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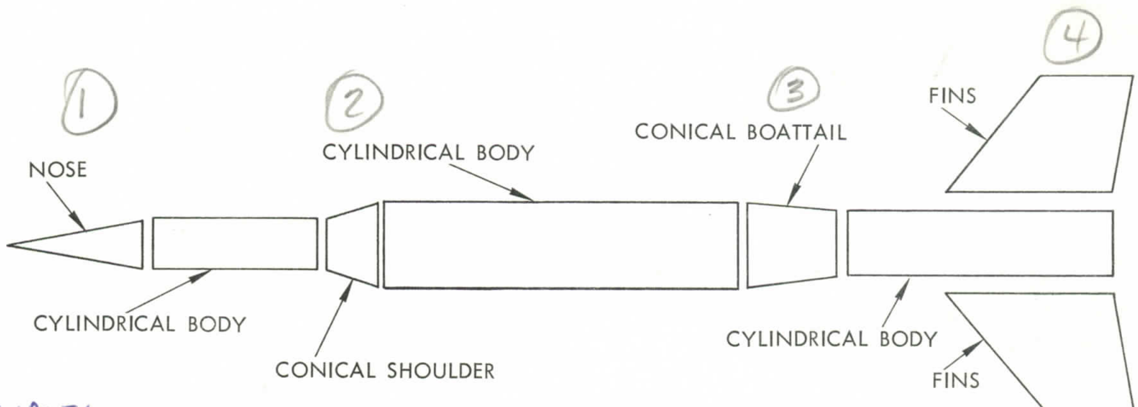
INFORMATION FOR YOU

CALCULATING THE CENTER OF PRESSURE OF A ROCKET

One problem faced by all students of model rocketry is the accurate determination of the center of pressure of their newly designed or constructed rocket. This paper specially prepared by James Barrowman, Aerospace Technologist in Fluid and Flight Dynamics, in the Sounding Rocket Branch of the Goddard Space Flight Center, concentrates on the method and the equations necessary for calculating the center of pressure. No attempt is made to reproduce the theoretical derivations for the equations as this would require a knowledge of the calculus by the reader. The notations used have been simplified, but at the same time are accurate.

In order to determine the center of pressure of a rocket, the rocket is divided into regions, and each region is analysed separately. Then the separate results are combined to obtain the value for the entire rocket. The particular set of equations in this paper is for a rocket that can be divided as shown in Figure 1. If there is more than one conical shoulder, conical boattail, and/or set of fins on the rocket, these should also be analysed separately and then included in the combination calculations.



ON INPUT:
N=1 - NOSE CONE INFO
N=2 - CONICAL SHOULDER
N=3 - " BOATTAIL
N=4 - FINS
N=5 - END OF CASE CARD

Figure 1

YOU MAY HAVE AS MANY INPUT SECTIONS FOR EACH ROCKET i.e. 2 CONICAL SHOULDERS AND 2 BOATTAILS.

Force

The normal force acting on a rocket is the component of the total force acting on the rocket which is perpendicular to the longitudinal axis of the rocket. In aerodynamic theory, the normal force acting on any part of the rocket is nondimensionalized (in simple terms - corrected for) with respect to the air density, velocity, and a vehicle reference dimension. The symbol for such a nondimensionalized force is C_N. In addition, when correcting for the angle of attack (α), it becomes C_{N_α}. At low speeds, C_{N_α} is a constant and depends only upon the shape of the vehicle. Even though C_{N_α} itself isn't normally of interest to the rocketeer, it is very important in the determination of the center of pressure. For simplicity, C_{N_α} will be called force in the rest of this paper.

Center of Pressure

In order to be meaningful, the center of pressure locations of all the portions of the rocket must be measured from the same reference point on the rocket. In this discussion, the forward tip of the nose is used as the common measuring point. The distance of the center of pressure from the nose tip is represented by the symbol, $\bar{X}$.

Subscripts

The subscripts added to C_{N_α} or $\bar{X}$ indicate to which part of the rocket the symbol refers. For example, the force on the nose is indicated by $(C_{N_\alpha})_N$. If a symbol has no subscript, then it refers to the entire rocket. The subscripts used in this report and their meanings are as follows:

CB = Conical Boattail
CS = Conical Shoulder
F = Fins
N = Nose
T(B) = Fins in the presence of the body.

Presentation of Equations

A diagram of the forces acting on a rocket and their associated centers of pressure is shown in Figure 2a. The locations of the conical shoulder, conical boattail and fins are defined in Figure 2b.

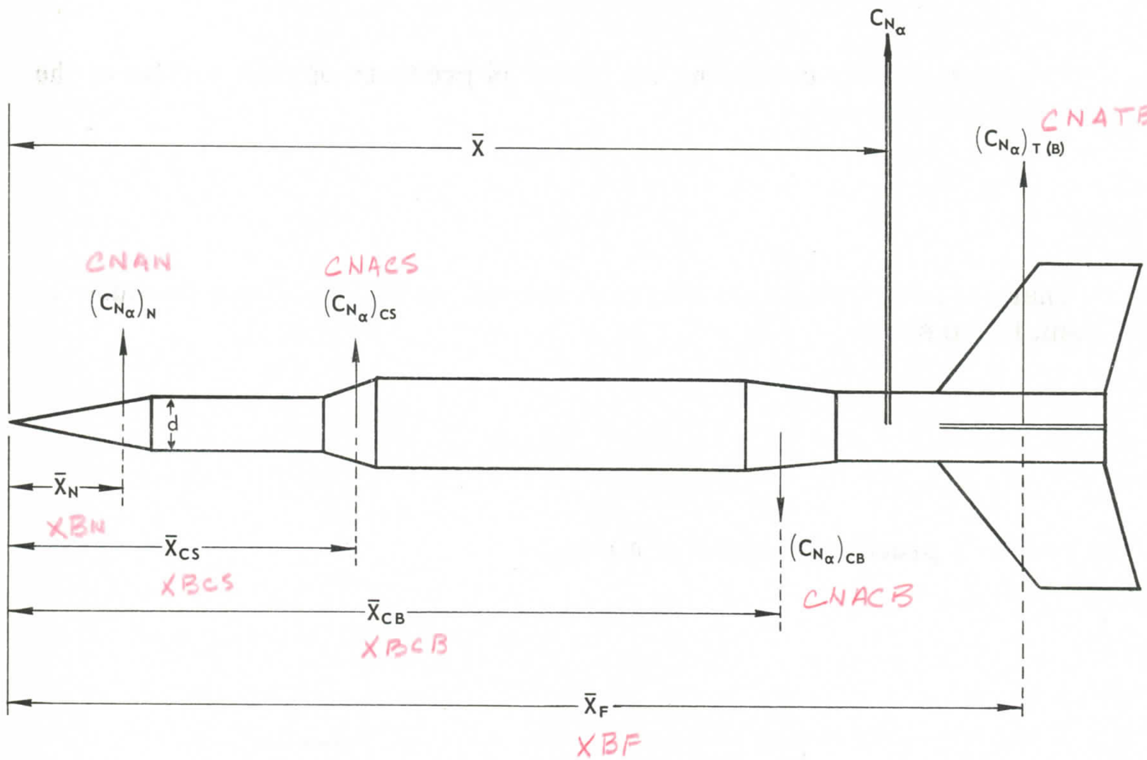


Figure 2a

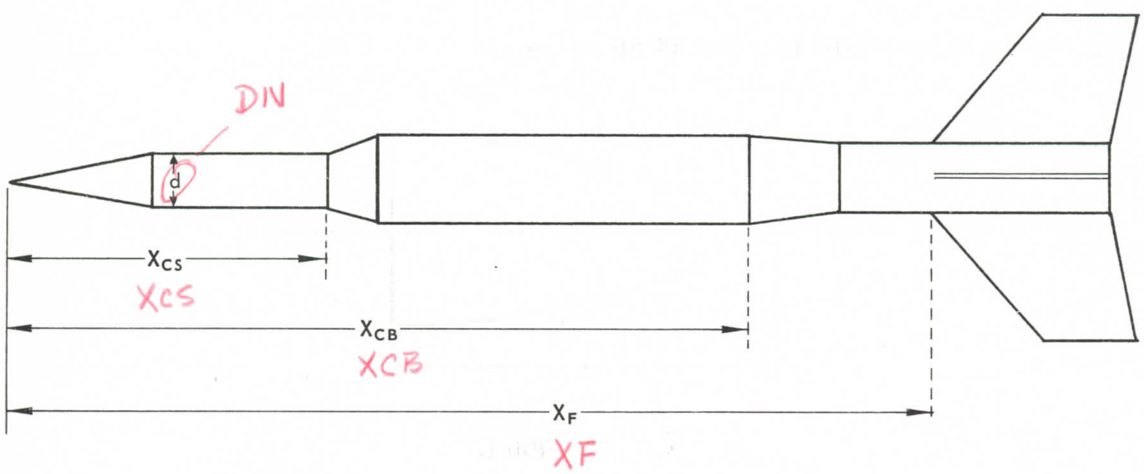


Figure 2b

The equations for computing the center of pressure of each section of the rocket are now presented.

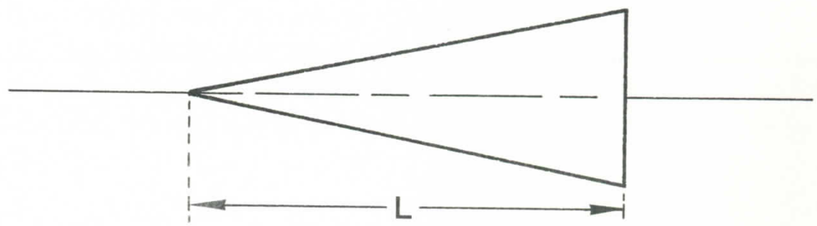
Nose

There are two basic nose shapes, cone and ogive. The force on either one of them is the same:

$$\left(C_{N_\alpha}\right)_N = 2$$

The center of pressure location of a cone,

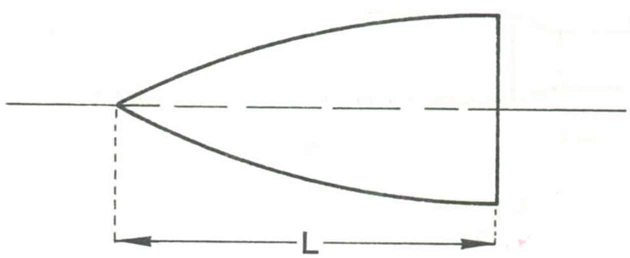
IN=1



LNC

is $\bar{X}_N = \frac{2}{3} L$

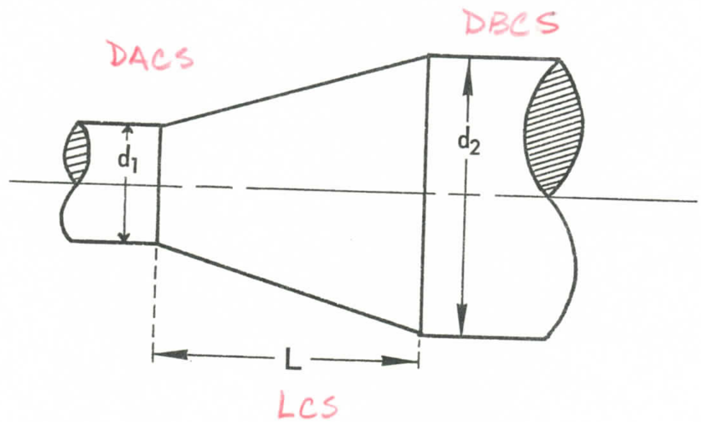
The center of pressure location of an ogive,



is $\bar{X}_N = .466\ L$

LNC

Conical Shoulder



The force on a conical shoulder is,

$$\left(C_{N_\alpha}\right)_{cs} = 2\left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2\right]$$

DN = d

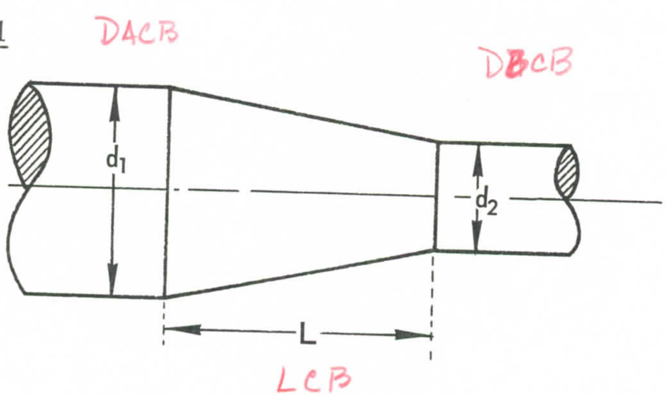
where "d" is the diameter at the base of the nose.

The center of pressure location of a conical shoulder is,

$$\bar{X}_{cs} = X_{cs} + \frac{L}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$

where X_{cs} is the distance from the tip of the nose to the front of the conical shoulder. (See Figure 2b)

Conical Boattail



The force on a conical boattail is,

$$\left(C_{N_\alpha}\right)_{CB} = 2\left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2\right]$$

where "d" is the diameter at the base of the nose. The force on a conical boattail should be negative.

The center of pressure location of a conical boattail is,

$$\bar{X}_{CB} = X_{CB} + \frac{L}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$

where X_{CB} is the distance from the tip of the nose to the front of the conical boattail. (See Figure 2b)

Cylindrical Body

For small angles of attack, the force on any cylindrical body portion is so small it can be neglected.

Fins

Any fin that is not too complicated in shape may be simplified to an idealized shape that has only straight line edges. Such an idealized fin and the dimensions associated with it are shown in Figure 3. Obviously, the force on the fins will depend on the number of fins on the rocket.

In terms of the dimensions, the force acting on the fins of a four finned rocket is:

4x4

$$\left(C_{N_\alpha}\right)_F = \frac{16\left(\frac{s}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

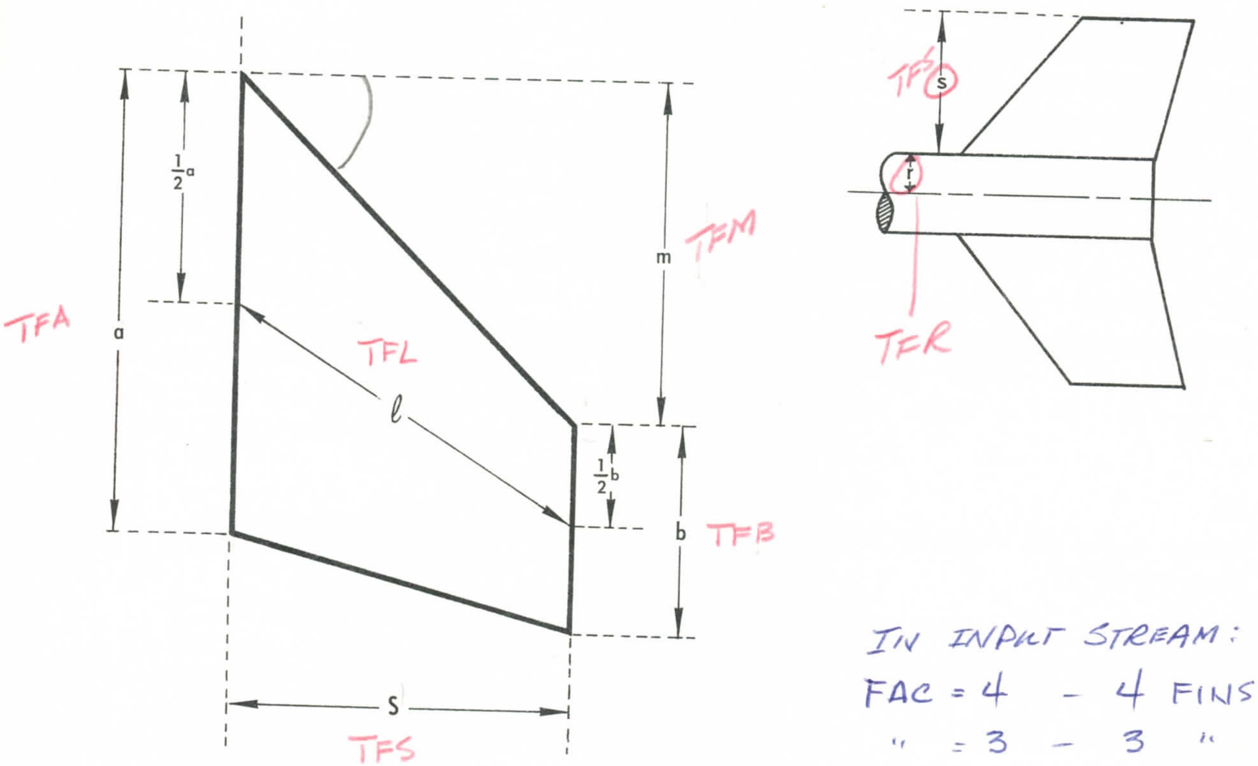


Figure 3

The force acting on the fins of a <u>three finned</u> rocket is:

$$\left(C_{N_\alpha}\right)_F = \frac{12\left(\frac{s}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

To account for the effect of the fins being attached to the body, the fin force is multiplied by an interference factor,

$$K_{T(B)} = 1 + \frac{r}{s + r}$$

where "r" is the radius of the body between the fins and "s" is shown in Figure 3. The total force on the tail in the presence of the body is then:

$$\left(C_{N_\alpha}\right)_{T(B)} = K_{T(B)} \left(C_{N_\alpha}\right)_F$$

The center of pressure location of the tail does not depend on the number of fins.

$$\bar{X}_F = X_F + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

where X_F is the distance from the nose tip to the front edge of the fin root. (See Figure 2b)

Combination Calculations

The total force on the entire rocket is the sum of all the forces on the separate regions, therefore:

$$C_{N_\alpha} = \left(C_{N_\alpha}\right)_N + \left(C_{N_\alpha}\right)_{CS} + \left(C_{N_\alpha}\right)_{CB} + \left(C_{N_\alpha}\right)_{T(B)}$$

The center of pressure of the entire rocket is found by taking a moment balance about the nose tip and solving for the total center of pressure location.

$$\bar{X} = \frac{\left(C_{N_\alpha}\right)_N \bar{X}_N + \left(C_{N_\alpha}\right)_{CS} \bar{X}_{CS} + \left(C_{N_\alpha}\right)_{CB} \bar{X}_{CB} + \left(C_{N_\alpha}\right)_{T(B)} \bar{X}_F}{C_{N_\alpha}}$$

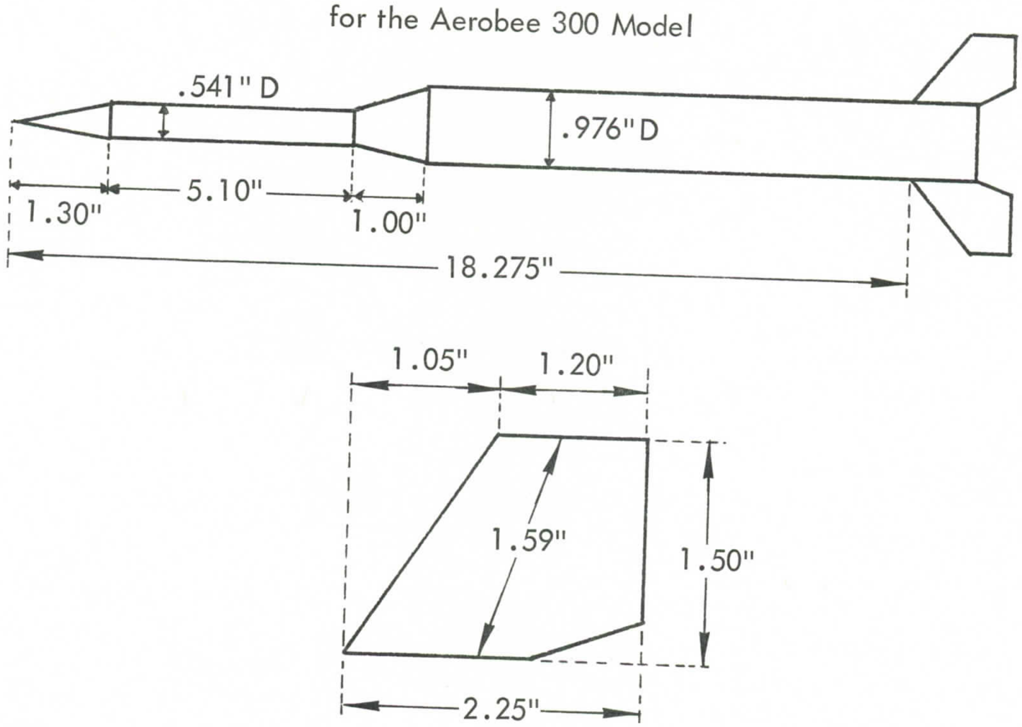
Using the Equations

There are two uses for the center of pressure equations. They can be used - to either analyse an existing rocket or design a new rocket. To illustrate the analysis of an existing rocket, a sample calculation for a model of the Aerobee 300 follows this discussion. Designing a new rocket is basically a problem of designing the fins. First, determine an initial rocket design which fits such requirements as the desired body tube size, payload compartment, nose cone shape, and any other special features desired. Second, calculate the center of gravity of the design using a standard technique. Third, calculate the center of pressure of the design. Fourth, compare the result with the center of gravity and see if the desired static margin has been obtained. If the desired static margin has not been obtained, then alter the fin configuration and re-analyse the rocket. Check the static margin again. Keep changing the fins until the desired static margin is obtained. The changes that should be made each time will be indicated by the previous result. This is essentially a method of trial and error. The more experience you have doing it, the better and faster you'll become. There are no hard and fast rules for designing anything. You must use your own judgement. The center of pressure equations are just a tool to help you make judgements in design.

NOTE OF CAUTION

THE EQUATIONS IN THIS REPORT ARE GOOD ONLY IF THE ROCKET FLIES AT A SMALL ANGLE OF ATTACK. BE SURE THAT THE STATIC MARGIN IS AT LEAST ONE MAXIMUM BODY DIAMETER LONG TO INSURE SMALL ANGLES OF ATTACK.

Center of Pressure Calculation
for the Aerobee 300 Model



Nose

Shape-Cone

$$\left(C_{N_\alpha}\right)_N = 2$$

$$\overline{X}_N = \frac{2}{3}(1.30) = 2(0.433)$$

$$\overline{X}_N = 0.866 \text{ in.}$$

Conical Shoulder

$$\left(C_{N_\alpha}\right)_{CS} = 2\left[\left(\frac{.976}{.541}\right)^2 - \left(\frac{.541}{.541}\right)^2\right]$$

$$= 2\left[(1.085)^2 - 1\right] = 2(3.26 - 1)$$

$$\left(C_{N_\alpha}\right)_{CS} = 4.52$$

$$\overline{X}_{CS} = 6.40 + \frac{1}{3}\left[1 + \frac{1 - \frac{.541}{.976}}{1 - \left(\frac{.541}{.976}\right)^2}\right]$$

$$= 6.40 + \frac{1}{3}\left[1 + \frac{1 - .554}{1 - .306}\right]$$

$$= 6.40 + \frac{1}{3}\left[1 + \frac{.446}{.694}\right]$$

$$= 6.40 + \frac{1}{3}\left[1 + .643\right]$$

$$= 6.40 + \frac{1}{3}(1.643)$$

$$= 6.40 + .548$$

$$\underline{\bar{X}_{cs} = 6.95 \text{ in.}}$$

Fins

Three Fins

$$\left(C_{N_\alpha}\right)_F = \frac{12\left(\frac{1.50}{.541}\right)^2}{1 + \sqrt{1 + \left(\frac{2 \times 1.59}{2.25 + 1.2}\right)^2}}$$

$$= \frac{12(2.77)^2}{1 + \sqrt{1 + \left(\frac{3.18}{3.45}\right)^2}}$$

$$= \frac{12(7.69)}{1 + \sqrt{1 + (.922)^2}} = \frac{92.1}{1 + \sqrt{1 + .851}}$$

$$= \frac{92.1}{1 + \sqrt{1.851}} = \frac{92.1}{1 + 1.361} = \frac{92.1}{2.361}$$

$$\underline{\left(C_{N_\alpha}\right)_F = 39.1}$$

$$K_{T(B)} = 1 + \frac{.488}{.488 + 1.5} = 1 + \frac{.488}{1.988}$$

$$= 1 + .245$$

$$K_{T(B)} = 1.245$$

$$C_{N_{\alpha}\ T(B)} = 1.245(39.1)$$

$$\underline{C_{N_{\alpha}\ T(B)} = 48.6}$$

$$\bar{X}_F = 18.275 + \frac{1.05}{3}\ \frac{(2.25 + 2.40)}{(2.25 + 1.20)} + \frac{1}{6}\left[2.25 + 1.20 - \frac{2.25(1.20)}{2.25 + 1.20}\right]$$

$$= 18.275 + .350\left(\frac{4.65}{3.45}\right) + \frac{1}{6}\left[3.45 - \frac{2.70}{3.45}\right]$$

$$= 18.275 + .350(1.348) + \frac{1}{6}(3.45 - .782)$$

$$= 18.275 + .473 + \frac{1}{6}(2.668)$$

$$= 18.748 + .445$$

$$\underline{\bar{X}_F = 19.19 \text{ in.}}$$

<u>Total Values</u>

$$C_{N_{\alpha}} = 2 + 4.52 + 48.6$$

$$\underline{\underline{C_{N_{\alpha}} = 55.1}}$$

$$\bar{X} = \frac{2(.866) + 4.52(6.95) + 48.6(19.19)}{55.1}$$

$$= \frac{1.73 + 31.4 + 932}{55.1} = \frac{965}{55.1}$$

$$\boxed{\bar{X} = 17.5 \text{ in.}}$$