MARGY
THIS IS THE
STUFF THAT
HAD TO BE DONE
ABOUT 18 YRS AGO
I SURE HOPE
THINGS HAVE
CHANGED
GW

ROTATING DISCS WITH TEMPERATURE GRADIENTS

by

G. W. Proctor

My grandfather
Howard Percival
Proctor held onto it
then gave it to my
sister.

Don McLaren

University of Toronto

1952

ROTATING DISCS WITH TEMPERATURE GRADIENTS

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G. W. Proctor

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of the requirements for the degree of
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CONTENTS

PREFACE

Due to the increasing practical applications of rotating discs with temperature gradients, it has been considered advisable by the Author to approach the topic from a general point of view and to select a most suitable application for a more detailed study. In this way it is to be hoped that the extent of the discussion will be sufficiently broad to apply successfully to all cases.

INTRODUCTION

The rapidly increasing power demand of modern times has led to the preferred use of large rotating machines because they have great advantages from a technical as well as from an economical point of view.

Far from the least important of the revolving components of these machines is the rotating disc, which plays a most coveted role in superchargers, axial compressors, and steam and gas turbines. While design problems exist in all rotating bodies, only in "thin" discs do they become laborious, time-consuming and complex.

With the trend of higher operating temperatures found in recent steam and gas turbine developments, thermal stresses as well as centrifugal stresses are becoming more and more important and cannot be neglected. The investigation and research associated with the additional thermal stresses have necessitated a great increase in time spent on design, and have revealed in some cases that these stresses outweigh those due to centrifugal forces. Complications in design have also arisen due to the demand for disc materials to undergo plastic deformation by yielding as a normal experience.

It has become apparent that disc stresses, under the high temperature working conditions of today, have few parallels in severity in normal engineering practice and only limited assistance can be obtained from the known properties of materials, which in other circumstances, are usually sufficient for design.

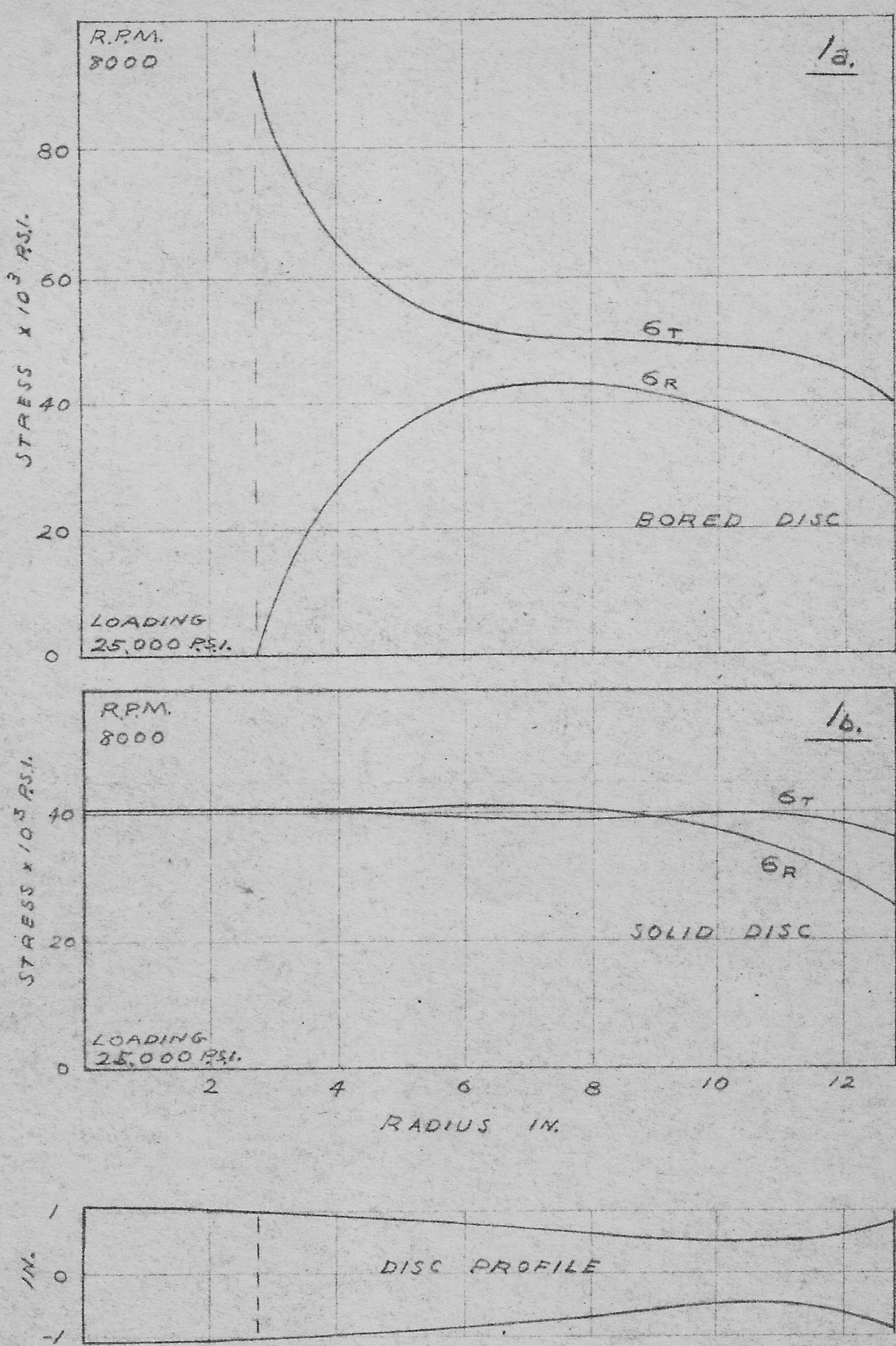
Chapter I

CENTRIFUGAL STRESSES IN ROTATING DISCS

Rotating disc wheels are subject to the triple action of radial, tangential and axial stresses. If there is no sudden change in thickness at the hub, and if the disc is axially "thin" in relation to its diameter, axial stresses may be neglected. The general assumption is therefore accepted that thin rotating discs are subjected to two principal stresses, the radial and the tangential. These biaxial stresses are, in the case of a disc with no temperature gradient, due to the centrifugal forces of the wheel itself and to any peripheral loading which the disc may support. In the case of steam and gas turbines there exists a rim loading consisting of blades or "buckets" fitted around the periphery by means of welding or suitable machined slots. The effect of this loading is to increase the radial stress throughout the disc and to a lesser degree, the tangential.

The general stress distribution for a rotating disc of non uniform cross-section with a rim loading is shown in diagrams (1a) and (1b). The disc is a gas turbine rotor of Inconel X with a speed of 8000 R.P.M. and a rim loading due to blades of 25,000 P.S.I.[1]. The radial and tangential stress distribution for a solid

CENTRIFUGAL STRESSES



AXIAL FLOW GAS TURBINE DISC — SOLID AND BORED

σ_R - CENT. RAD. STRESS

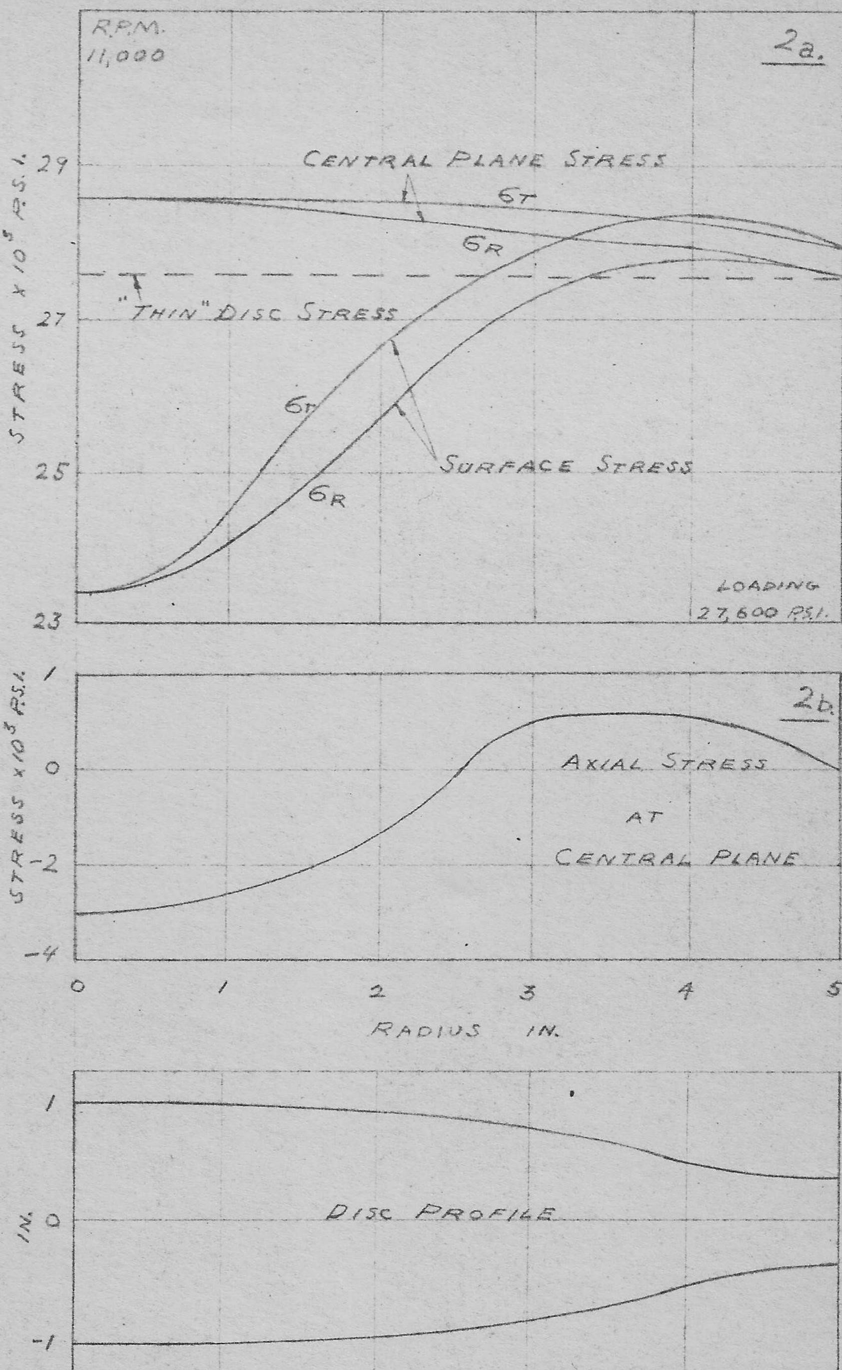
σ_T - CENT. TAN. STRESS

disc with no bored hole at the centre is given in diagram (1b). The two principal stresses are tensile and equal a maximum at the disc centre or axis of rotation. The radial stress drops to equal the rim loading stress at the periphery. The tangential stress drops to a value in tension that is determined by the general disc contour, the speed and the radial loading. In diagram (1a) the centrifugal stresses for a disc with a central bored hole of 5.5 inches diameter are shown. The effect of the hole is to reduce the radial stress to zero at the inner radius and to act as a stress raiser with respect to the tangential stresses, producing a concentration of extremely high tensile stress.

The action of the hub is important in its strengthening effect on the rest of the disc and the restraint it offers may be more clearly understood if three separate hollow cylinders, one within the other, are considered. The radial expansion, being a function of the tangential velocity of each ring or cylinder is greatest for the outer ring and least for the innermost. Thus if the rings were welded together, the extent of expansion of the outer cylinder ring would then depend upon the restraint offered by the next inner cylinder which in turn would depend upon the restraint offered by the innermost cylinder. From this reasoning it can be seen that the addition of material at any radius of the disc does not always produce a strengthening effect. There are three general conclusions that may be deduced, however, and these are that the hub is the restraining portion of a rotating disc, that the larger the ratio of hub strength to disc inertia forces, the stronger the disc and that a stronger hub restricts radial expansion to a great extent thus reducing the tangential stress[2].

Should the hub, however, be very large, coinciding with high disc stresses, axial stresses will be present as is shown in diagram (2b). The disc profile shown is that of a General Electric Supercharger rotor.[3] The design of the profile has been carried out in such a way as to produce an almost constant value of the radial and tangential stresses according to "thin" disc theory. This stress is 27,400 P.S.I. Analysing the disc, assuming there is a three dimensional distribution of stresses, reveals the errors of "thin" disc analysis. At the central plane of the disc the radial and tangential stresses are both greater than that predicted for a "thin" disc and at the surface, the stresses are much lower. The axial stress of diagram (2b) is calculated at the central plane of the disc and has a maximum of 3000 P.S.I. in compression at the hub. This stress, while almost negligible, aids the radial and tangential tensile stresses

THICK DISC STRESSES



SOLID, CONSTANT STRESS, SUPERCHARGER DISC

σ_R CENT. RAD. STRESS
σ_T - CENT. TAN. STRESS

in causing yielding. It can be seen that for all discs with the same or a smaller value of hub thickness to rim diameter ratio "thick" disc analysis may be replaced by a "thin" disc theory assuming, of course, sound material, and that the material where the highest stresses occur in unbored discs is favourably situated, being unaffected by machining. The "thin" disc stresses will represent the average stress distribution across the axial depth of the disc.

The problem of determining the biaxial stress distribution in discs is one of the most complicated in machine design. It has been approached from completely mathematical solutions, from graphical methods, and from finite difference tabular solutions.

The complete conditions which determine the centrifugal stresses in a disc and which must be satisfied are as follows:[4]

$$\frac{1}{h}\frac{d(rh\sigma_R)}{dr} - \sigma_T + \rho w^2 r^2 = 0 \qquad (1)$$

$$\frac{dU}{dr} = \frac{1}{E}\left(\sigma_R - \mu\sigma_T\right) \qquad (2)$$

$$\frac{U}{r} = \frac{1}{E}\left(\sigma_T - \mu\sigma_r\right) \qquad (3)$$

r - radius of any point in the disc

h - disc thickness at radius R (inches)

σ_R - radial centrifugal stress

σ_T - tangential centrifugal stress

ρ - density of disc material

ω - angular velocity of the disc

U - radial deformation

E - Young's Modulus

u - Poisson's Ratio

The complete stress distribution is found by the simultaneous solution of these three equations. Equation 1 represents the equilibrium conditions and is found by considering the equilibrium of a small element of a rotating disc with axial symmetry. Equations 2 and 3 represent the compatability conditions for a two-dimensional stress distribution with axial symmetry and are found by determining the relations between radial and tangential strains and the radial deformation. If the thickness can be given as a function of the radius (i.e. $h = f(r)$) the values of the radial and tangential stress can be found as functions of the radius. The effect of rim loading must be considered also and this requires a slightly more complicated solution.

For discs with flat parallel sides, that is is constant, stress distributions have been determined by Simoshenko[4]. In some cases where the thickness of a disc does not vary appreciably from hub to rim, an approximate stress analysis may be performed considering a flat plate.

To facilitate production, some discs are machined with flat, tapering sides. In this case the thickness h is still a simple function of the radius and the stress solution results in infinite series.[5]

Irregular profiles have been more accurately analysed by fitting a suitable hyperbolic function to the disc contour and determining its equation (i.e. $h = Kr^{-n}$). This method was proposed first by Grübler in 1906 and developed by Dr. A. Stodola for use in conjunction with turbine wheel stresses. Approximately forty-five hours are required to complete and to check the calculations which require long and involved mathematical manipulation before they can yield the desired design data.

For discs whose profiles were not so easily expressed by one single equation an adaption of the foregoing method was proposed by R. Krouse in 1947[6]. By matching portions of the disc profile with several different hyperbolae and solving the resulting equations of equilibrium and compatability a more accurate stress distribution was obtained. The length of time for the calculations was, of course, proportionately increased.

Dr. Carl G. P. Laval, during his studies of rotating discs, proposed an equation of a disc's profile which would allow a constant value of stress to exist throughout the disc. This equation is of the type —

$$h = h_o e^{m} \qquad \text{WHERE} \qquad m = \frac{-\rho \omega^2 r^2}{2 6_T}$$

h_o = HUB THICKNESS

and, of course, applies only to solid unbored discs.

Several various graphical and tabular finite difference solutions have been proposed in recent years. Possibly the shortest method of all these, for an experienced mathematician, is that of C. H. Hartree. The method is applicable to highly irregular profiles of any shape and is based upon a finite difference solution of equations 1, 2 and 3. Trial and error attempts are necessary and a keen mathematical mind is required to reduce the time consuming calculations to a minimum by accurate guesses.[3]

The outstanding graphical method is by C. M. McDowell.[7] A radius of curvature type of integration is carried out graphically and while the method is one of trial and error, only three or four trials are necessary for an experienced person.

The finite difference solution, in a tabular form is probably the most widely accepted method of today. It is flexible in that adjustments can be made so that other distributions such as thermal stresses can also be calculated and superimposed on the centrifugal stresses. Important contributions to this method have been made by A. S. Thompson, W. Leopold, S. S. Manson, M. B. Millenson and M. Donath. The methods are, for the most part, similar in many respects but each has its own characteristics. All involve the simplest of tabular calculations and while lengthy, can be performed by a person with only limited mathematical knowledge.

It may be concluded that, in general, discs under the action of centrifugal forces only, lend themselves to complete mathematical solutions. Errors are, of course, introduced, where the disc profile cannot be accurately approximated but these are usually small and if of a necessity higher accuracy is required, one of the finite difference solutions may be used.

Chapter II

THERMAL GRADIENTS

When a disc has heat applied to its rim or to its centre of rotation, a temperature gradient is set up and the heat flow is in a radial direction. Due to the differential expansion within the disc, thermal stresses are produced, which vary in severity with the magnitude of the gradient and with its steepness or slope. Supercharger wheels, although quite small, have radial temperature gradients produced by hot gases in passing through the peripheral blading at the rim. Axial compressor discs on the other hand, have either no gradient or a negative one, with heating taking place at the hub. As atmospheric air enters the axial compressor of a gas turbine aircraft engine, the rim and blading of the first few stages are cooled. Due to the compression of the air, the last stages of the compressor are heated at the rim. The hubs of all stages of the compressor are, however, heated by conduction from the rear of the engine and may be as high as 250° C. at maximum engine speed. Since the rim temperature of the last few stages is also about this temperature, no gradient exists here. The first stage rotor, however, will have a negative temperature gradient of about 200° C. or even greater during cold weather operation.

Steam turbine and gas turbine discs are heated from the rim by hot gases and have positive temperature gradients. The hub temperature of a steam turbine disc is usually maintained as close to the rim temperature as is feasibly possible to reduce the gradient. With rim temperatures as high 470° C. gradients of 200° C. would usually exist. The gradient in a gas turbine disc is of the order of 400° C. for present day operation, with blade temperatures up to 730° C. and the rim at 600° C. The gradient in a gas turbine rotor is by far the severest and gives rise to stresses greater than those in any other rotating disc. The rate of change of the temperature near the rim may be proportional to the second, third or even fourth power of the radius at that point. The gradient will also vary during the engine operation. For a fast acceleration upon starting up, the temperature difference between the rim and hub will be a maximum. At maximum engine speed, the rim is hottest but in this case the hub of the turbine disc is also correspondingly hotter.

The effect of the hot rim and cold hub, in the case of the gas turbine disc, so overshadows all other cases of rotating discs with positive temperature gradients, that the majority of the discussion that will follow will be with reference to the gas turbine disc.

All conclusions that are drawn, however severe the conditions, will apply to all positive gradient discs, but to a lesser degree of severity.

In the case of negative gradients, there are only a few instances of practical importance, notably the first stage of an axial compressor of a turbo-jet engine, and the stresses induced in this case are low in value. The effect, however, of negative gradients, will be referred to from time to time and will be used purposely as a contrast to the positive gradient effects.

Temperature gradients are drawn in diagrams (3) and (4) of chapter III as a function of the radius for a compressor first stage disc and for a gas turbine disc. The compressor disc is typical of those presently in operation in axial flow jet engines of the 5000 to 8000 lb. thrust class. The turbine disc is the same bored disc as shown in diagram (1). The curves for both discs represent the assumed design conditions and are not necessarily the gradients that would be obtained at cruising speeds of the engines.

The measurement of the actual gradients on discs while rotating in operation is extremely difficult and is not usually attempted. Test beds for discs have been built at most engine manufacturing plants and temperatures measured provide a reasonable approximation to the true conditions. The temperatures have been measured by thermocouples, optical pyrometers, and thermionic paint with the thermocouples being the most successful. Thermionic paints, although most attractive, have not been developed to the stage, as yet, where they are completely reliable and accurate.

The hub and rim temperatures are, in most cases of discs, easily obtainable. In cases of low valued gradients of less than 200° C. the effect of the rate of change of temperature with radius is not so important that a linear or second degree functional relation cannot be assumed. In gradients greater than this, a close approximation to the true gradient is necessary. It has been generally accepted in the case of steam and gas turbines that the temperature distribution is proportional to the fourth power of the radius. In some severe cases, however, the fifth power has been known to be used.[3]

Although it is desirable to reduce the magnitude of gradients in discs, it is not always possible. If the hub of a disc with a positive gradient were heated, there would be excessive radial thermal expansion in most cases. A slight increase would also be produced in the centrifugal stresses. More important than the restriction of radial expansion is the proximity of the hub to a journal bearing.

In gas turbine jet engines, the rear bearing is immediately in front of the turbine disc. A hot bearing may be realized in stationary engines but not in the relatively light weight jet engines. To facilitate the rear bearing in this case, cooling air is introduced and the upstream face of the disc's hub is maintained at a suitable bearing temperature. Temperature gradients in the axial direction of up to 300° C. have often existed since the down-stream face of the disc is usually heated by leakage of combustion gases.

The effect of this gradient causes symmetrical bending of the disc in the lateral direction of its axis.[8] Fortunately this deflection is counteracted by the effect of the gas bending moment on the disc's blading which causes bending in the opposite direction. This condition is also present in steam turbines where the temperature difference of the faces is less but the effective diameter of the discs are much larger.

Chapter III

THERMAL STRESSES IN ROTATING DISCS

The effect of thermal gradients is to cause differential expansion within a disc thus setting up a stress distribution. These stresses may be considered as a biaxial distribution as in the case of centrifugal stresses. As a result, the thermal stresses may be added algebraicly to the centrifugal stresses to determine the total biaxial principal stresses.

The calculation of thermal stresses is, for the most part, a repetition of the method used for centrifugal stresses. In the equilibrium equation (1) the term representing the angular velocity of the disc, is set to zero. The increase in the radial and tangential strain, αT, where α equals the thermal coefficient of expansion and T is the temperature, is added directly to the two compatability equations of radial deformation. If T is replaced by a function of the radius, as $T = T_0 + Kr^n$ and if α is assumed constant, there will be no new variables used, and no difficulties will be introduced.

The thermal and centrifugal stresses may be calculated separately assuming an elastic material, but the latter is also dependent to some extent upon the temperature distribution because of the variation with temperature of Young's Modulus and possibly also of Poisson's Ratio.

In recent years the general trend has been to determine the two stress distributions by finite difference tabular methods. The majority of those listed in Chapter I have been developed to accommodate the calculation of both stress systems.

As an example, the Donath Method may be discussed briefly.[10] The disc is divided into a number of rings whose thicknesses are equal to the mean value of the thicknesses of the corresponding elements. The temperature, thickness, coefficient of expansion, Poisson's Ratio and Young's Modulus are considered as changing in abrupt steps but being constant for any one ring. By an ingenious sum and difference of stresses method, Donath obtains a tabular solution. In general, this tabulation consists of evaluating the complementary solutions for centrifugal stresses and thermal stresses and a particular solution governed by the rim and hub loading conditions. The accuracy of the method depends upon the relative sizes of the rings into which the disc is divided and the reliability of the values of temperature, coefficient of expansion and Young's Modulus. The thermal and centrifugal stresses are calculated separately and it is desirable to perform the calculations this way because on one hand, it is possible to modify the thermal stresses by a change in the disc cooling, without producing much change in the centrifugal stresses; on the other hand, the centrifugal stresses can be modified by a change in speed or of disc profile, without producing much change in the thermal stresses.

The tabular type of solution has the advantage that it may be adapted to punch card electronic computors. These machines eliminate an excess of 90% of the man hours expended in calculation.

A 30 to 50 man hour calculation can be obtained in a matter of minutes from a computing machine, the majority of the time being spent in setting up the given data and necessary information on special punch cards. A second advantage of tabular solutions is that the accuracy of the stresses determined, will be higher. The disc may be divided into any number of rings or stations, and as the number is increased, the disc's profile is more accurately approximated. It is customary to have the minimum spacing between stations at the hub and the rim where the profile changes rapidly.

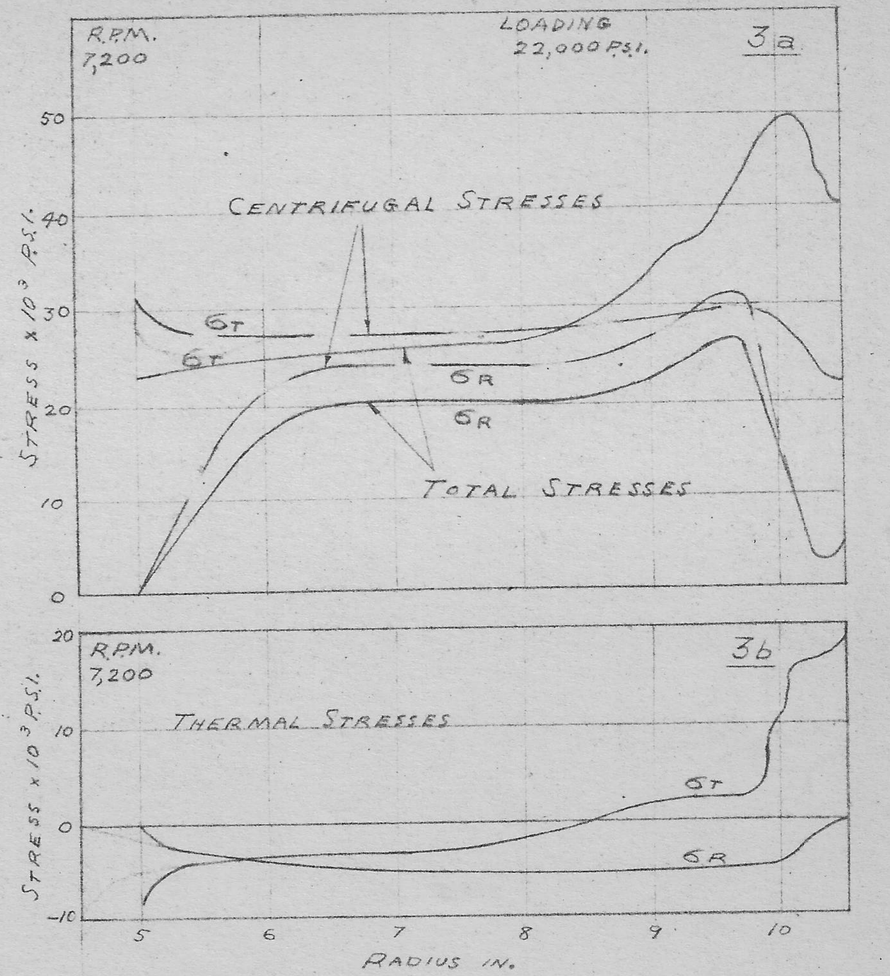
The tabular type of solution applies equally to both positive and negative temperature gradients and the only difference will be the distribution of stresses determined.

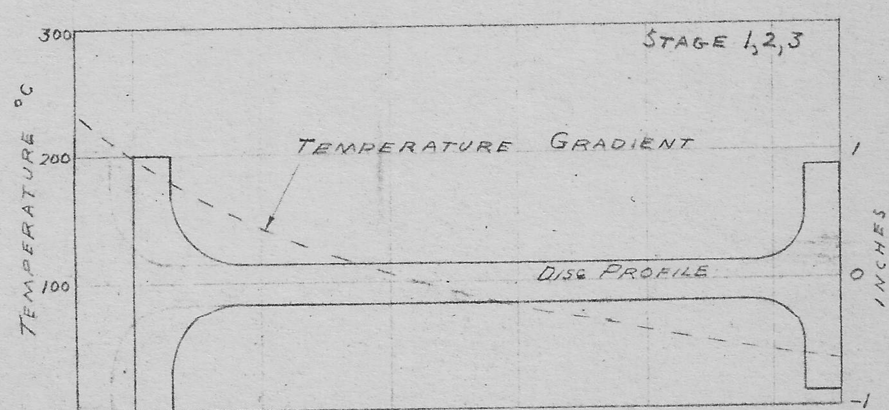
The effect of present day negative gradients in discs is not serious and relatively low stresses are produced. Taking the first stage compressor disc of an axial gas turbine engine as an example, the centrifugal and thermal stresses are shown in diagrams (3a) and (3b). Due to the cold rim and hot hub, there is less restriction in the disc's central region and expansion at the rim is promoted. The cool rim in its role of preventing radial expansion, causes thermal radial compressive stresses to occur throughout the disc, except at the bore and the rim where it must be zero. The thermal tangential stresses progress from compression due to expansion at the bore, to high tension at the rim.

Upon combining the thermal and centrifugal stress distributions, it is found that the total radial stresses are decreased throughout the disc and the resultant tangential stresses are decreased at the hub and increased considerably at the rim.

Due to the nature of the operation of the first few stages of the compressor, it is necessary only to design on the basis of the tensile tangential stress at the rim. It can be seen from the diagrams that stresses in other portions of the disc are much less in comparison. In an acceptable design, this hoop stress at the rim must not exceed the elastic limit of the disc material taken

THERMAL STRESSES – NEG. GRAD.





AXIAL FLOW GAS TURBINE COMPRESSOR DISC

σ_R - RAD. STRESS
σ_T - TAN. STRESS

at the temperature of the rim. Factors of safety are usually not considered for aeroplane gas turbine discs since the weight of a disc is an important factor and the disc thickness will increase appreciably if a design stress less than the elastic limit is used. In actual practice, should this stress be exceeded, plastic flow would take place at the rim only but the stress distribution throughout the whole disc would be altered. Disc failure would only occur when the average tangential stress over a diametral section is equal to the yield strength of the material.[11]

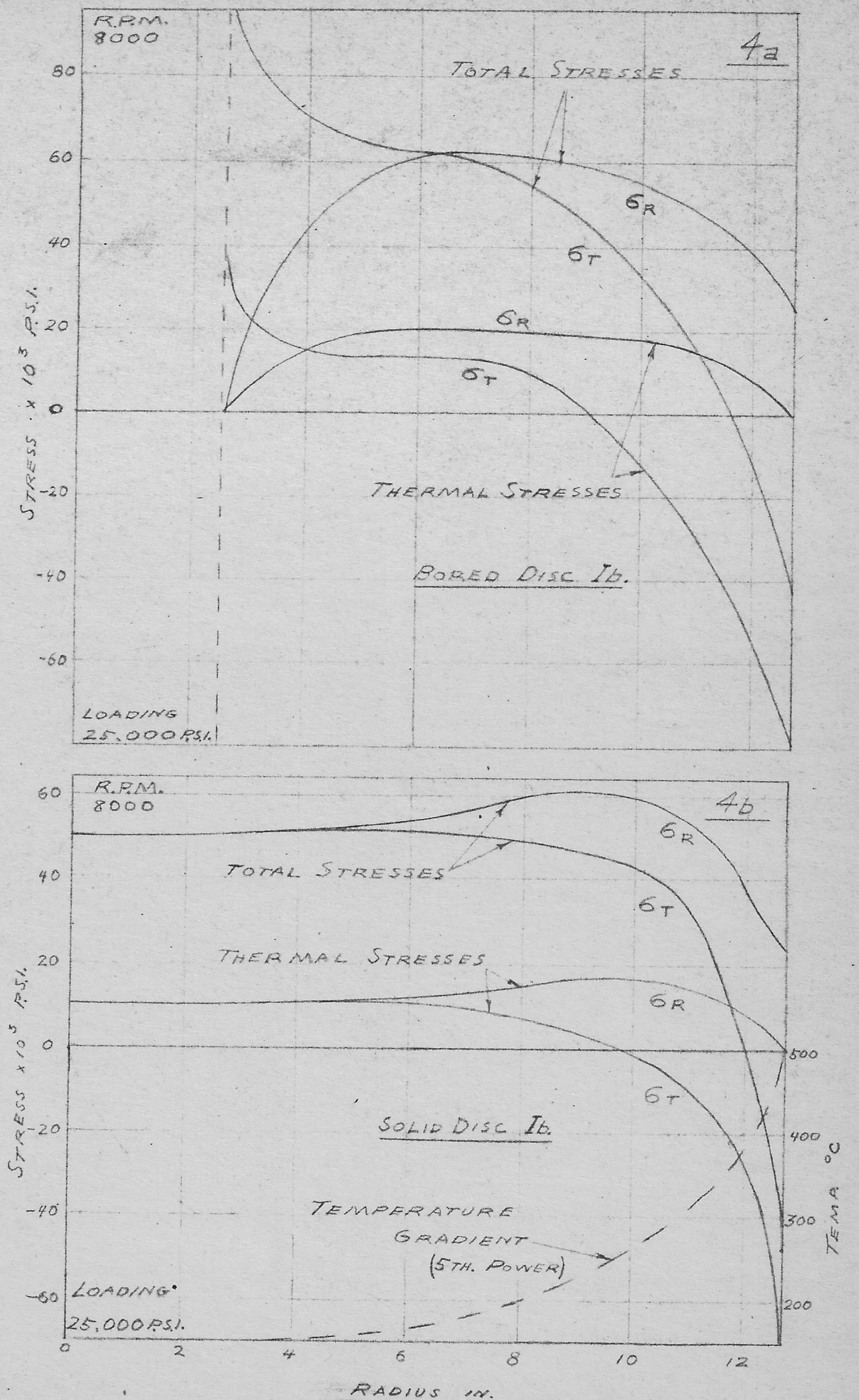
In the disc of diagrams (3a) and (3b) the neck at the rim might easily be thickened with no adverse effects on strength. The hoop stress would not be decreased greatly at the rim and would be increased near the centre of the disc. The radial stress would be decreased at the rim and increased at the hub. In this case more use would be made of the available understressed portions of the disc. Also, with a decrease in hub thickness as well, the total stress distribution at all points of the disc would fall.

Since in the disc shown, the stresses are not severe, the contour or profile is designed in such a way as to be easily machined and mass produced. Thus a parallel sided centre profile is chosen and all curves are circle arcs.[12] Also, a disc which has a slim hub and tapers toward an increasing thickness at the rim is not a practical design since there is a greater distribution of mass at a large radius which adds to the total weight of the disc and, therefore, to the weight of the engine as a whole.

The design of discs with negative gradients will not depend appreciably upon strength considerations alone but will tend to be a problem based upon a correct balance of the most desirable design variables.

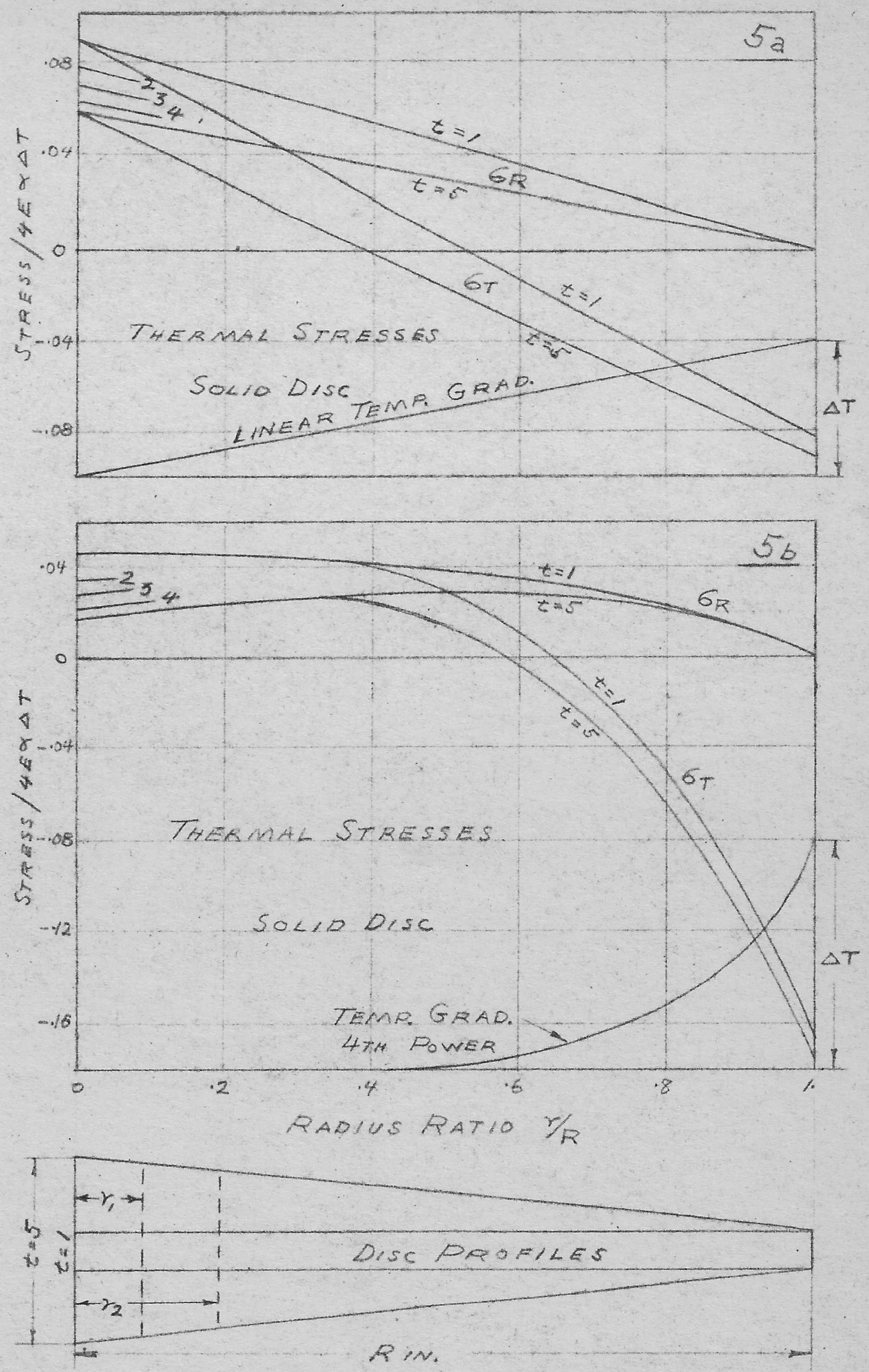
The effect of positive gradients in discs is exactly opposite to that of negative gradients. The turbine disc which is the important example of this type of gradient has a cool hub and a hot rim, and as a result expansion at the rim is restricted by the hub. Thus the radial stress is tensile throughout the disc and is zero at the rim and bore. If there is no bore the stress remains fairly constant for all values of the radius except near the rim. The tangential thermal stress is tensile at the hub reaching a stress concentration peak at the bore and is highly compressive at the rim. The rim compression is due to the cool hub's restriction on the expansion of the rim. Typical curves are shown for an axial flow jet engine turbine disc in diagrams (4a) and (4b).[1] The centrifugal stresses are also shown

THERMAL STRESSES - POS. GRAD.



σ_R, σ_T - RAD., TAN. STRESSES and the total elastic stress distribution is drawn. With this type of gradient the thermal stresses increase the centrifugal radial thus necessitating thicker discs. The thermal tangential tensile stress concentration at the hub (for this bored disc case) adds to the similar centrifugal tangential tensile stress concentration and a dangerous stress peak is reached. In the diagram it is 80,000 P.S.I. At the rim, the thermal hoop stress is reduced by the centrifugal hoop stress but is still in very high compression. This peak value of compressive stress is due to the degree of the temperature variation with the radius. In diagrams (5) and (6)[5] the variation in thermal stresses is shown, for different positive temperature gradients, and also for different hub thicknesses. In (5a) a linear temperature gradient is used and the variation in hub thickness is from one to a factor of five. The stresses are for all purposes linear and the rim compression is not excessive. In (5b) the temperature variation is to the fourth power and the rim compression is highly concentrated. Since there is very little gradient at the hub, the radial and tangential stresses are almost constant in value. In diagram (6a) an eighth power temperature gradient is shown. The stresses at the centre and at the hub are quite constant in this case and the concentration of compression at the hub is extremely

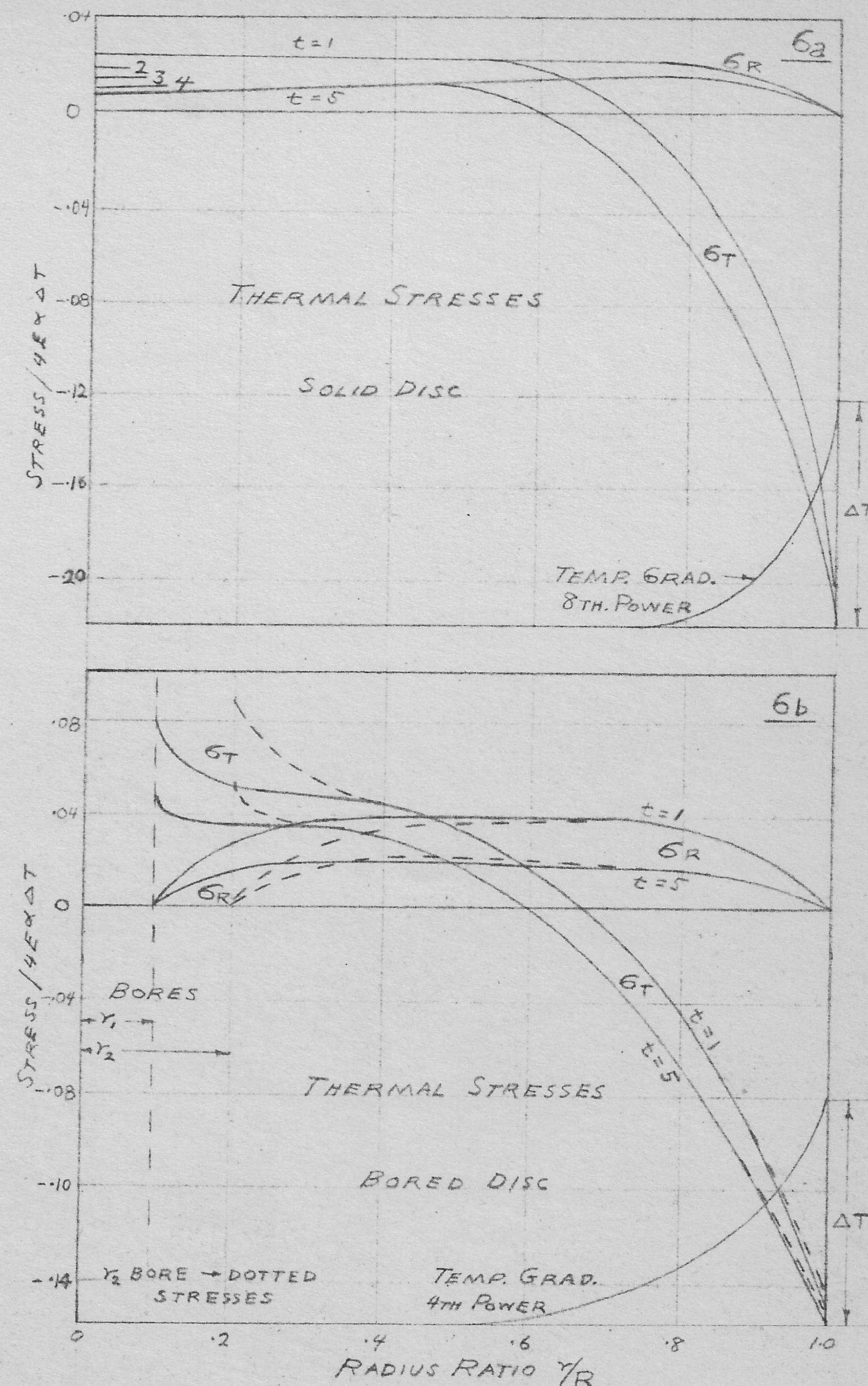
THERMAL STRESSES



STRAIGHT SIDED PROFILE & TEMP. GRADIENT VARIATION

σR, σT - RAD., TAN. STRESSES

THERMAL STRESSES



DISC PROFILES – DIAG. 1.

6R, 6T – RAD., TAN. STRESSES

great. In diagram (6b) the thermal stresses for a bored disc of diagram (5a) are shown. Two diameters of bores are used to illustrate the stress rise due to the larger bore. It may be seen that average disc stresses from the rim inward are lowest for the eighth power gradient and greatest for the linear gradient. The effect of hub thickening is greatest in the case of the linear gradient and reduces hub stresses while increasing rim stresses. These curves clearly illustrate the effect of hub stiffening. In the case of a bored disc, hub thickening plays an important role in that it removes the tensile hub stress concentration while only slightly increasing the rim compression.

Mr. W. Leopold states four conclusions for rotating discs with positive temperature gradients:[1]

1. The slight temperature gradient is beneficial as long as the slight compression values obtained do not become greater than the tensile stresses due to rotation.

2. Large temperature gradients impose large values of compressive stress in the disc and can only be counteracted by allowing the disc to expand under centrifugal force.

3. To increase the strength of a disc functioning under temperature conditions, directly opposite steps must be taken from those necessary to strengthen the disc without a temperature gradient.

4. Low values of the modulus of E and coefficient of expansion result in lower values of thermal stress.

The first conclusion applies to rotating discs with hot rims where thermal stresses are low. This might well apply to the majority of steam turbine discs. Gas turbine discs have such a large gradient that the thermal compression at the rim may equal seven times the centrifugal tensile stresses at the rim.

Conclusions two and three are true but cannot successfully be applied to the gas turbine disc. A thinning of the hub section to reduce thermal stresses promotes very high centrifugal stresses by virtue of the disc's ability to expand. If the centrifugal stresses were of secondary importance in the region of the hub and the centre of the profile, this would be a partial solution to the problem. The centrifugal stresses constitute at least 3/4 of total stresses in the disc, excluding the rim region.

Chapter IV

COMPARISON OF POSITIVE AND NEGATIVE TEMPERATURE STRESSES

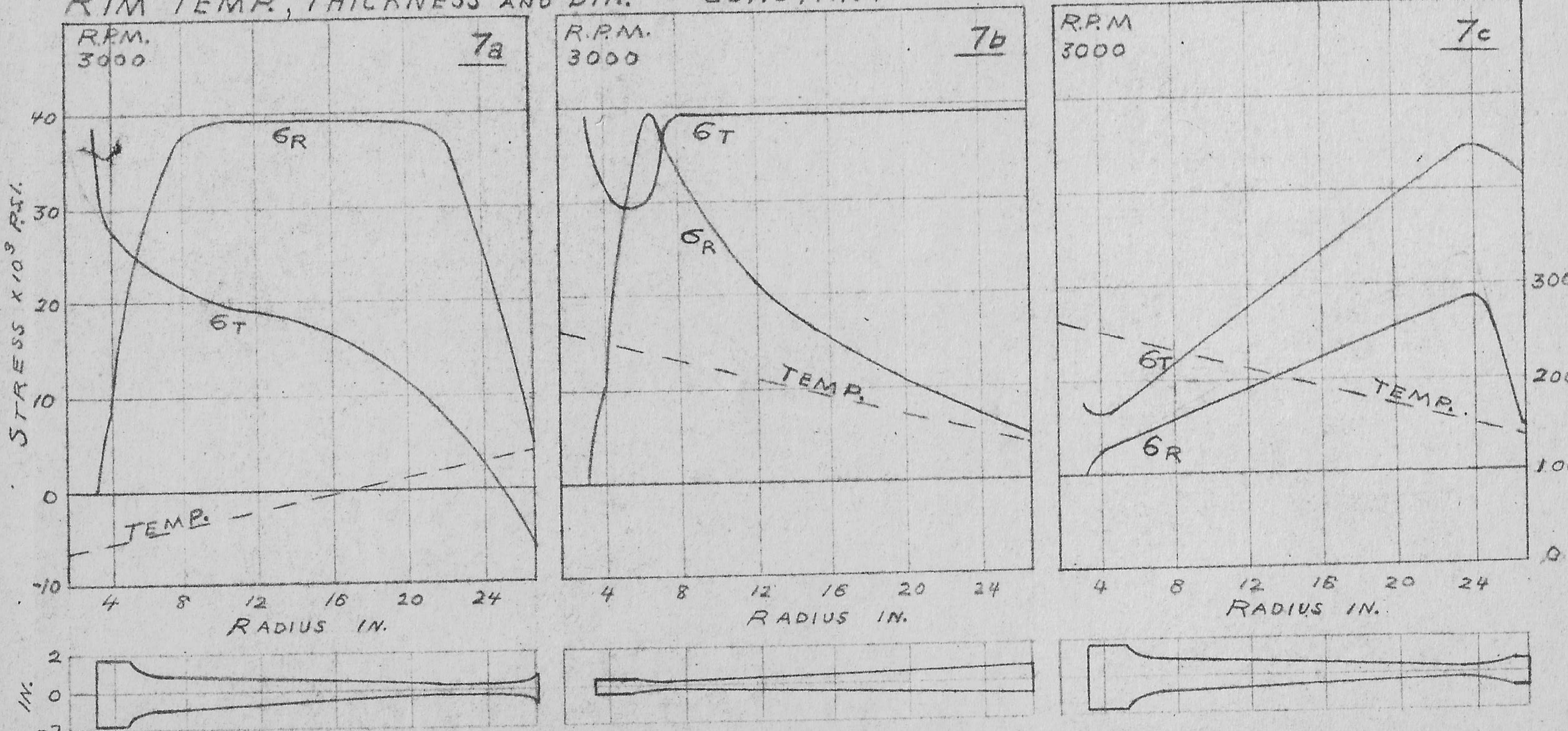
It is convenient at this point to observe the difference in the disc profiles and in stresses, due to positive and negative temperature gradients.

In the accompanying diagrams (7a) and (7b)[10] two bored discs are shown, designed on similar bases but with opposite temperature gradients. The discs are for a steam turbine and are about 52 inches in diameter excluding bucket loading. The stress study was carried out to determine the effect of a hot or cold hub, the rim temperature being held constant at 138° C. The temperature gradient was about 100° C. in both cases and the engine speed was 3000 R.P.M. The design calculations were based upon a rim thickness of 1.10 inches, a bored hub of 3.15 inches diameter, and a radial centrifugal rim loading of 5000 P.S.I. The maximum allowable disc stress for the material used was to be 38,400 P.S.I. The method of design used was that proposed by Holzer in 1913, an extremely lengthy, trial and error type of calculation determining total stresses only.

It can be seen that the hub of the positive gradient disc is necessarily large compared to that of

POS. AND NEG. GRAD. STRESSES

RIM TEMP., THICKNESS AND DIA. — CONSTANT



6_R, 6_T — RAD., TAN. STRESSES

BORED DISCS FOR STEAM TURBINE

the negative gradient. The small hub is necessary in the case of the negative gradient to reduce the tensile loading at the rim which supports the disc. The outer regions of this disc are well utilized with respect to tangential stresses and the unusually slim hub, even with the maximum stress of 38,400 P.S.I., proves sufficient.

Both discs are approximately the same weight despite the effects of opposite gradients. The cooled hub disc has much of its mass at the axis of rotation and the hot hub disc has a larger mass at a greater radius.

The effect of widening the hot hub is seen in diagram (7c). This operation would, of course, be necessary to support the disc against gas bending forces and axial vibration. Increasing the thickness to 3.89 inches, the thickness of the cooled hub disc, there is a decided reduction in radial stresses at the hub and in tangential stresses in the same region.

If the temperature gradients shown were not linear but varied as some power of the radius, the tangential compression at the rim of the cooled hub disc would become larger and the hub stresses less. In this case some material could be removed from the hub. If the gradient varied to some power above one in the hot hub disc the hub tangential stress would drop appreciably and the total tangential would be lower with the total radial stress higher at the rim. Any rise in total radial stress in this case can be reduced by addition of material at the hub.

The cold hub disc cannot be successfully designed as is the one mentioned, when the temperature gradient increases by another 300° C. Adjustment of the profile can do little to counteract the tremendous forces associated with large scale differential thermal expansion.

Chapter V

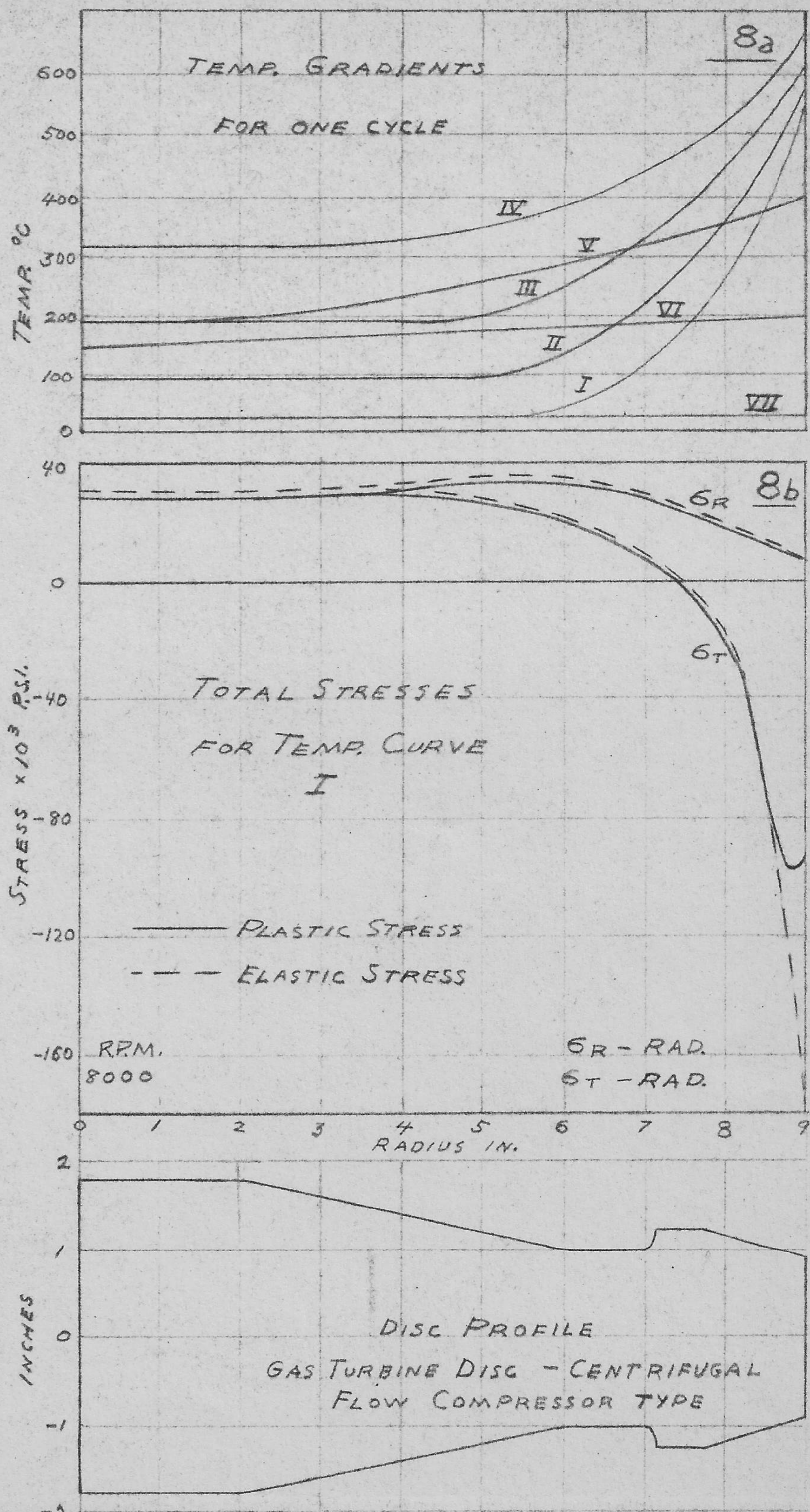
THE EFFECT OF VARYING TEMPERATURE GRADIENTS IN GAS TURBINE DISCS DURING ONE ENGINE CYCLE

A gas turbine disc, whether from an axial or centrifugal flow engine, is taken, before its first run to be free of residual stress. The operation of an engine from warm-up and take-off to landing and engine cooling may be considered as one complete cycle. The temperature gradient imposed upon the turbine disc is constant only for steady flow conditions such as cruising or idling. During the remainder of the cycle, the gradient changes continually. In diagram (8a) the temperature curves are shown for the disc of a centrifugal flow gas turbine engine of American design.[13] The curves are numbered to show their sequence in a cycle.

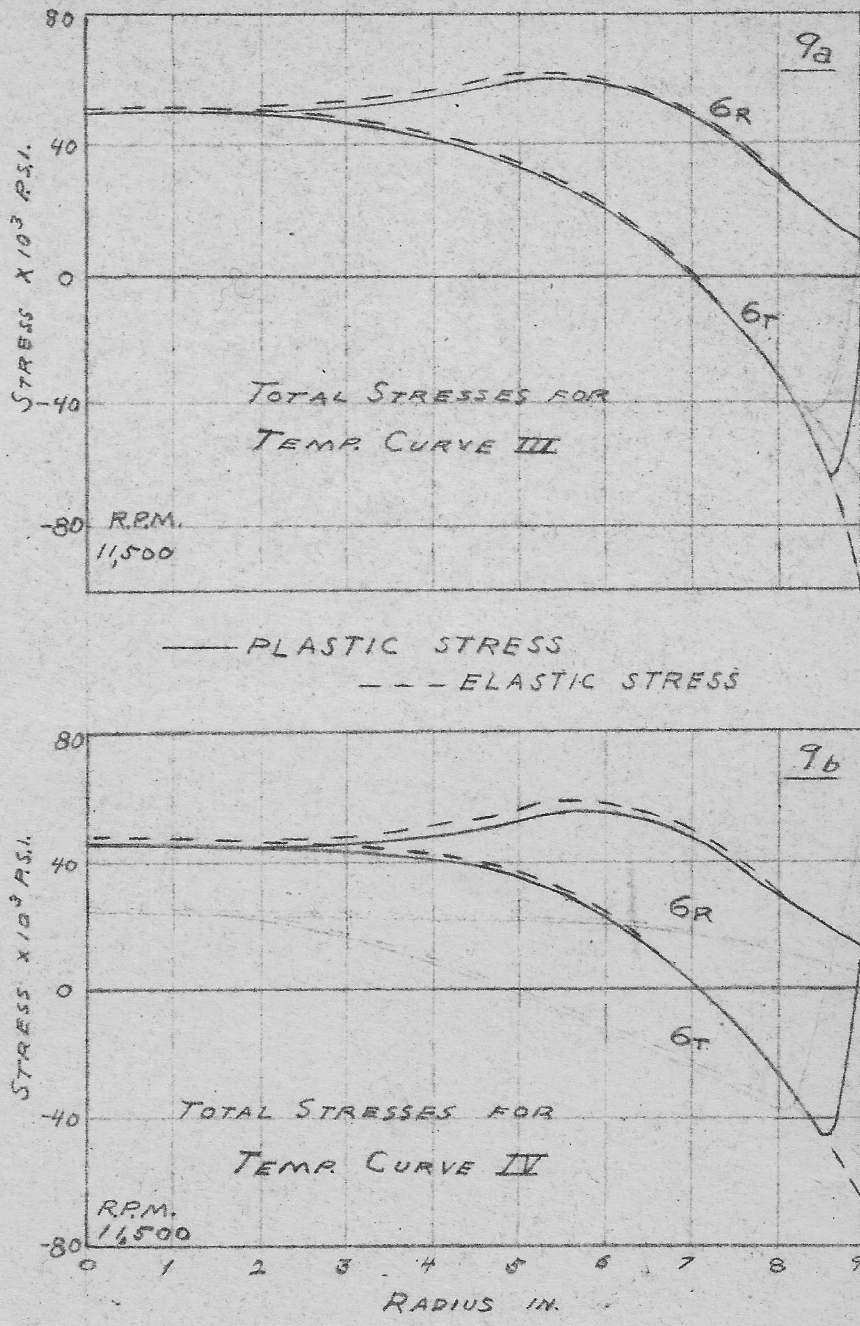
CONDITIONS	CURVE NO.	TURBINE R.P.M.	BLADE LOADING P.S.I.
STARTING	I	8,000	4600
STARTING	II	10,000	7060
STEADY-STATE	III, IV	11,500	9340
STOPPING	V	5,000	1770
STOPPING	VI, VII	0	0

Usually because of the high temperature and high compressive stress at the rim of such a disc plastic

POS. GRAD. STRESSES FOR ONE CYCLE

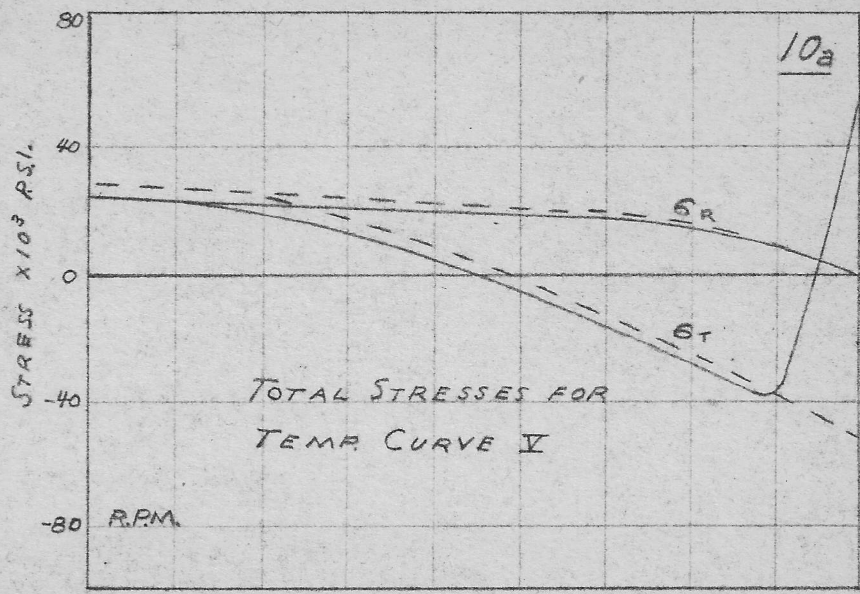


POS. GRAD. STRESSES FOR ONE CYCLE

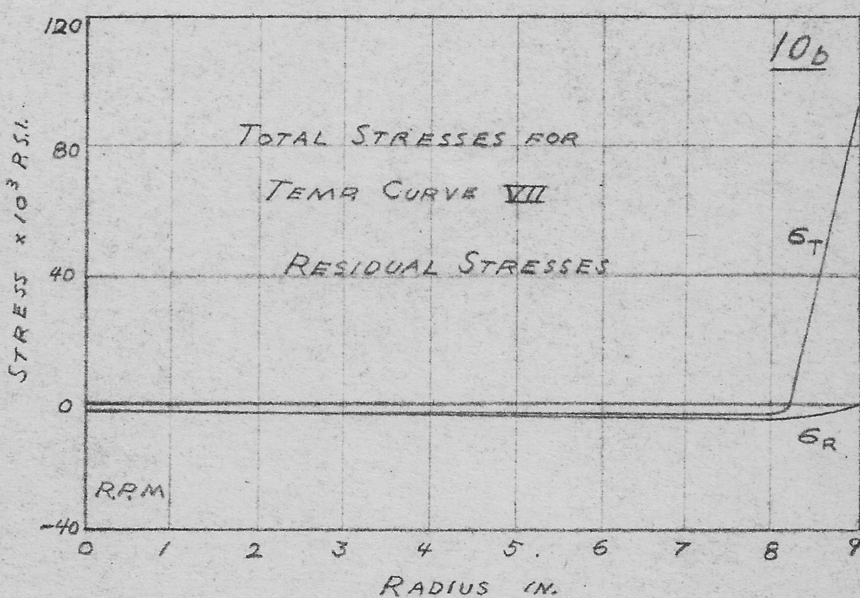


σ_R, σ_T - RAD., TAN. STRESS

POS. GRAD. STRESSES FOR ONE CYCLE



——— PLASTIC STRESS
- - - ELASTIC STRESS



σ_R, σ_T - RAD., TAN. STRESS yielding should occur there and indeed for the disc shown in diagram (8b) operating under the above temperature gradients this does happen. The elastic limit for this material is 80,000 P.S.I. at 260° C. and 60,000 P.S.I. at 570° C. The plastic flow that takes place at the rim alters the remaining stress distribution throughout the disc, and the calculated elastic stresses are not reached.

Plastic and elastic disc stresses (centrifugal plus thermal) are shown in diagrams (8b) to (10b) and are those distributions corresponding to the respective temperature curves for one engine cycle.

The plastic stresses illustrated have been calculated by a method attributed to M. B. Millenson and S. S. Manson.[13] Successive approximations with known stress strain curves were utilized over a range of four temperatures. Since plastic stresses have arisen to such importance in the last few years several methods of calculation have been proposed. The general procedure, however, is to include any departure from linear elasticity in the compatability conditions which are dependent upon stress strain phenomena. When this modification (which differs for each method of approach), to allow for any possible departure from Hooke's Law is made, the compatability equations become true for any values of stress.

The equations of compatability, together with the equilibrium equation, are treated by finite difference methods to produce a solution. The complex nature of the mathematics involved in this stress study is left to more complete and highly technical reports.[13,14,15]

In diagram (8b) the elastic and plastic stresses are shown for the temperature conditions of Curve I of diagram (8a). The gradient is a maximum in this case and the rim compression is also, reaching a maximum of 180,000 P.S.I. Plastic yielding takes place under the high temperature at the rim as shown and as a result the stresses decrease throughout the remainder of the disc.

As the temperature difference drops after the warm up stage, the rim elastic compression reduces, as seen in diagram (9a) corresponding to Curve III. The decrease in stress at the edge of the rim is greatest due to its least resistance to plastic flow. Slightly in from the rim edge, the plastic stresses remain equal to the elastic stresses and are still in high compression. The temperature at this point is not sufficient to cause a yielding at that value of stress.

The centrifugal stresses in this diagram have also increased due to the increase in engine speed. In diagram (9b) the steady state stresses are shown. The reduction in the temperature gradient has caused a still further reduction in rim compression and the plastic stress, due to previous yielding is seen reversing into tension. Diagram (10b) representing engine stopping or slowing down, reveals the decrease in speed by a drop in average stresses over the disc. Rim cooling has caused the reversed plastic stress to become highly tensile and the difference in plastic and elastic stress has increased over the hub and centre of the disc to a value of approximately 5000 P.S.I. Diagram (10b) illustrates the extent of the reversed plastic stress distribution at the rim. The peak concentration is 100,000 P.S.I. There are no centrifugal stresses since the engine has stopped and only those due to the action of plastic flow at the rim are left. This stress distribution is called the residual stress distribution after the first cycle and constitutes the starting stress system for the second cycle. The tensile tangential stresses at the rim are balanced by the radial and tangential compression throughout the remainder of the disc.

It is of interest to note that a distribution of stresses similar to this may be obtained by a rotating disc of approximately the same profile operating under a negative gradient where the disc is uniformly hot throughout but decreases rapidly to a cool rim. That is the temperature gradient as ordinarily drawn would not be concave upward but downward.

Chapter VI

THE EFFECT OF EXCESSIVE POSITIVE TEMPERATURE GRADIENTS

In the construction of gas-turbine wheels, welding has been used at times to attach the blades to the rim of the disc. The reason has usually been, lower costs, more rapid fabrication and a stronger joint. The experience of American and British manufacturers with the welded-blade construction has shown that frequently cracks have formed at the rim of the wheel and have as a result, limited the useful life of welded-blade turbine discs.

It has also been found that turbine discs with inserted blades of the "fir-tree", "dovetail" or cylindrical type root, have discontinuous rims and are not subject to cracking. The portion of the disc's rim between the roots of two blades cannot support any tangential stress and as a result there is only radial plastic flow and no residual stresses. The tangential concentration of residual stress will be below or inside the root serrations of the disc, and at this point the temperature is sufficiently low as to restrict excessive residual stress.[16]

The cracks that have formed in welded blade design follow radial lines as is suggested by the fact that it is the tangential component of the residual stress which becomes excessive. In some axial flow jet engine discs of 28 inches diameter the cracks have grown to a depth of some .25 inches in the first cycle.

The extent of this depth depends upon the magnitude of the residual stress peak and the tensile strength of the material. While these cracks do not lead to immediate failure of the disc, they do tend to grow during the cooling period following each successive cycle of operation. The growth of the cracks, however, becomes less and less due to the fact that the rim region containing the cracks can support no tangential stress during succeeding cycles, and there is less probability of plastic flow and large residual stresses in the region inside the crack roots where the temperature is less severe.

If the ductility of the disc material is high as in carbon-steel, the resistance to cracking is great. The rim temperature is sufficient to cause the disc material at the rim to lose its hardness and to allow plastic flow even after cooling.

In discs of low ductility such as tool steel where the hardness number is large, rim cracking occurs most easily. The brittle resistance offered to plastic flow causes fine cracks to begin at any rim surface irregularity and to grow quickly. In bored discs with central holes where the hub section is thin, cracks may also be propagated. The radial stress is zero at the bore and the tangential tensile stress is very high. Cracks in the hub are not common with ductile steels but may occur at unpolished bores that have become strain hardened in some way.

Chapter VII

THE EFFECT OF PLASTIC FLOW DURING CONTINUED CYCLIC OPERATION

If the disc material were elastic, the stress distribution resulting from engine stopping and disc cooling would be obtained by subtracting the total elastic stresses from the final running stress distribution. It has been seen, however, that plastic flow takes place over the period of a cycle and there is a modification of the stress distribution. Due to the high tensile residual stress that results, plastic flow may occur again at room temperature.

Beginning with the resultant plastic residual stress at room temperature, the effect of the second engine operation cycle may be determined. The complete stress distribution of the second cycle will be altered in accordance with the magnitude of the final plastic residual stresses of the first cycle. Similarly, the stress state of the third and fourth cycles will depend upon the extent of the residual stresses of the second and third.

In diagrams (11b), (12a) and (12b) the tangential elastic and plastic stresses for three engine cycles are shown. The profile of the disc was

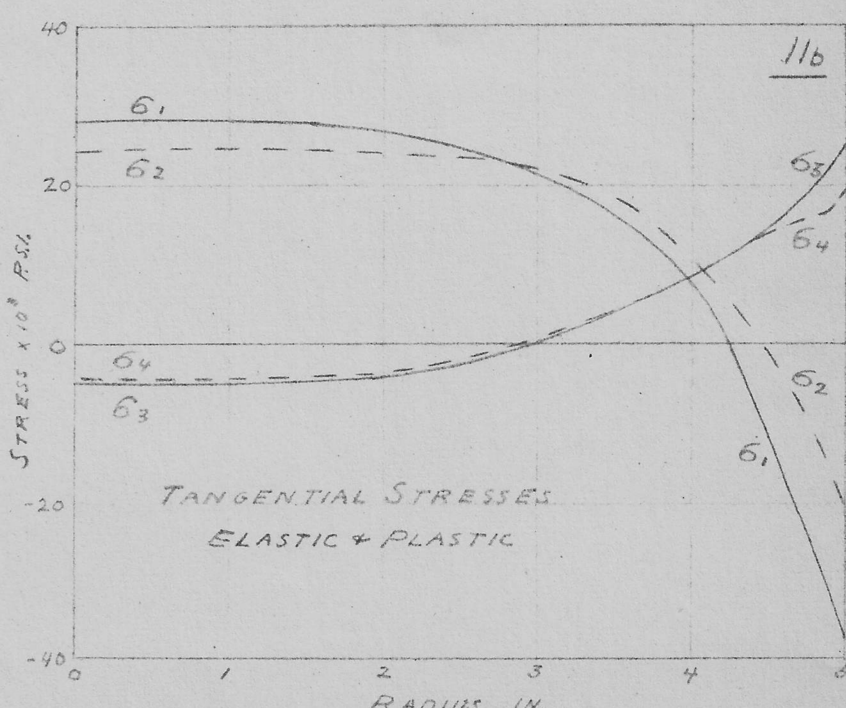
POS. GRAD. STRESSES FOR CONSECUTIVE CYCLES

σ_1 TOTAL PRINCIPAL ELASTIC TANGENTIAL STRESS - AFTER HEATING.

σ_2 TOTAL PRIN. PLASTIC TAN. STRESS - AFTER HEATING AND YIELDING.

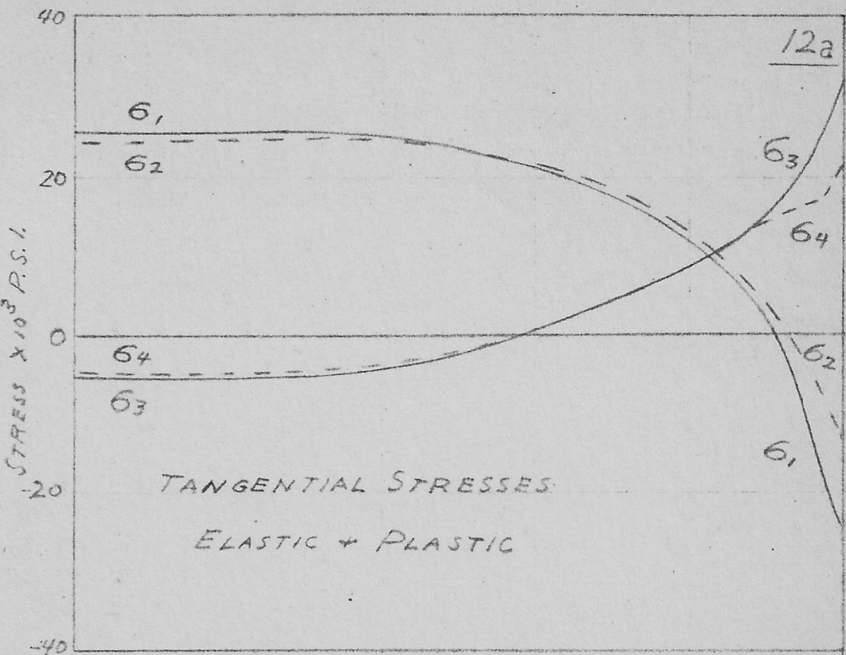
σ_3 TOTAL PRIN. ELASTIC TAN. STRESS - AFTER COOLING.

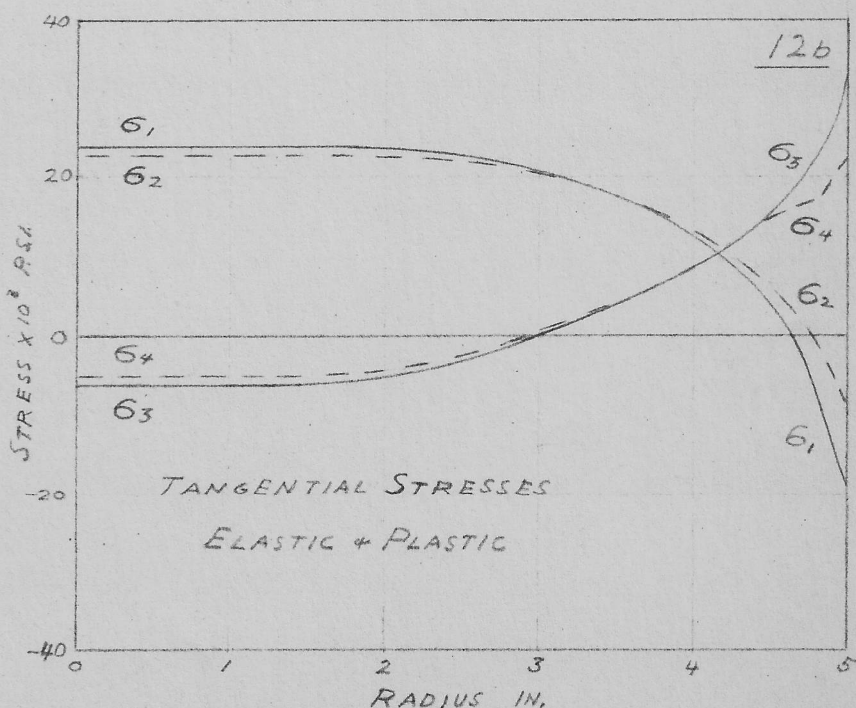
σ_4 TOTAL PRIN. PLASTIC TAN. STRESS - AFTER COOLING AND YIELDING



DISC PROFILE SIMILAR TO DIAG 2.

POS. GRAD. STRESSES FOR CONSECUTIVE CYCLES





the same as that of diagram (2) and the material was "Stayblade", an austenitic steel of low elastic limit.[3] The cyclic temperature distribution was almost identical with that of diagram (8a) and the engine speed was approximately 11,000 R.P.M.

In the first cycle diagram the plastic yielding at room temperature results in a drop of 6000 P.S.I. in the rim residual stress. Starting with this stress for the second cycle the mean elastic stresses throughout the disc are lower with less compression at the rim. There is still plastic flow during engine running but of a smaller magnitude than in the first cycle. The residual stresses immediately upon cooling are greater at all points than in the previous cycle and the elastic tensile at the rim has become 26,000 P.S.I., an increase of 6000 P.S.I. This is due to the cumulative effect of plastic flow at the rim. The reduction of rim stress due to plastic flow at room temperature for the second cycle is, of course, greater and the starting stress distribution for the third engine cycle is, as a result, larger than for the second or first cycles.

In the third cycle the same procedure takes place but in this case the change in the respective stress distributions is less and it could be guessed that the fourth cycle stresses would be almost identical to those of the third.

All stresses were calculated on the basis of the initial stress-strain diagram of the material omitting any strain hardening arising from previous cycles. Although it was not assumed in this case that the strain was cyclic the information present at the date of the investigation suggested that under the present disc temperatures, cyclic strain would usually follow, as did the cyclic stress.[3]

The procedure used in the plastic stress calculations as applied to the stress strain diagrams for the material was only approximate, and while it leads after a few cycles to a condition of stress, the calculated strain would not generally be cyclic even if in practice it proved to be so. This would occur due to the errors introduced by the assumption that at heating up, all the plastic strain occurs at an upper distribution of temperature and when stopping all the plastic strain occurs at the low temperature. Also, another most important difference from practice arises from the fact that the strain and temperature cycles alter the stress strain characteristics of the metal. This factor has recently been the subject of much investigation.

Until the effects of both plastic strain and operating temperatures in modifying the stress-strain characteristics of a material are known with fair accuracy, some conventional use of stress-strain diagrams in the initial condition was found to be necessary. It was recommended at that time that the equivalent tensile plastic strain calculated for each previous cycle be treated as cumulative in finding the part of the stress-strain diagram to be employed for the cycle under consideration.[3] Thus there would be a strain hardening effect represented by the sum of all equivalent tensile plastic strains, for the cycles preceeding the one under examination. Further, it was recommended that only strains for the same part of the cycle be taken together, or in other words, the strain occurring upon heating and cooling are assumed to be quite independent and affect only their respective stress-strain diagrams. No allowance, however, was made for strain hardening in deriving the stress for diagrams (11b), (12a) and (12b) which, as previously noted, were based upon the initial stress strain diagrams unaffected by previous plastic strain.

Referring to diagram (12a) it will be seen that if yielding occurs during the first cycle at the inner part of the disc, as it does with "Stayblade", the lower stresses in the second and third cycles would not cause further plastic deformation and the inner part of the disc would behave thereafter in an elastic manner. This is because the stress is less than the new elastic limit of the material brought about by the previous yielding and provided for by the convention recommended above. At the outer part of the disc plastic strain occurs at each cycle unless it is suppressed by an increase in the elastic limit brought about by the preceeding cycles of temperature and strain. The rim is subjected to alternate compression and extension in which compression occurs largely at the higher rim temperatures and extension at the lower temperatures. An idea of the order of tangential strain to be accommodated is afforded by noting the differential thermal expansion of the rim inside the blade root due to its temperature in excess of the axis temperature. This was taken to be 250° C. for the "Stayblade" disc. The differential expansion resulting was .0039 inches per inch.[3]

If all disc materials behave similarly to the "Stayblade" disc with the exception that they may probably have a higher elastic limit, it appears that except for creep, which has not been discussed, virtually elastic conditions would be developed in all discs operating with an upper temperature of 500° C. at the inside of the blade roots. Cumulative creep would, if present, exercise a disturbing effect and as this factor would assume increasing importance at higher temperatures it must not be lost to view.

If at the beginning of a running period, that is, after heating and yielding, the stress distribution were shown by (12b) any creep would supplement the plastic strain produced by yielding and reduce the tangential compressive stress at the rim. The tangential creep strain would be clearly compressive since like plastic strain, its type would be determined by the algebraic sign of twice the tangential stress minus the radial stress.[3] The radial stress is tensile and positive and the tangential stress is compressive and negative. Thus both principal stresses contribute compressive tangential strain. There would be zero compressive strain when the tangential stress is tensile and equal to one half the radial stress. If it becomes greater than one half the radial stress the tangential creep strain would be positive and tensile. The effect of the tensile creep strain in this case would be to reduce the tangential stress at the rim of the disc. In both cases, creep results in a reduction in the stresses causing the creep and the general behaviour at first approaches creep relaxation conditions.

Generally unless very high rim temperatures and radial loading were used, creep would slow up rapidly. If plastic strain of the outer part of a disc occurs when the disc cools down, it would operate in the direction of neutralizing the preceeding creep as is evident from diagram (12b).

In the "Stayblade" disc problem examined, it was concluded that because of the slowing up of the creep rate, and action of plastic strain in cancelling creep, the radial loading and temperature at the outer part of the disc were not high enough to produce excessive creep. This is likely to be the position, generally, in all well designed discs, because if the blade fastenings are adequate from a creep standpoint the conditions in the disc adjacent would usually be more suitable.

CONCLUSION

The thermal stresses imposed on a rotating disc by the action of temperature gradients may be superimposed directly upon the principal centrifugal stresses along with any residual stresses that may have existed previous to rotation. Positive and negative temperature gradients produce exactly opposite thermal stress distributions, each of which have their own desirable or undesirable characteristics depending upon the slope and magnitude of the gradients. In general thermal stresses due to slight gradients that approach linearity are in most applications considered beneficial while those due to excessive gradients of the "high power" type tend to produce high concentrated stresses dangerous to disc strength.

Due to the nature of the use of the discs in high temperature power plants, positive temperature gradients of excessive magnitude have become by far the most important. The most severe conditions of peak stress, plastic flow, and creep, appear at the rim where the temperature is highest and where it has the greatest range. Given the temperature distribution, very little can be done by design to ease the working conditions because their severity arises largely from the temperature distribution alone and changes in the disc profile, within moderate limits, has negligible influence upon thermal stress.

If the temperature distribution is cyclic in nature, as in most applications it is, the disc rim may undergo plastic deformation by yielding as a normal experience, either for a limited number of cycles or, under more severe conditions, throughout its life.

It is at the rim, therefore, that the working conditions differ so much from normal engineering practice, and new information concerning disc material properties and performance under these conditions are necessary. Examination of the problem reveals a need for more complete high temperature stress strain characteristics of disc materials, and extensive tests and studies of the ability of disc materials to endure for an appropriate life, the expected cycles of plastic as well as elastic stress and strain.

REFERENCES

No.

1. "Steam And Gas Turbines" by A. Stodola, published by McGraw Hill Book Co., 1927. Vol. 1, pp 388-393.

2. "Stresses In Rotating Disks At High Temperatures", by A. Stanley Thompson, A.S.M.E., Preview No. A6, 1945.

3. "First Report On Turbine Discs For Jet Propulsion Units", compiled by the Disc Sub-committee of the M.A.P. High Temperature Materials Research Dept., National Physical Lab., England, 1948.

4. "Theory Of Elasticity" by S. Timoshenko, published 1934 by McGraw Hill Book Co.

5. "Centrifugal And Thermal Stresses In Straight Sided Tapering Discs", By P. G. Gray, Royal Airforce Establishment, England, Report No. E 3956, 1945.

6. "Stresses And Deformations Of Rotating Discs Under The Action Of Temperature Gradients, Thrust, and Precessional Loads", by R. Krouse, S.A.E. Preview No. 7, 1945.

7. "Stresses In Rotating Discs By Radius Of Curvature Integration" by C.M. McDowell, S.A.E. Preview No. 13, 1947.

8. "Study Of Stress States In Gas Turbine Disks As Determined From Measured Operating Temperature Distributions", by J. E. Farmer and M. B. Millenson and S. S. Manson, N.A.C.A. Research Memorandum No. E8C16.

9. "Centrifugal And Thermal Stresses In Rotating Discs", by W. R. Leopold, A.S.M.E. Trans., Vol. 15, 1948.

10. "Steam And Gas Turbines" by A. Stodola, published by McGraw Hill Book Co., 1927. Vol. 1, pp 393-404.

REFERENCES (cont'd)

11. "Effects Of Temperature Distribution And Elastic Properties Of Materials On Gas Turbine Disk Stresses" by A. G. Holmes and R. D. Faldetta, N.A.C.A. Report No. 864, 1947.

12. "Residual Stress Analysis Of Overspeeded Disks With Central Hole By X-Ray Defraction" by James N. Good, N.A.C.A. Research Memorandum No. E8E11, 1948.

13. "Determination Of Stresses In Gas Turbine Disks Subjected To Plastic Flow And Creep" by M. B. Millenson and S. S. Manson, N.A.C.A. Technical Note No. 1637, 1948.

14. "Rotating Disks In The Region Of Permanent Deformation" by F. Laszlo, N.A.C.A. Technical Memorandum No. 1192, 1948.

15. "General Plastic Behavior And Approximate Solutions Of Rotating Disks In The Strain Hardening Range" by M. H. Lee Wu, N.A.C.A. Technical Note No. 2367, 1949.

16. "Investigation Of Rim Cracking In Turbine Wheels With Welded Blades" by M. B. Millenson and S. S. Manson, N.A.C.A. Research Memorandum No. E6L17, 1947.

17. Experimental Investigation Of Rim Cracking In Disks Subjected To High Temperature Gradients" by P. I. Wilterdink, N.A.C.A. Research Memorandum No. 9F16, 1949.