau} K_q q_1 \left[\int (K_4' P - K_4 p)\, dt + K_5' P \right] + \Delta_3 \frac{C_{Y_p}\, p}{4\mu_2}$$

WHERE $\Delta_3 C_{Y_p} = -K_5 K_q . C_{Y_{\delta T}} . q_1 \frac{2V}{b}$

FORCING ROLLING MOMENT DUE TO $(\delta_a . q_1)$

IS $$\frac{Cl_{\delta T}}{i_x} \cdot K_q q_1 \left[\int (K_4' P - K_4 p)\, dt + K_5' P \right] + \Delta_3 \frac{Cl_p}{i_x} \frac{b}{2V} \cdot p$$

WHERE $\Delta_3 Cl_p = -K_5 . K_q . Cl_{\delta T} . q_1 . \frac{2V}{b}$

FORCING YAWING MOMENT DUE TO $(\delta_a . q_1)$

IS $$\frac{Cn_{\delta T}}{i_z} \cdot K_q q_1 \left[\int (K_4' P - K_4 p)\, dt + K_5' P \right] + \Delta_3 \cdot \frac{Cn_p}{i_x} \cdot \frac{b}{2V} \cdot p$$

WHERE $\Delta_3 Cn_p = -K_5 . K_q . Cn_{\delta T} . q_1 . \frac{2V}{b}$

APPROXIMATION TO CF 105 LATERAL DAMPER SYSTEM

$$\beta\left[-\frac{C_{y_\beta}}{2\tau}\right]+\phi\left[-\lambda\frac{C_{y_p}'}{[illegible]}+\frac{g\lambda^2}{V}\right]+\psi\left[-\lambda\frac{C_{y_r}'}{[illegible]}-\frac{g\lambda^2}{V}\right]+a_y\left[\frac{C_{y_{a}}}{2\tau}+\phi\right]=\left[\frac{C_{y_{\delta}}'}{2\tau}+\frac{C_{y_{\delta r}}K_4}{2\tau}\right]\left[\int(K_1'p-K_1q)dt+K_2'p\right]+C_{y_{\delta r}}K_{10}R$$

$$\beta\left[-\frac{C_{l_\beta}}{i_A}\right]+\phi\left[-\lambda\frac{C_{l_p}'}{i_A}\frac{b}{2V}-\lambda q\frac{K_{xz}}{K_x^2}+\lambda^2\right]+\psi\left[-\lambda\frac{C_{l_r}'}{i_A}\frac{b}{2V}-\lambda^2\frac{K_{xz}}{K_x^2}+\lambda\frac{K_z^2-K_y^2}{K_x^2}q\right]+a_y\left[-\frac{C_{l_{a}}}{i_A}\right]=\left[\frac{C_{l_{\delta}}'}{i_A}+\frac{C_{l_{\delta r}}K_4}{i_A}\right]\left[\int(K_1'p-K_1q)dt+K_2'p\right]+\frac{C_{l_{\delta r}}}{i_A}K_{10}R$$

$$\beta\left[-\frac{C_{n_\beta}}{i_C}\right]+\phi\left[\lambda\frac{K_y^2-K_x^2}{K_z^2}q-\lambda^2\frac{K_{xz}}{K_z^2}-\lambda\frac{C_{n_p}'}{i_C}\frac{b}{2V}\right]+\psi\left[-\lambda\frac{C_{n_r}'}{i_C}\frac{b}{2V}+\lambda\frac{K_{xz}}{K_z^2}q+\lambda^2\right]+a_y\left[-\frac{C_{n_{a}}}{i_C}\right]=\left[\frac{C_{n_{\delta}}'}{i_C}-\frac{C_{n_{\delta r}}K_4}{i_C}\right]\left[\int(K_1'p-K_1q)dt+K_2'p\right]+\frac{C_{n_{\delta r}}}{i_C}K_{10}R$$

$$\beta\left[\lambda\right]+\phi\left[-\frac{g}{V}\cos\theta_1\cos\Theta-\lambda\alpha_1\frac{g\lambda^2}{V}\right]+\psi\left[\lambda\cos\alpha_1+\frac{g\lambda^2}{V}\right]+a_y\left[-\frac{1}{g}\right]=0$$

$C_{y_p}'=C_{y_p}+\Delta_1C_{y_p}+\Delta_2C_{y_p}+\Delta_3C_{y_p}$ $\quad\Delta_1C_{y_p}=-K_3C_{y_{\delta a}}\frac{2V}{b}$; $\Delta_2C_{y_p}=-(K_6\bar{x}_3+\cdot 0\,[illegible]\,K_7)\frac{2V}{b}C_{y_{\delta r}}$; $\Delta_3C_{y_p}=-K_5K_{9}C_{y_{\delta r}}\,q\frac{2V}{b}$

$C_{y_r}'=C_{y_r}+\Delta C_{y_r}$ $\quad\Delta C_{y_r}=K_7C_{y_{\delta r}}\frac{2V}{b}$

$C_{y_{\delta a}}'=C_{y_{\delta a}}+\Delta C_{y_{\delta a}}$ $\quad\Delta C_{y_{\delta a}}=K_6C_{y_{\delta r}}$

$C_{l_p}'=C_{l_p}+\Delta_1C_{l_p}+\Delta_2C_{l_p}+\Delta_3C_{l_p}$ $\quad\Delta_1C_{l_p}=-K_3C_{l_{\delta a}}\frac{2V}{b}$ $\quad\Delta_2C_{l_p}=-(K_6\bar{x}_3+\cdot 0\,[illegible]\,K_7)\frac{2V}{b}C_{l_{\delta r}}$; $\Delta_3C_{l_p}=-K_5K_9C_{l_{\delta r}}\,q\frac{2V}{b}$

$C_{l_r}'=C_{l_r}+\Delta C_{l_r}$ $\quad\Delta C_{l_r}=K_7\frac{2V}{b}C_{l_{\delta r}}$

$\bar{x},\bar{z},$ POSITION OF TRANSVERSE ACCELEROMETER IN FEET

$C_{l_{\delta a}}'=C_{l_{\delta a}}+\Delta C_{l_{\delta a}}$ $\quad\Delta C_{l_{\delta a}}=K_6C_{l_{\delta r}}$

$C_{n_p}'=C_{n_p}+\Delta_1C_{n_p}+\Delta_2C_{n_p}+\Delta_3C_{n_p}$ $\quad\Delta_1C_{n_p}=-K_3C_{n_{\delta a}}\frac{2V}{b}$ $\quad\Delta_2C_{n_p}=-(K_6\bar{x}_3+\cdot 0\,[illegible]\,K_7)\frac{2V}{b}C_{n_{\delta r}}$ $\quad\Delta_3C_{n_p}=-K_5K_9C_{n_{\delta r}}\,q\frac{2V}{b}$

$C_{n_r}'=C_{n_r}+\Delta C_{n_r}$ $\quad\Delta C_{n_r}=K_7\frac{2V}{b}C_{n_{\delta r}}$ $\quad\delta_{ra}$ = INCREMENTAL RUDDER ANGLE DUE TO INCREMENTAL NET RUDDER FROM DAMPER(S)

$K_6=\frac{\delta_{ra}}{\delta_a}$

$C_{n_{\delta a}}'=C_{n_{\delta a}}+\Delta C_{n_{\delta a}}$ $\quad\Delta C_{n_{\delta a}}=K_6C_{n_{\delta r}}$

$C_{y_{a_y}}=K_8C_{y_{\delta r}}$ $\quad C_{l_{a_y}}=K_8C_{l_{\delta r}}$ $\quad C_{n_{a_y}}=K_8C_{n_{\delta r}}$

$K_1=\delta_a/-\int p\,dt$ $\quad K_3=\delta_a/\dot{p}$ $\quad\delta_{r_5}=K_5\delta_a$, $K_5=K_5'+K_5''$ $\;K_5'=K_{5(t=0)}$ $\;K_5''=-\frac{g}{\delta_a}\int\delta_{r_5}\,dt$

$\delta_{r_7}=K_7(r-\alpha_1 p)$ $\quad K_7=K_7'+K_7''$, $K_7'=K_{7(t=0)}$; $K_7''=-\frac{g}{r-\alpha_1p}\int\delta_{r_7}\,dt$

ALL OTHER SYMBOLS IN CURRENT AVRO NOTATION

$K_1'=\delta_a/\int p\,dt$ $\quad K_2'=\delta_a/\dot{p}$ $\quad K_8=\delta_{r}/a_y$ $\quad K_9=\delta_r/[illegible]$

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NOTE THAT THE δ_r's AND δ_a's ARE NOT TOTAL INCREMENTS BUT OUTPUTS DUE TO THE RELEVANT RESPONSE CONSTANT

FROM THE SIDE FORCE EQUATION

$$a_Y = \frac{\beta \frac{C_{Y_\beta}}{2\tau} + p \frac{C'_{Y_p}}{4\mu_2} + r \frac{C'_{Y_r}}{4\mu_2} + \left[\frac{C'_{Y_{\delta a}}}{2\tau} + \frac{C_{Y_{\delta r}} K_9 q_1}{2\tau}\right]\left[\int (K'_4 P - K_4 p)\,dt + K'_5 P\right] + \frac{\bar{X}}{V}\dot{r} - \frac{\bar{Z}}{V}\dot{p}}{F}$$

$$+ \frac{C_{Y_{\delta r}} . K_{10} . P_E}{2\tau . F} \qquad \text{WHERE } F = \left[\frac{1}{V} - \frac{C_{Y_{a_Y}}}{2\tau}\right] = \left[\frac{1}{V} - \frac{K_8 C_{Y_{\delta r}}}{2\tau}\right]$$

ALSO $\dfrac{a_Y}{V} = -\dfrac{\bar{Z}\dot{p}}{V} + \dfrac{\bar{X}\dot{r}}{V} - p\alpha_1 - \phi\dfrac{g}{V}.\cos\phi_1.\cos\beta_1.\cos\Theta + \dot{\beta} + r.\cos\alpha_1$

$$\therefore a_Y\left(-\frac{C_{Y_{a_Y}}}{2\tau} + \frac{1}{V}\right) = \dot{\beta} - \beta\frac{\Delta C_{Y_\beta}}{2\tau} - \phi\frac{g}{V}.\cos\phi_1.\cos\beta_1.\cos\Theta - p\alpha_1$$

$$- p\frac{\Delta_4 C_{Y_p}}{4\mu_2} - \dot{p}\left\{\bar{Z}\left[\frac{1}{V} + \frac{[K_8 . C_{Y_{\delta r}}]}{V 2\tau F}\right]\right\} - \frac{K_8 C^2_{Y_{\delta r}}}{4\tau^2 F} . K_{10} . P_E$$

$$+ r.\cos\alpha_1 - r\frac{\Delta_2 C_{Y_r}}{4\mu_2} + \dot{r}\left\{\bar{X}\left[\frac{1}{V} + \frac{[K_8 . C_{Y_{\delta r}}]}{V 2\tau F}\right]\right\}$$

$$- \frac{K_8 . C_{Y_{\delta r}}}{4\tau^2 F}\left[C'_{Y_{\delta a}} + C_{Y_{\delta r}} . K_9 q_1\right]\left[\int (K'_4 P - K_4 p)\,dt + K'_5 P\right]$$

$$\Delta C_{Y_\beta} = \frac{K_8 . C_{Y_{\delta r}} . C_{Y_\beta}}{2F\tau} \;;\; \Delta_4 C_{Y_p} = \frac{K_8 . C_{Y_{\delta r}} . C'_{Y_p}}{2F\tau} \;;\; \Delta_2 C_{Y_r} = \frac{K_8 . C_{Y_{\delta r}} . C'_{Y_r}}{2F\tau}$$

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$$-a_r\frac{Cl_{a_r}}{i_x} = -\beta\frac{\Delta Cl_\beta}{i_x} - p\cdot\Delta_4\frac{Cl_p}{i_x}\cdot\frac{b}{2V} - r\cdot\Delta_2\frac{Cl_r}{i_x}\cdot\frac{b}{2V}$$

$$-\frac{K_8 Cl_{\delta_r}}{F i_x 2\Upsilon}\left[C'_{Y_{\delta a}} + C_{Y_{\delta r}}\cdot K_9\cdot q_1\right]\left[\int(K'_4 P - K_4 p)\,dt + K'_5 P\right]$$

$$-\frac{\bar{X}\dot{r}K_8 Cl_{\delta_r}}{FVi_x} + \frac{\bar{Z}\dot{p}K_8 Cl_{\delta_r}}{FVi_x}$$

$$-\frac{K_8 Cl_{\delta_r}}{i_x}\cdot\frac{C_{Y_{\delta_r}}K_{10}P_E}{2\Upsilon F}$$

WHERE $\Delta Cl_\beta = \dfrac{K_8 Cl_{\delta_r}\,C_{Y_\beta}}{2\Upsilon F}$

$$\Delta_4 Cl_p = \frac{K_8 Cl_{\delta_r}\,C'_{Y_p}}{2\Upsilon F}$$

$$\Delta_2 Cl_r = \frac{K_8 Cl_{\delta_r}\,C'_{Y_r}}{2\Upsilon F}$$

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$$-a_r \cdot \frac{C_{n_{a_r}}}{i_z} = -\beta \cdot \frac{\Delta C_{n_\beta}}{i_z} - r \cdot \frac{\Delta_2 C_{n_r}}{i_z} \cdot \frac{b}{2V} - p \cdot \frac{\Delta_4 C_{n_p}}{i_z} \cdot \frac{b}{2V}$$

$$-\frac{K_8 \cdot C_{n_{\delta r}}}{F i_z 2\Upsilon}\left[C_{Y_{\delta a}}' + C_{Y_{\delta r}} \cdot K_9 q_k\right]\left[\int (K_4' p - K_7 p)\,dt + K_5' p\right]$$

$$-\frac{\bar{X}\dot{r} K_8 C_{n_{\delta r}}}{F V i_z} + \frac{\bar{Z}\dot{p} K_8 C_{n_{\delta r}}}{F V i_z} - \frac{K_8 \cdot C_{n_{\delta r}} \cdot C_{Y_{\delta r}} K_{10} \cdot P_E}{i_z \cdot 2\Upsilon F}$$

WHERE $\Delta C_{n_\beta} = \frac{K_8 \cdot C_{n_{\delta r}} \cdot C_{Y_\beta}}{2\Upsilon F}$

$$\Delta_4 C_{n_p} = \frac{K_8 \cdot C_{n_{\delta r}} \cdot C_{Y_p}'}{2\Upsilon F}$$

$$\Delta_2 C_{n_r} = \frac{K_8 C_{n_{\delta r}} \cdot C_{Y_r}'}{2\Upsilon F}$$

IN ADDITION FOR SUBSTITUTING FOR a, THE FOLLOWING NOTATION IS NOW USED

$$C_{y_{\bar\beta}} = \left(1 + \frac{K_g C_{y_{\delta r}}}{2F\tau}\right) C_{y_\beta} : C_{l_{\bar\beta}} = C_{l_\beta} + \frac{K_g C_{l_{\delta r}} \cdot C_{y_\beta}}{2\tau F} : C_{n_{\bar\beta}} = C_{n_\beta} + \frac{K_g C_{n_{\delta r}}}{2\tau F} \cdot C_{y_\beta}$$

$$C_{y_{\bar p}} = \left(1 + \frac{K_g C_{y_{\delta r}}}{2F\tau}\right) C'_{y_p} : C_{l_{\bar p}} = C'_{l_p} + \frac{K_g C_{l_{\delta r}} \cdot C'_{y_p}}{2\tau F} : C_{n_{\bar p}} = C'_{n_p} + \frac{K_g \cdot C_{n_{\delta r}} \cdot C'_{y_p}}{2\tau F}$$

$$C_{y_{\bar r}} = \left(1 + \frac{K_g C_{y_{\delta r}}}{2F\tau}\right) C'_{y_r} : C_{l_{\bar r}} = C'_{l_r} + \frac{K_g \cdot C_{l_{\delta r}} \cdot C'_{y_r}}{2\tau F} : C_{n_{\bar r}} = C'_{n_r} + \frac{K_g C_{n_{\delta r}} \cdot C'_{y_r}}{2\tau F}$$

$$C_{y_{\overline{\delta a}}} = C'_{y_{\delta a}} \qquad C_{l_{\overline{\delta a}}} = C'_{l_{\delta a}} \qquad C_{n_{\overline{\delta a}}} = C'_{n_{\delta a}}$$

THE LATERAL EQUATIONS ARE NOW EXPRESSED IN THE FINAL NOTATION ON THE FOLLOWING PAGE.

APPROXIMATION TO CF105 LATERAL DAMPER SYSTEM

$$p\left[-\frac{C_{Y_p}}{2\mu}+\lambda\right]+\phi\left[-\frac{g}{V}\cos\theta\ [illegible] -\lambda\left(\frac{C_{Y_\phi}}{[illegible]}+\alpha\right)+\frac{\lambda^2\,[illegible]}{V\,[illegible]}\right]+\psi\left[\lambda\left(\frac{C_{Y_\psi}}{[illegible]}-[illegible]\right)\frac{[illegible]}{V\,[illegible]}\right]=\frac{[illegible]}{[illegible]}\left\{\left[C_{Y_\xi}+K_q C_{Y_\zeta}\right]\left[\int\left(K_\phi P-K_\phi\right)dt+K_\phi' P\right]+K_\zeta\,[illegible]\,C_{Y_\zeta}\right\}$$

$$p\left[-\frac{C\ell_p}{[illegible]}\right]+\phi\left[-\lambda\frac{C\ell_\phi}{[illegible]}\frac{b}{2V}+\lambda^2\left(1+[illegible]\right)[illegible]\right]+\psi\left[-\lambda\frac{C\ell_\psi}{[illegible]}\frac{b}{2V}-\lambda^2\left([illegible]\right)[illegible]\right]=\frac{1}{[illegible]}\left\{\left[C\ell_\xi+bC_{Y_\xi}+K_q\left(C\ell_\zeta+bC_{Y_\zeta}\right)\right]\left[\int\left(K_\phi P-K_\phi\right)dt+K_\phi' P\right]+aK_\zeta\,[illegible]\,C\ell_\zeta\right\}$$

$$p\left[-\frac{C_{n_p}}{[illegible]}\right]+\phi\left\{\lambda^2\left(\frac{[illegible]}{[illegible]}-\frac{[illegible]}{[illegible]}\right)-\lambda\frac{C_{n_\phi}}{[illegible]}\frac{b}{2V}+\lambda q\,\frac{[illegible]}{[illegible]}\right]+\psi\left\{\lambda^2\left(1-\frac{[illegible]}{[illegible]}\right)-\lambda\frac{C_{n_\psi}}{[illegible]}\frac{b}{2V}\,[illegible]\right]=\frac{1}{[illegible]}\left\{\left[C_{n_\xi}+cC_{Y_\xi}+K_q\left(C_{n_\zeta}+cC_{Y_\zeta}\right)\right]\left[\int\left(K_\phi P-K_\phi\right)dt+K_\phi' P\right]+aK_\zeta\,[illegible]\,C_{n_\zeta}\right\}$$

$C_{Y_\beta}^{*} = a\,C_{Y_\beta}$ $\quad C\ell_\beta^{*} = C\ell_\beta + b\,C_{Y_\beta}$ $\quad C_{n_\beta}^{*} = C_{n_\beta} + c\,C_{Y_\beta}$ $\quad a = \left[1+\frac{K_y C_{Y_\beta}}{[illegible]}\right]$

$C_{Y_p}^{*} = a\,C_{Y_p}'$ $\quad C\ell_p^{*} = C\ell_p' + b\,C_{Y_p}'$ $\quad C_{n_p}^{*} = C_{n_p}' + c\,C_{Y_p}'$ $\quad b = \frac{K_y C\ell_{Y}}{[illegible]}$

$C_{Y_r}^{*} = a\,C_{Y_r}'$ $\quad C\ell_r^{*} = C\ell_r' + b\,C_{Y_r}'$ $\quad C_{n_r}^{*} = C_{n_r}' + c\,C_{Y_r}'$ $\quad c = \frac{K_y C_{n_Y}}{[illegible]}$

$C_{Y_{\zeta}}^{*} = C_{Y_\zeta} + [illegible]$ $\quad C\ell_\zeta^{*} = C\ell_\zeta + K_\zeta C\ell_{[illegible]}$ $\quad C_{n_\zeta}^{*} = C_{n_\zeta} + K_\zeta C_{n_{[illegible]}}$

$C_{Y_r}' = C_{Y_r} + [illegible]$ $\quad C\ell_r' = C\ell_r + K_\zeta C\ell_{[illegible]}\,[illegible]$ $\quad C_{n_r}' = C_{n_r} + K_\zeta C_{n_{[illegible]}}\,[illegible]$

$C_{Y_p}' = C_{Y_p} - \frac{[illegible]}{b}\left\{K_y C_{Y_\zeta} + C_{Y_\zeta}\left[K_3\left(K_1+K_q\right)+[illegible]K_1\right]\right\}$

$C\ell_p' = C\ell_p - \frac{2V}{b}\left\{K_3 C\ell_\zeta + C\ell_\zeta\left[K_3\left(K_1+K_q\right)+[illegible]K_1\right]\right\}$

$C_{n_p}' = C_{n_p} - \frac{2V}{b}\left\{K_3 C_{n_\zeta} + C_{n_\zeta}\left[K_3\left(K_1+K_q\right)+[illegible]K_1\right]\right\}$

$K_\phi = \xi_\phi \big/ -\int p\,dt$ $\quad K_\phi' = \xi \big/ \int P\,dt$ $\quad K_2 = \xi_\phi \big/ -p$ $\quad K_\phi' = \xi \big/ P$ $\quad$ [illegible]

$[illegible] = K_6\,[illegible]$ $\quad K_6 = K_6' + K_6''$ $\quad K_6' = K_{[illegible]}$ $\quad K_6'' = -\frac{[illegible]}{[illegible]}\int [illegible]\,dt$ $\quad$ [illegible]

$[illegible] = K_7\left(1-[illegible]\right)$ $\quad K_7 = K_7' + K_7''$ $\quad K_7' = K_{7_{[illegible]}}$ $\quad K_7'' = -\frac{[illegible]}{[illegible]}\int [illegible]\,dt$

[illegible] K_6, K_7 are functions of time

$K_q = \xi / [illegible]$ $\quad K_1 = \xi_1 / [illegible]$

[illegible] (R) : $K_{[illegible]} = \xi_{[illegible]} / R$

$\bar{Y}, \bar{Z}$ [illegible] in feet

P = roll rate command

[illegible]

M.S. [illegible] Jan '58

The principle of the time solution of the lateral equations of motion may be seen by the step by step solution of the following differential equation.

$$a\ddot{x} + b\dot{x} + c\,x = \int_{t=0}^{t=t} (K\dot{X} - K\dot{x})\,dt = I$$

where a, b and c are constants and $\dot{X}$ is the command rate of displacement.

The values of the variables are studied at successive constant time intervals of $\Delta t = 2\,h$. The notation used is demonstrated by the following example.

$x_{3/2}.h$ is the value of $\dot{x}$ at time $t = \frac{3\,\Delta t}{2}$ multiplied by $\frac{\Delta t}{2}$

The initial conditions at $t=0$ are $x = \dot{x} = \ddot{x} = 0$

The following is the sequence of steps taken for aolution

$x_{1/2} = x_0 + h\,\dot{x}_0$

$\dot{x}_{1/2} = \dot{x}_0 + h\,\ddot{x}_0$

$I_{1/2} = \frac{1}{2}\left\{K_1(\dot{X}_0 + \dot{X}_{1/2})\,h - K_2.(\dot{x}_0 + \dot{x}_{1/2}).\,h\right\}$

$$\ddot{x}_{1/2} = \frac{I_{1/2} - b\,\dot{x}_{1/2} - c\,x_{1/2}}{a}$$

$\dot{x}_1 = \dot{x}_0 + 2\,h\,\ddot{x}$

$x_1 = x_0 + h\,(\dot{x}_0 + \dot{x}_1)$

$I_1 = \frac{1}{2}\left\{2\,I_{1/2} + K_1.(\dot{X}_{1/2} + \dot{X}_1)\,h - K_2.(\dot{x}_{1/2} + \dot{x}_1)\,h\right\}$

$$\ddot{x}_1 = \frac{I_1 - b\,\dot{x}_1 - c\,x_1}{a}$$

$x_{1\frac{1}{2}} = x_1 + \dot{x}_1\,h$

$\dot{x}_{1\frac{1}{2}} = \dot{x}_1 + \ddot{x}_1.\,h$

... continued...

$$I_{1\frac{1}{2}} = \frac{1}{2}\left\{ 2\ I_1 + K_1\ (\dot{X}_1 + \dot{X}_{1\frac{1}{2}})\ h - K_2 (\dot{x}_1 + x_{1\frac{1}{2}}) \right.$$

$$\ddot{x}_{1\frac{1}{2}} = \frac{I_{1\frac{1}{2}} - b\ \dot{x}_{1\frac{1}{2}} - c\ x_{1\frac{1}{2}}}{a}$$

$$\dot{x}_2 = \dot{x}_1 + 2\ h\ \ddot{x}_{1\frac{1}{2}}$$

$$x_2 = x_1 + h\ (\dot{x}_1 + \dot{x}_2)$$

$$I_2 = \frac{1}{2}\left\{ 2\ I_{1\frac{1}{2}} + K_1 \cdot (\dot{X}_{1\frac{1}{2}} + \dot{X}_2)\ h - K_2\ (\dot{x}_{1\frac{1}{2}} + \dot{x}_2) \right.$$

$$\ddot{x}_2 = \frac{I_2 - b\ \dot{x}_2 - c\ x_2}{a}$$

etc.

INCREMENTAL AILERON AND RUDDER MOVEMENTS

WITH LATERAL DAMPER ENGAGED

$$\delta a = K'_4 \int_{t=0}^{t=t} P.dt - K_4 \int_{t=0}^{t=t} p.\, dt + K'_5 P - K_5 p$$

$$\delta r = K_6.\delta a + K_7.r - .061\, K_7 P$$

$$+ K_8 \left\{ - \bar{Z}.\dot{p} + \bar{X}.\dot{r} - p\,\alpha_1.V - \phi.g \cos\phi_1 \cos\beta_1 \cos \textcircled{H} + \frac{V.d\beta + r.\cos\alpha_1 V}{dt} \right\}$$

$$+ K_9.\delta a.q_1 + K_{10} P_E$$

FOR DEFINITIONS SEE PAGE 12; K_6, K_7 ARE FUNCTIONS OF TIME

DETERMINATION OF DERIVATIVES WITH LATERAL DAMPER ENGAGED

With the above in mind the side force equation is more conveniently expressed as follows.

$$-\beta\frac{C_{\bar{Y}_\beta}}{2\tau} - p\frac{C_{\bar{Y}_p}}{4\mu_2} - r\frac{C_{\bar{Y}_r}}{4\mu_2} + \left[\frac{\bar{Z}\dot{p}}{V} - \frac{\bar{X}\dot{r}}{V}\right]\left[1 + \frac{K_6 C_{Y_{\delta r}}}{2\tau F}\right] + \frac{a_Y}{V}$$

$$= \frac{a}{2\tau}\left\{\left[C_{\bar{Y}_{\delta a}} + K_9 q_{f_1} C_{Y_{\delta r}}\right]\left[\int(K'_4 P - K_4 p)\,dt + K'_5 P\right] + K_{10} P_E C_{Y_{\delta r}}\right\}$$

Let it be supposed that the above equation exactly expresses the side force equation and that physical measurements are 100% accurate.

The unknowns in the equation are, $C_{\bar{Y}_\beta}$, $C_{\bar{Y}_p}$, $C_{\bar{Y}_r}$, $C_{Y_{\delta r}}$, $C_{\bar{Y}_{\delta a}}$

Now if the values of the variables β, p, r, $\dot{r}$, $\dot{p}$, a_Y, P_E, P are sampled simultaneously at five different times, t_1, t_2, t_3, t_4, t_5 and the values are substituted into the side force equation to form eqn:- OY_{t_1}, OY_{t_2}, OY_{t_3}, OY_{t_4} : OY_{t_5} simultaneous solution of these equations will yield values of $C_{\bar{Y}_\beta}$, $C_{\bar{Y}_p}$, $C_{\bar{Y}_r}$, $C_{Y_{\delta r}}$, $C_{\bar{Y}_{\delta a}}$

However in practice physical measurements are not 100% accurate. Hence some of the afore-mentioned derivatives would be evaluated with doubtful accuracy. This situation may be remedied by the formation of additional cases.

It is intended that in many cases flight recording frequency will be 20 values per second. However if the damper is engaged and the impulse command signal is quickly reduced to a constant zero value, the variables will approach steady values about 0.5 seconds after the initial impulse command signal. Hence in many cases it is only possible to sample the values at N distinct times where $N < 0.5/.05$. Hence the number of equations is restricted to Eqn. t_1, Eqn. t_2 ——————— Eqn. t_7 where $(t_2 - t_1) = (t_3 - t_2) = (t_4 - t_3)$ etc. = 0.05 seconds.

.... continued....

As time increases from the instance of the initial impulse command signal, the variation in the values of the variables becomes less as the command signal approaches a constant value. Hence if it is decided to form 10 sets of 5 equations it is advantageous to utilise combinations of Eqn. t_1 --------- Eqn. t_6, to the maximum. Equation t_7 is used to form an additional four cases, although more are possible. It is thought that increased accuracy will not be attained by more cases.

Hence the formation of the following selected cases on the next page:-

CASE 1 Eqns. $t_1, t_2, t_3, t_4, t_5 \rightarrow (C\bar{y} : C\bar{y}_p : C\bar{y}_r : Cy_{\delta r} : C\bar{y}_{\delta a})_1$

" 2 " t_2, t_3, t_4, t_5, t_6 " (" " " " " $)_2$

" 3 " t_1, t_3, t_4, t_5, t_6 " (" " " " " $)_3$

" 4 " t_1, t_2, t_4, t_5, t_6 " (" " " " " $)_4$

" 5 " t_1, t_2, t_3, t_5, t_6 " (" " " " " $)_5$

" 6 " t_1, t_2, t_3, t_4, t_6 " (" " " " " $)_6$

" 7 " t_1, t_2, t_3, t_4, t_7 " (" " " " " $)_7$

" 8 " t_2, t_3, t_4, t_5, t_7 " (" " " " " $)_8$

" 9 " t_1, t_3, t_4, t_5, t_7 " (" " " " " $)_9$

" 10 " t_1, t_2, t_4, t_5, t_7 " (" " " " " $)_{10}$

$$C\bar{y}_\beta = 1 / n \{ (C\bar{y}_\beta)_1 + (C\bar{y}_\beta)_2 \text{ ---------------- } + (C\bar{y}_\beta)_n$$

$$C\bar{y}_p = 1 / n \{ (C\bar{y}_p)_1 + (C\bar{y}_p)_2 \text{ ---------------- } + (C\bar{y}_p)_n$$

etc. where $n \leqslant 10$ and n is the number of cases formed.

The same process may be applied to the rolling and yawing equations.

Similarly

$$C\bar{l}_\beta = 1 / n \left\{ (C\bar{l}_\beta)_1 + (C\bar{l}_\beta)_2 + \text{------------} + (C\bar{l}_\beta)_n \right.$$

$$C\bar{l}_p = 1 / n \left\{ (C\bar{l}_p)_1 + (C\bar{l}_p)_2 + \text{------------} + (C\bar{l}_p)_n \right.$$

etc. where $n \leqslant 10$ and n is the number of cases formed

Similarly

$$C\bar{n}_\beta = 1 / n \left\{ (C\bar{n}_\beta)_1 + (Cn_\beta)_2 + \text{------------} + (C\bar{n})_n \right.$$

$$Cn_p \; 1 / n \left\{ (C\bar{n}_p)_1 + (Cn_p)_2 + \text{------------} + (C\bar{n}_p)_m \right.$$

etc. where $n \leqslant 10$ and n is the number of cases formed.

The solution of the three lateral equations of motion yields the values of $Cy_{\delta r}$, $Cy_{\delta a}$, $Cl_{\delta r}$, $Cl_{\delta a}$, $Cn_{\delta r}$, $Cn_{\delta a}$ From these values and the evaluation of the effective derivatives $C\bar{n}_\beta$, $C\bar{l}_p$ etc., the values of the free aircraft derivatives Cn_β, Cl_p etc. may be determined. Now if the gains of the servo- mechanism system are altered, new values of the effective derivatives may be evaluated and the consequent response of the aircraft to an arbitrary command signal, can be calculated by the afore-mentioned step by step method of response determination.